2012 – 2013 Log1 Contest Round 1
Theta Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	
2	If $2^{6x+1}=16$, find the value of $\log_2 x$.	
3	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of $f\left(g\left(h\left(j\left(e^{81}\right)\right)\right)\right)$.	
4	Which is larger: $\log_4 130-\log_4 5$ or $\dfrac{\log_4 130}{\log_4 5}$?	
5	A circle whose radius has length $\dfrac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	

	5 points each	
6	Solve for x: $\ln(1-x)-\ln(1+x)=3$	
7	If $f(x)=\log_x(4x)$, find the value of a satisfying $f(a)=9$.	
8	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	
9	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	
10	Find the value of x satisfying $\left(2^{3^4}\right)^2=4^{9^x}$.	

	6 points each	
11	Find the value of x satisfying $x^{x^{x^{\cdots}}}=\log_x(\log_x 2)$.	
12	Find the domain of the function $f(x)=\log_4\left(x^3-2x^2-9\right)$, written in interval notation.	
13	If $(4\log_{17} m)(\log_6 17)=12$, find the value of $\log_{36}\sqrt{m}$.	
14	Find the value of $27^{\log_3 12}$.	
15	Find all real values of x such that $\left(x^2-9x+19\right)^{x^2+2x-24}=1$.	

2012 – 2013 Log1 Contest Round 1
Alpha Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	
2	If $2^{6x+1}=16$, find the value of $\log_2 x$.	
3	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of a that satisfies $f(g(h(j(a))))=0$.	
4	A circle whose radius has length $\frac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	
5	Solve for x: $\ln(1-x)-\ln(1+x)=3$	

	5 points each	
6	Solve for x: $16^{x^2}=2^{3x+1}$	
7	If $f(x)=\log_x(4x)$, find the value of a satisfying $f(a)=9$.	
8	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	
9	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	
10	Find the value of x satisfying $x^{x^{x^{\cdots}}}=\log_x(\log_x 2)$.	

	6 points each	
11	Solve for x: $4^{16^x}=16^{4^x}$	
12	Find the domain of the function $f(x)=\log_4(x^3-2x^2-9)$, written in interval notation.	
13	Solve for positive x: $x^{x^{x+3}}=(x^{81})^{x^2}$	
14	Find the value of $27^{\log_3 12}$.	
15	Find all real values of x such that $(x^2-9x+19)^{x^2+2x-24}=1$.	

2012 – 2013 Log1 Contest Round 1
Mu Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	
2	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of a that satisfies $f\left(g\left(h\left(j(a)\right)\right)\right)=0$.	
3	A circle whose radius has length $\dfrac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	
4	Solve for x: $\ln(1-x)-\ln(1+x)=3$	
5	Find the value of the slope of the tangent to $y=x^x$ at the point $(3,27)$.	

	5 points each	
6	If $f(x)=\log_x(4x)$, where $x>2$, find the range of f, written in interval notation.	
7	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	
8	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	
9	Find the value of x satisfying $x^{x^{x^{x^{\cdots}}}}=\log_x(\log_x 2)$.	
10	Find the average value of the function $f(x)=x^3$ on the interval $[2,4]$.	

	6 points each	
11	The solution to $4^{32^x}=32^{4^x}$ is $x=-1+\log_2 a$, where $a>0$. Find the value of a^3.	
12	Find the domain of the function $f(x)=\log_{x^2-5}\left(x^3+3x^2-4x-12\right)$, written in interval notation.	
13	Solve for positive x: $x^{x^{x+3}}=\left(x^{81}\right)^{x^2}$	
14	Find the value of a if $\int_1^2(\log_2 x)dx=\log_2 a$.	
15	Find all real values of x such that $\left(x^2-9x+19\right)^{x^2+2x-24}=1$.	

2012 – 2013 Log1 Contest Round 1
Theta Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	24
2	If $2^{6x+1}=16$, find the value of $\log_2 x$.	-1
3	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of $f\left(g\left(h\left(j\left(e^{81}\right)\right)\right)\right)$.	0
4	Which is larger: $\log_4 130-\log_4 5$ or $\dfrac{\log_4 130}{\log_4 5}$?	$\dfrac{\log_4 130}{\log_4 5}$
5	A circle whose radius has length $\dfrac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	π^3

	5 points each	
6	Solve for x: $\ln(1-x)-\ln(1+x)=3$	$\dfrac{1-e^3}{1+e^3}$
7	If $f(x)=\log_x(4x)$, find the value of a satisfying $f(a)=9$.	$\sqrt[4]{2}$
8	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	0.71534
9	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	Napier
10	Find the value of x satisfying $\left(2^{3^4}\right)^2=4^{9^x}$.	2

	6 points each	
11	Find the value of x satisfying $x^{x^{x^{x^{\cdots}}}}=\log_x(\log_x 2)$.	$\sqrt{2}$
12	Find the domain of the function $f(x)=\log_4\left(x^3-2x^2-9\right)$, written in interval notation.	$(3,\infty)$
13	If $(4\log_{17} m)(\log_6 17)=12$, find the value of $\log_{36}\sqrt{m}$.	$\dfrac{3}{4}$
14	Find the value of $27^{\log_3 12}$.	1728
15	Find all real values of x such that $\left(x^2-9x+19\right)^{x^2+2x-24}=1$.	3, 4, ± 6

2012 – 2013 Log1 Contest Round 1
Alpha Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	24
2	If $2^{6x+1}=16$, find the value of $\log_2 x$.	-1
3	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of a that satisfies $f\left(g\left(h\left(j(a)\right)\right)\right)=0$.	e^{81}
4	A circle whose radius has length $\dfrac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	π^3
5	Solve for x: $\ln(1-x)-\ln(1+x)=3$	$\dfrac{1-e^3}{1+e^3}$

	5 points each	
6	Solve for x: $16^{x^2}=2^{3x+1}$	$1, -\dfrac{1}{4}$
7	If $f(x)=\log_x(4x)$, find the value of a satisfying $f(a)=9$.	$\sqrt[4]{2}$
8	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	0.71534
9	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	Napier
10	Find the value of x satisfying $x^{x^{x^{x^{\cdots}}}}=\log_x(\log_x 2)$.	$\sqrt{2}$

	6 points each	
11	Solve for x: $4^{16^x}=16^{4^x}$	$\dfrac{1}{2}$
12	Find the domain of the function $f(x)=\log_4\left(x^3-2x^2-9\right)$, written in interval notation.	$(3,\infty)$
13	Solve for all positive values of x: $x^{x^{x+3}}=\left(x^{81}\right)^{x^2}$	1, 3
14	Find the value of $27^{\log_3 12}$.	1728
15	Find all real values of x such that $\left(x^2-9x+19\right)^{x^2+2x-24}=1$.	3, 4, ± 6

2012 – 2013 Log1 Contest Round 1
Mu Logs & Exponents

Name: ________________

	4 points each	
1	Evaluate: $(\log_7 4)(\log_2 125)(\log_5 2401)$	24
2	If $f(x)=\log_5 x$, $g(x)=\log_4 x$, $h(x)=\log_3 x$, and $j(x)=\ln x$, find the value of a that satisfies $f\left(g\left(h\left(j(a)\right)\right)\right)=0$.	e^{81}
3	A circle whose radius has length $\dfrac{\log_3 27^{\pi}}{\log_9 27^2}$ encloses what area?	π^3
4	Solve for x: $\ln(1-x)-\ln(1+x)=3$	$\dfrac{1-e^3}{1+e^3}$
5	Find the value of the slope of the tangent to $y=x^x$ at the point $(3,27)$.	$27+27\ln 3$

	5 points each	
6	If $f(x)=\log_x(4x)$, where $x>2$, find the range of f, written in interval notation.	$(1,3)$
7	Using $\log 5=0.69897$, find the value of $\log_{25} 10$, rounded accurately to five decimal places.	0.71534
8	What is the last name of the mathematician to whom the introduction of logarithms in the early 17th century is generally attributed?	Napier
9	Find the value of x satisfying $x^{x^{x^{x^{\cdots}}}}=\log_x(\log_x 2)$.	$\sqrt{2}$
10	Find the average value of the function $f(x)=x^3$ on the interval $[2,4]$.	30

	6 points each	
11	The solution to $4^{32^x}=32^{4^x}$ is $x=-1+\log_2 a$, where $a>0$. Find the value of a^3.	20
12	Find the domain of the function $f(x)=\log_{x^2-5}\left(x^3+3x^2-4x-12\right)$, written in interval notation.	$\left(-3,-\sqrt{6}\right)\cup\left(-\sqrt{6},-\sqrt{5}\right)\cup\left(\sqrt{5},\sqrt{6}\right)\cup\left(\sqrt{6},\infty\right)$
13	Solve for all positive values of x: $x^{x^{x+3}}=\left(x^{81}\right)^{x^2}$	1, 3
14	Find the value of a if $\int_1^2(\log_2 x)dx=\log_2 a$.	$\dfrac{4}{e}$
15	Find all real values of x such that $\left(x^2-9x+19\right)^{x^2+2x-24}=1$.	3, 4, ± 6

2012 – 2013 Log1 Contest Round 1
Logs & Exponents Solutions

Mu	Al	Th	Solution
1	1	1	$(\log_7 4)(\log_2 125)(\log_5 2401)=(\log_2 4)(\log_5 125)(\log_7 2401)=2\cdot 3\cdot 4=24$
	2	2	$6x+1=4\Rightarrow x=\frac{1}{2}\Rightarrow \log_2\frac{1}{2}=-1$
2	3		$f\left(g\left(h\left(j(a)\right)\right)\right)=0\Rightarrow g\left(h\left(j(a)\right)\right)=1\Rightarrow h\left(j(a)\right)=4\Rightarrow j(a)=81\Rightarrow a=e^{81}$
		3	$f\left(g\left(h\left(j\left(e^{81}\right)\right)\right)\right)=f\left(g\left(h(81)\right)\right)=f\left(g(4)\right)=f(1)=0$
		4	$\log_4 130-\log_4 5=\log_4 26<\log_4 32=2.5$ and $\frac{\log_4 130}{\log_4 5}=\log_5 130>\log_5 125=3$, so $\frac{\log_4 130}{\log_4 5}$ is larger.
3	4	5	$\frac{\log_3 27^{\pi}}{\log_9 27^2}=\frac{\pi\log_3 27}{2\log_9 27}=\frac{3\pi}{2(1.5)}=\pi$, so the enclosed area is $\pi(\pi)^2=\pi^3$
4	5	6	$3=\ln(1-x)-\ln(1+x)=\ln\frac{1-x}{1+x}\Rightarrow e^3=\frac{1-x}{1+x}\Rightarrow x\left(1+e^3\right)=1-e^3\Rightarrow x=\frac{1-e^3}{1+e^3}$
5			For $y=x^x$, using logarithmic differentiation, $y'=x^x(1+\ln x)$, so the slope of the tangent at that point is $3^3(1+\ln 3)=27+27\ln 3$.
	6		Since $16=2^4$, $4x^2=3x+1\Rightarrow 0=4x^2-3x-1=(4x+1)(x-1)\Rightarrow x=1$ or $-\frac{1}{4}$.
6			$\log_x(4x)=\frac{\ln 4x}{\ln x}=\frac{\ln 4+\ln x}{\ln x}=\frac{\ln 4}{\ln x}+1=\frac{1}{\log_4 x}+1$, and since $\log_4 x$ takes on all values $\left(\frac{1}{2},\infty\right)$ when $x>2$, $\frac{1}{\log_4 x}+1$ takes on all values $\left(0+1,\frac{1}{1/2}+1\right)=(1,3)$.
	7	7	$9=\log_x(4x)=\frac{\ln 4x}{\ln x}=\frac{\ln 4+\ln x}{\ln x}=\frac{\ln 4}{\ln x}+1=\frac{1}{\log_4 x}+1\Rightarrow \log_4 x=\frac{1}{8}\Rightarrow x=\sqrt[8]{4}=\sqrt[4]{2}$
7	8	8	$\log_{25}10=\frac{1}{\log 25}=\frac{1}{2\log 5}=\frac{1}{2(0.69897)}=\frac{1}{1.39794}=0.715338...$, so rounded to five decimal places, the value would be 0.71534.
8	9	9	John Napier is generally credited with the introduction of logarithms in the early 1600s.
		10	$2^{2\left(9^x\right)}=\left(2^2\right)^{9^x}=4^{9^x}=\left(2^{3^4}\right)^2=\left(2^2\right)^{3^4}=2^{2\left(3^4\right)}\Rightarrow 9^x=3^4=81\Rightarrow x=2$

9	10	11	Using the definition of logs, $x^{\left(x^{x^{x^{\cdots}}}\right)} = \log_x 2 \Rightarrow 2 = x^{\left(x^{\left(x^{x^{x^{\cdots}}}\right)}\right)} = x^{x^{x^{\cdots}}} = x^2 \Rightarrow x = \sqrt{2}$ since $x > 0$
10			The average value is $\frac{1}{4-2}\int_2^4 x^3 dx = \frac{1}{2}\left(\frac{1}{4}x^4\right)\Big\vert_2^4 = \frac{1}{8}\left(4^4 - 2^4\right) = \frac{1}{8}(256-16) = \frac{1}{8}(240) = 30$.
	11		$4^{\left(4^x\right)^2} = 4^{16^x} = 16^{4^x} = \left(4^2\right)^{4^x} = 4^{2\left(4^x\right)} \Rightarrow \left(4^x\right)^2 = 2\left(4^x\right)$, and since $4^x \neq 0$, $4^x = 2 \Rightarrow x = \frac{1}{2}$.
11			$2^{2\left(2^x\right)^5} = \left(2^2\right)^{\left(2^x\right)^5} = 4^{32^x} = 32^{4^x} = \left(2^5\right)^{\left(2^x\right)^2} = 2^{5\left(2^x\right)^2} \Rightarrow 2\left(2^x\right)^5 = 5\left(2^x\right)^2$, and since $2^x \neq 0$, $\left(2^x\right)^3 = \frac{5}{2} \Rightarrow 2^x = \sqrt[3]{\frac{5}{2}} = \frac{\sqrt[3]{20}}{2} \Rightarrow x = \log_2 \frac{\sqrt[3]{20}}{2} = \log_2 \sqrt[3]{20} - \log_2 2 = -1 + \log_2 \sqrt[3]{20}$, so $a = \sqrt[3]{20} \Rightarrow a^3 = 20$.
	12	12	$\log_4\left(x^3 - 2x^2 - 9\right) = \log_4\left(\left(x-3\right)\left(x^2+x+3\right)\right)$, and since $x^2 + x + 3 > 0$ for all real x, we simply need $x - 3 > 0 \Rightarrow x > 3$, so the answer in interval notation is $(3, \infty)$.
12			For the base, we must have $x^2 - 5 > 0$ and $x^2 - 5 \neq 1$, so x is in $\left(-\infty, -\sqrt{6}\right) \cup \left(-\sqrt{6}, -\sqrt{5}\right) \cup \left(\sqrt{5}, \sqrt{6}\right) \cup \left(\sqrt{6}, \infty\right)$. For the argument of the log, $x^3 + 3x^2 - 4x - 12 = (x+3)(x-2)(x+2) > 0$, so x is in $(-3,-2) \cup (2, \infty)$. The intersection of these intervals is $\left(-3, -\sqrt{6}\right) \cup \left(-\sqrt{6}, -\sqrt{5}\right) \cup \left(\sqrt{5}, \sqrt{6}\right) \cup \left(\sqrt{6}, \infty\right)$.
		13	$12 = \left(4\log_{17} m\right)\left(\log_6 17\right) = 4\log_6 m \Rightarrow \log_6 m = 3 \Rightarrow m = 6^3 = 216$, $\log_{36}\sqrt{216} = \log_{6^2} 6^{\frac{3}{2}} = \log_6 6^{\frac{3}{4}} = \frac{3}{4}$
13	13		Since $x \neq 0$, $x^{x^{x+3}} = \left(x^{81}\right)^{x^2} = x^{81x^2} \Rightarrow 81x^2 = x^{x+3} = x^x x^3 \Rightarrow 81 = x^x x = x^{x+1}$, and by inspection, $x = 3$. This works by taking $\log_x$ of both sides, which you can only do if $x \neq 1$. By checking $x = 1$ separately, it is easy to verify that that is also a solution.
	14	14	$27^{\log_3 12} = 12^{\log_3 27} = 12^3 = 1728$
14			$\int_1^2 \left(\log_2 x\right) dx = \frac{1}{\ln 2}\int_1^2 (\ln x) dx = \frac{1}{\ln 2}\left(x \ln x - x\right)\Big\vert_1^2 = \frac{(2\ln 2 - 2) - (1\ln 1 - 1)}{\ln 2} = \frac{2\ln 2 - 1}{\ln 2} = \frac{\ln 4 - \ln e}{\ln 2} = \frac{\ln\frac{4}{e}}{\ln 2} = \log_2 \frac{4}{e}$, so $a = \frac{4}{e}$

15	15	15	$\left(x^2-9x+19\right)^{x^2+2x-24}=1$ in three ways: the exponent is 0 and the base isn't, the base is 1, or the base is -1 and the exponent is even. For the first case, $0=x^2+2x-24$ $=(x+6)(x-4)\Rightarrow x=-6$ or $x=4$, neither of which make the base 0. For the second case, $1=x^2-9x+19\Rightarrow 0=x^2-9x+18=(x-3)(x-6)\Rightarrow x=3$ or $x=6$. For the third case, the exponent is even only if x is, and $-1=x^2-9x+19\Rightarrow 0=x^2-9x+20$ $=(x-4)(x-5)\Rightarrow x=4$ or $x=5$, so only $x=4$ works in this case. Therefore, the solutions are $3,\ 4,\ 6,$ and -6.