Name ______________________________________

Rocket City Math League
Senior Division

2016-2017
Inter-School Solutions

Answers must be written inside the corresponding box on the answer sheet. All answers must be written in exact, reduced, simplified, and rationalized form. Figures are not necessarily drawn to scale. **No calculators, books, or other aides may be used.**

1. The area of the original launch pad is 25. The area of a circle is $A = \pi r^2$. So, $25 = \pi r^2$, which gives $r = \frac{\mathbf{5}}{\sqrt{\pi}} = \frac{\mathbf{5}\sqrt{\pi}}{\pi}$. The diameter is twice the radius, so the diameter is $\frac{\mathbf{10}\sqrt{\pi}}{\pi}$.

2. $x = \sin^{-1} 0.28 = \sin^{-1} \frac{7}{25}$. This is a 7, 24, 25 right triangle. The side opposite of x has a length of 7, so $\tan x = \frac{7}{24}$.

3. $_5P_4 = \mathbf{5 \bullet 4 \bullet 3 \bullet 2 = 120}$

4. John's average net gain per game is $\frac{1}{4}(30 - 50 + 20 + 10) - 10 = -7.5$. After forty games, his net gain would be $40(-7.5) = -300$, or $-\$300$.

5. Jack can fill $\frac{3}{5}\%$ of the tank per minute. Jill can fill $\frac{3}{2}\%$ of the tank per minute. Combined, they can fill $\frac{21}{10}\%$ of the tank per minute. Dividing 100 by $\frac{21}{10}$ gives us about 47.6 minutes, which rounds to 48 minutes.

6. The factor pairs of 24 are $(1,24),(2,12),(3,8),(4,6)$. Given that A is odd and not 1, then $A = 3$ and $B = 8$. Plugging in for A and B gives us, $3\begin{bmatrix} 2 & 9 \\ 8 & 6 \end{bmatrix} = 8\begin{bmatrix} w & x \\ y & z \end{bmatrix}$, so $\begin{bmatrix} 6 & 27 \\ 24 & 18 \end{bmatrix} = \begin{bmatrix} 8w & 8x \\ 8y & 8z \end{bmatrix}$. Solving each element individually shows that $w = \frac{3}{4}$, $x = \frac{27}{8}$, $y = 3$, and $z = \frac{9}{4}$. The sum is $\frac{3}{4} + \frac{27}{8} + 3 + \frac{9}{4} = \frac{6}{8} + \frac{27}{8} + \frac{24}{8} + \frac{18}{8} = \frac{75}{8}$, which is $9\frac{3}{8}$ or 9.375.

7. Use long division: $3x^2 - 3x - 8\overline{\smash{)}12x^4 - 9x^3 - 26x^2 - 17x - 24}$. The answer is $4x^2 + x + 3$.

8. Using logarithm rules, we have:

$$(6\log 3)\left(4\log_9 2 - 2\log_9 \sqrt{6}\right) - 3\left(4\log 2 - 2\log 2\sqrt{15}\right)$$
$$= \log\left(3^6\right)\left[\log_9\left(2^4\right) - \log_9\left(\sqrt{6}^2\right)\right] - 3\left[\log\left(2^4\right) - \log\left(\left(2\sqrt{15}\right)^2\right)\right]$$
$$= \log 729(\log_9 16 - \log_9 6] - 3[\log 16 - \log 60]$$
$$= \log 729\left(\log_9 \frac{8}{3}\right) - 3\log\frac{4}{15}$$
$$= \log 729\left(\frac{\log\frac{8}{3}}{\log 9}\right) - 3\log\frac{4}{15}$$

$$= 3\log\frac{8}{3} - 3\log\frac{4}{15} = 3\log\left(\frac{\frac{8}{3}}{\frac{4}{15}}\right) = 3\log 10 = 3.$$

9. Let $c =$ total cost. We have $\frac{c}{12} = \frac{c}{13} + 3$.

Multiplying both sides by 13, we get $\frac{13c}{12} = c + 39$.

So, $\frac{c}{12} = 39$; $c = 468$ or $\$468$.

10. Use the formula $\genfrac{}{}{0pt}{}{n}{r} x^{n-r} y^{r}$, where n is the degree and r is the term minus one, to find the answer. Plugging n and r into the equation gives us $\frac{12 \cdot 11 \cdot 10 \cdot 9!}{3!9!}(3a)^{12-9} b^{9}$. This simplifies to $220 \cdot 27a^{3}b^{9}$, or $5940a^{3}b^{9}$.

11. The function $y = -10x^2 + 35x + 6$ describes a parabola, so the vertex is the only point on the graph that has a horizontal tangent line. We can find the vertex by converting the quadratic equation into vertex form.

Factor out -10 : $-10\left(x^2 - 3.5x\right) + 6$

Complete the square: $-10\left(x^2 - 3.5x + 3.0625\right) + 6 + 30.625$

Simplify: $-10(x - 1.75)^2 + 36.625$

Therefore, the vertex is $(1.75, 36.625)$, which can also be written as $\left(\frac{7}{4}, \frac{293}{8}\right)$ or $\left(1\frac{3}{4}, 36\frac{5}{8}\right)$.

12. The polynomial $x^4 + 8x^3 + x^2 + 78x - 72 = 0$ can be factored into $(x+3)(x-1)(x-4)(x-6) = 0$. Since $a < b < c < d$, then $a = -3, b = 1, c = 4, d = 6$. Plugging these values into the equation gives us $4(-3)^2 - (6^2) + 3(1)$ which evaluates to 3.

13. Answer: **2352**

Solution: The number of roses after each round of picking forms a geometric sequence. The number of petals and leaves on 1 rose both form an arithmetic sequence.

Round	Number of roses	Number of leaves on 1 rose	Number of petals on 1 head
1	1	2	177
2	2	3	171
3	4	4	165
4	8	5	159
5	16	6	153

The total number of leaves after **15** roses are picked is found by multiplying the number of roses (**16**) by the number of leaves on one rose (**6**).

$\mathbf{16 \bullet 6 = 96}$

The total number of petals after **15** roses are picked is found by multiplying the number of roses (**16**) by the number of petals on one rose (**153**).

$\mathbf{16 \bullet 153 = 2448}$

The difference between petals to leaves is $\mathbf{2448 - 96 = 2352}$.

14. Let r_A equal the speed at which Rocket A travels (in feet per minute). Let r_B equal the speed at which Rocket B travels (in

feet per minute). Therefore, $r_B = \frac{5}{4} r_A$. After 70 minutes, Rocket A travels $70r_A + 1450$ feet, while Rocket B travels $70r_B = \frac{350}{4} r_A$ feet. Since Rocket A and Rocket B reach the same elevation after 70 minutes, $70r_A + 1450 = \frac{350}{4} r_A$. Solving for r_A gives $r_A = \frac{580}{7}$. Substituting this into $r_B = \frac{5}{4} r_A$ gives $r_B = \frac{725}{7}$. The time it takes for Rocket A to reach 10,000 feet is $\frac{10{,}000 - 1{,}450}{580/7} = 8{,}550 \cdot \frac{7}{580} = \frac{5985}{58}$ minutes, while the time it takes for Rocket B to reach 10,000 feet is $\frac{10{,}000}{725/7} = \frac{70{,}000}{725} = \frac{5{,}600}{58}$ minutes. The difference between these times is $\frac{385}{58}$, or $6\frac{37}{58}$, minutes.

15. A number is divisible by 9 if the sum of the digits is divisible by 9.
Joan must use one of the digits twice to form a 4-digit number. If Joan uses the digit 6 twice, then the remaining two digits must add up to 6 or 15. This gives us six possibilities: (1, 5), (2, 4), (4, 2), (5, 1) for 6, and (7, 8), (8, 7) for 15.
If Joan uses the digit 6 once, then twice one digit plus the other digit must add up to 3, 12, or 21. This gives us eight possibilities: none for 3; (2, 8), (2, 5), (5, 2), (8, 2) for 12; and (3, 9), (5, 8), (8, 5), (9, 3) for 21.

There are $8 \cdot 7 = 56$ possibilities total, so the probability is $\frac{14}{56} = \frac{1}{4}$ or 0.25.

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