Name _____

Rocket City Math League

Senior Division

2016-2017
Inter-School Test

Answers must be written inside the corresponding box on the answer sheet. All answers must be written in exact, reduced, simplified, and rationalized form. Figures are not necessarily drawn to scale. **No calculators, books, or other aides may be used.**

Time limit: 30 minutes

1. Intergalactic Inc. wishes to change the shape of a launch pad; however, they would like to keep the area of the launch pad the same. The original launch pad is square; they want to create a new circular launch pad. The original pad has a side length of 5 meters. What would be the diameter of the new launch pad? *(1 point)*

2. Find $\tan x$ if $x = \sin^{-1} 0.28$ and $0 \le x < \frac{\pi}{2}$. *(1 point)*

3. The launch codes for a rocket are comprised of four characters, and the order of the characters is important. The characters are chosen from five symbols and each symbol is used only once. How many possible launch codes are there? (*1 point*)

4. John is playing a spinner game. The spinner is colored in equal parts red, blue, yellow, and green. To play the spinner game, John must pay $10. If the spinner lands on red, John wins $30. If the spinner lands on blue, John loses $50. If the spinner lands on yellow, John wins $10. If the spinner lands on green, John wins $20. What is John's expected gain after playing this game 40 times? *(1 point)*

5. Jack and Jill are fetching pails of water to fill a tank. Jack can fill 3% of the tank in 5 minutes. Jill can fill 3% of the tank in 2 minutes. How long will it take them to fill the tank, rounded to the nearest minute? *(1 point)*

6. If $A\begin{bmatrix} 2 & 9 \\ 8 & 6 \end{bmatrix} = B\begin{bmatrix} w & x \\ y & z \end{bmatrix}$, where A and B are scalars and $AB = 24$, $A \neq 1$, and A is odd and B is even, find the sum of $w, x, y,$ and z. *(2 points)*

7. Divide $12x^4 - 9x^3 - 26x^2 - 17x - 24$ by $3x^2 - 3x - 8$. *(2 points)*

8. Evaluate: $(6\log 3)(4\log_9 2 - 2\log_9 \sqrt{6}) - 3(4\log 2 - 2\log 2\sqrt{15})$. *(2 points)*

9. 12 people are splitting the cost of a meal. If the meal was instead split among 13 people, each person would pay $3 less. What is the total cost of the meal? *(2 points)*

10. Find the tenth term in the expansion of $(3a + b)^{12}$ when written with like terms combined and in descending order of exponents of a. *(3 points)*

11. The path of a particle can be modeled in two-dimensional space by the function $y = -10x^2 + 35x + 6$. Find all the points at which the path has a horizontal tangent line. Give your answer in (x, y) form. *(3 points)*

12. The polynomial $x^4 - 8x^3 + x^2 + 78x - 72 = 0$ has roots a, b, c, d, where $a < b < c < d$. Evaluate $a^2c - d^2 + 3b$. *(3 points)*

13. Tim the alien is picking space roses, a space flower from a bush that instantaneously regrows two roses every time one rose is picked. The rose bush begins with one rose that has two leaves and 177 petals. Each time the rose bush regrows its roses, each regenerated rose has 1 more leaf and 6 less petals than the previous rose it replaces. Tim picks the initial rose and every following regrown rose. If he stops picking roses after he has picked a total of 15 roses and the rose bush regrows its roses once more, how many fewer leaves than petals does the rose bush have in total on its remaining roses? *(4 points)*

14. Interstellar Agencies has developed two new rockets and plans to test them. Rocket A is to be launched vertically from Site A, which is 1450 feet above sea level. Rocket B is to be launched vertically from Site B, which is at sea level. Rocket B's speed is 1.25 times Rocket A's speed. If both rockets are launched at the same time, it will take 70 minutes for the rockets to reach the same elevation from sea level. What is the difference between the times, in minutes, that it takes each rocket to reach an elevation of 10,000 feet? *(4 points)*

15. Joan wants to make a 4-digit number that is divisible by 9. She randomly draws 3 cards, without replacement, from a box containing 9 cards with each of the non-zero digits written exactly once. She then forms the 4-digit number, using each digit at least once. If the first number she draws is a 6, what is the probability that Joan is able to form a 4-digit number that is divisible by 9? *(5 points)*