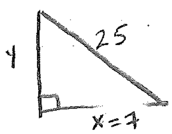
4.6 – 8.1 Review Questions

These are questions to prepare you for related rates and L'Hopital's Rule.

1. A 25 foot ladder is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. $\frac{dx}{dt}=2$
   a. How fast is the top of the ladder moving down the wall when its base is 7 feet from the wall?



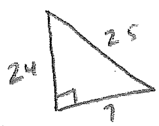
$x^2+y^2=c^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2c\frac{dc}{dt}$

$7(2)+24\frac{dy}{dt}=0$

$\frac{dy}{dt}=\frac{-14}{24}$ ft/sec. $=-\frac{7}{12}$ ft/sec.

   b. Consider the triangle formed by the side of the house, the ladder and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.



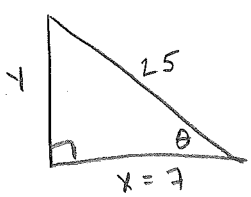
$A=\frac{1}{2}xy$

$2(7)\cdot 2+2\cdot 24\frac{dy}{dt}=0$ $\quad \left.\frac{dy}{dt}\right|_{x=7}=\frac{-28}{48}=-\frac{7}{12}$

$\frac{dA}{dt}=\frac{1}{2}x\frac{dy}{dt}+\frac{1}{2}y\frac{dx}{dt}$

$=\frac{1}{2}(7)\cdot-\frac{7}{12}+\frac{1}{2}(24)(2)=\frac{-49}{24}+24=\frac{-49+576}{24}=\frac{527}{24}$ ft²/sec.

   c. Find the rate at which the angle between the ladder and wall of house is changing when the base of the ladder is 7 feet from the wall.



$\sin\theta=\frac{y}{25}$ $\quad \cos\theta=\frac{7}{25}$

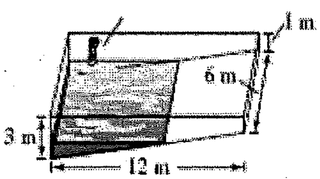
$\cos\theta\frac{d\theta}{dt}=\frac{1}{25}\frac{dy}{dt}$

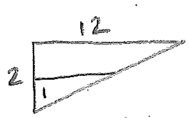
$\frac{7}{25}\cdot\frac{d\theta}{dt}=\frac{1}{25}\cdot\frac{-7}{12}$

$\frac{d\theta}{dt}=\frac{1}{\cancel{25}}\cdot\frac{-\cancel{7}}{12}\cdot\frac{\cancel{25}}{\cancel{7}}=-\frac{1}{12}$

$-\frac{1}{12}$ rads/sec.

2. A swimming pool is 12 meters long, 6 meters wide, 1 meter deep at the shallow end and 3 meters deep at the deep end. Water is being pumped into the pool at ¼ cubic meters per minute, and there is 1 meter of water at the deep end. At what rate is the water level rising?





$\frac{dV}{dt}=\frac{1}{4}$ $\quad$ w = constant = 6

$V=\frac{1}{2}hL\cdot w$

$V=3hL$

$V=3h\cdot(6h)=18h^2$

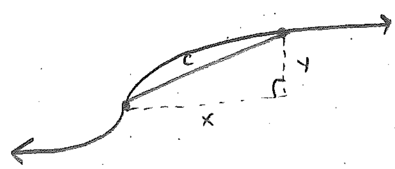
$\frac{h}{L}=\frac{2}{12}$ $\quad L=6h$

$\frac{dV}{dt}=36h\frac{dh}{dt}$

$\frac{1}{4}=36\cdot 1\cdot\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{1}{144}$ m/min.

3. A particle is moving the graph of $y=\sqrt[3]{x}$. When $x=8$, the y component of the position of the particle is increasing at the rate of 1 cm per second. $y=\sqrt[3]{x}$
   a. How fast is the x component changing at this moment?



$x=8$ $\quad y=x^{1/3}$

$\frac{dy}{dt}=1$ $\quad \frac{dy}{dt}=\frac{1}{3}x^{-2/3}\cdot\frac{dx}{dt}$

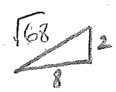
$1=\frac{1}{3}\cdot\frac{1}{8^{2/3}}\frac{dx}{dt}$

$1=\frac{1}{12}\frac{dx}{dt}$ $\quad \frac{dx}{dt}=12$ cm/sec.

   b. How fast is the distance from the origin changing at this moment?

$x^2+y^2=c^2$ $\quad y(8)=2$

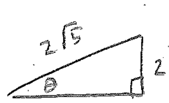
$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}c\frac{dc}{dt}$



$8\cdot 12+2\cdot 1=2\sqrt{68}\frac{dc}{dt}$

$\frac{dc}{dt}=\frac{98}{2\sqrt{68}}=\frac{49}{\sqrt{17}}$ cm/sec.

   c. How fast is the angle of inclination θ changing at this moment?



$\sin\theta=\frac{y}{c}$

$\cos\theta\frac{d\theta}{dt}=\frac{c\frac{dy}{dt}-y\frac{dc}{dt}}{c^2}$

$\frac{8}{2\sqrt{17}}\frac{d\theta}{dt}=\frac{2\sqrt{17}\cdot 1-2\cdot\frac{49}{\sqrt{17}}}{4\cdot 17}$

$\frac{d\theta}{dt}=\frac{2\sqrt{17}\cdot\frac{18}{\sqrt{17}}\cdot\frac{\sqrt{17}}{4}}{20}=\frac{\frac{34}{2}-\frac{98}{2}}{68}=\frac{-44}{40}=\frac{-11}{10}$ rads/sec.

L'Hopital's Rule Practice Problems

1. $\lim_{x\to 3} \frac{2x-6}{x^2-9} = \lim_{x\to 3} \frac{2}{2x} = \frac{1}{3}$

$\frac{0}{0}$

2. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{\frac{1}{2\sqrt{x+1}}}{1} = \frac{1}{4}$

$\frac{0}{0}$

3. $\lim_{x\to \infty} \frac{5x^2-3x+1}{3x^2-5}$

$\frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{10x-3}{9x}$ $\frac{\infty}{\infty}$ $= \lim_{x\to\infty} \frac{10}{9} = \frac{10}{9}$

4. $\lim_{x\to 2} \frac{x^3-x-2}{x-2}$

5. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x} = \lim_{x\to 0} \frac{\frac{-2x}{2\sqrt{4-x^2}}}{1} = 0$

$\frac{0}{0}$

6. $\lim_{x\to 0} \frac{e^x-(1-x)}{x} = \lim_{x\to 0} \frac{e^x+1}{1} = 2$

$\frac{0}{0}$

7. $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)} = \lim_{x\to 0} \frac{2\cos 2x}{3\cos 3x} = \frac{2}{3}$

$\frac{0}{0}$

8. $\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

$\frac{0}{0}$

9. $\lim_{x\to \infty} \frac{3x^2-2x+1}{2x^2+3} = \lim_{x\to\infty} \frac{6}{4} = \frac{3}{2}$

$\frac{\infty}{\infty}$ $\frac{6x-2}{4x}$ $\frac{\infty}{\infty}$

10. $\lim_{x\to \infty} \frac{x^2+2x+1}{x-1} = \lim_{x\to\infty} \frac{2x+2}{1} = \infty$

$\frac{\infty}{\infty}$

11. $\lim_{x\to \infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{1}{\frac{2x}{2\sqrt{x^2+1}}}$

$\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = 1$

$\frac{\infty}{\infty}$

12. $\lim_{x\to \infty} \frac{\ln x}{x}$

$\frac{\infty}{\infty} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\frac{1}{f(x)} = \frac{f(x)}{1}$

13. $\lim_{x\to \infty} \frac{(\ln x)^3}{x}$

$\frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{3(\ln x)^2 \cdot \frac{1}{x}}{1}$ $\frac{\infty}{\infty}$ $= \lim_{x\to\infty} \frac{6\ln x \cdot \frac{1}{x}}{1}$ $\frac{\infty}{\infty}$ $= \lim_{x\to\infty} \frac{\frac{6}{x}}{1} = 0$

14. $\lim_{x\to 0^+} (-x\ln x)$

15. $\lim_{x\to \infty} \left(x\sin\frac{1}{x}\right)$

16. $\lim_{x\to \infty} x^{1/x}$

17. $\lim_{x\to 0^+} (1+x)^{1/x}$

18. $\lim_{x\to 2^+} \left(\frac{8}{x^2-4} - \frac{x}{x-2}\right)$

19. $\lim_{x\to 1^+} \left(\frac{3}{\ln x} - \frac{2}{x-1}\right)$

20. $\lim_{x\to \infty} \frac{x^2}{e^{5x}} = \lim_{x\to\infty} \frac{2}{25e^{5x}} = 0$

$\frac{\infty}{\infty}$ $\frac{2x}{5e^{5x}}$ $\frac{\infty}{\infty}$

21. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x} = 0$

$\frac{0}{1}$