§7-4 CONT. HALF ANGLE IDENTITIES

{ I WILL NOT DERIVE THESE, YOU WILL HAVE TO TRUST ME!! IF NOT SEE PP. 450-451. }

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \quad ; \quad \cos\theta \neq -1$$

FOR OBVIOUS REASONS

SECRET IDENTITIES FOR $\tan\frac{\theta}{2}$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

AND

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

* NOTICE THESE DO NOT HAVE RADICALS OR +/- TO NEGOTIATE

THE HALF ANGLE IDENTITIES ARE ESPECIALLY USEFUL TO FIND <u>EXACT</u> <u>VALUES</u> OF ANGLE NOT ON THE <u>UNIT CIRCLE</u>

FOR EXAMPLE 15° OR $\frac{\pi}{8}$ ... ANGLES THA ARE EXACTLY HALF OF UNIT CIRCLE ANGLES.

EXAMPLES

USE A HALF ANGLE IDENTITY TO FIND THE EXACT VALUE OF EACH FUNCTION

a. $\cos \frac{\pi}{8} = ?$

* THINK $\frac{\pi}{8} = 22.5^\circ$ WHICH IS HALF OF 45° OR $\pi/4$ SO. $\pi/4$ IS OUR θ

$$\cos \frac{\pi/4}{2} = \pm\sqrt{\frac{1+\cos \pi/4}{2}}$$

$$= \sqrt{\frac{1+\cos \pi/4}{2}}$$

* SINCE $\pi/8$ IS IN QI COS IS POSITIVE IN QI WE USE "+"

$$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

* $\cos \pi/4 = \frac{\sqrt{2}}{2}$ FROM THE UNIT CIRCLE

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\frac{2}{1}}}$$

* NOW SIMPLIFY LIKE CRAZY!!!

$$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2}{1}}}$$

* FLIP AND MULT

$$= \sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

* $\sqrt{4} = 2$

$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

b. $\tan 15^\circ = ?$

$$\tan\frac{30^\circ}{2} = \frac{1-\cos 30^\circ}{\sin 30^\circ}$$

* THINK 15° IS HALF OF 30°, SO 30° IS OUR θ
* THEN PICK AN IDENTITY. THAT WORKS FOR YOU.

$$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

* $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\sin 30^\circ = \frac{1}{2}$

FROM THE GOOD 'OL UNIT CIRCLE

$$= \frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

* SIMPLIFY!

$$= \frac{\frac{2-\sqrt{3}}{2}}{\frac{1}{2}}$$

* FLIP AND MULTIPLY

$$= \frac{2-\sqrt{3}}{2}\cdot\frac{2}{1} = \boxed{2-\sqrt{3}}$$

TRY USING $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ OR

$$\frac{\sin\theta}{1+\cos\theta}$$

HALF FUN!

VERIFY $\sin\frac{x}{2}\cos\frac{x}{2} = \frac{\sin x}{2}$

$$\pm\sqrt{\frac{1-\cos\theta}{2}} \cdot \pm\sqrt{\frac{1+\cos\theta}{2}} = \frac{\sin x}{2}$$

* SUBSTITUTE IDENTITIES

$$\sqrt{\frac{(1-\cos\theta)(1+\cos\theta)}{4}} = \frac{\sin x}{2}$$

* MULTIPLY

$$\sqrt{\frac{1-\cos^2\theta}{4}} = \frac{\sin x}{2}$$

* FOIL TOP

$$\sqrt{\frac{\sin^2\theta}{4}} = \frac{\sin x}{2}$$

* PYTHAGOREAN IDENTITY

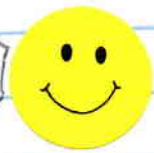
$$\frac{\sin\theta}{2} = \frac{\sin\theta}{2}$$

* SQUARE ROOT TOP & BOTTOM

DONE!

* SORRY I USED θ INSTEAD OF X ON THE LEFT SIDE ..

P. 454 # 14-20, 33-35, 38



* ALSO LOOK AT P450 EXAMPLE 1d. TO HELP WITH 34 & 35