7.3 Sum and Difference Identities

Sum and Diff. Identities for Cosine Function

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

these are opposites

or...

sum: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

diff: $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Ex.1) Use sum or diff. identities to find the exact value of $\cos 105°$.

$$\cos 105 = \cos(60+45) = \cos 60\cos 45 - \sin 60\sin 45$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

Sum and Difference Identities for Sine Function

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

same sign

or...

sum: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

diff: $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Ex.2) Find $\sin(x-y)$ if $\sin x = \frac{9}{41}$ and $\sin y = \frac{7}{25}$. Assume $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$.

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

✓ ? ? ✓

But what is $\cos x$ and $\cos y$?

Very handy for this chapter! → To find these, use the Pythagorean Identity.

$$\sin^2 x + \cos^2 x = 1$$

$\cos x$:

$$\sin^2 x + \cos^2 x = 1$$
$$\left(\frac{9}{41}\right)^2 + \cos^2 x = 1$$
$$\cos^2 x = 1 - \frac{81}{1681}$$
$$\cos^2 x = \frac{1600}{1681}$$
$$\cos x = \sqrt{\frac{1600}{1681}}$$
$$\cos x = \frac{40}{41}$$

$\cos y$:

$$\sin^2 y + \cos^2 y = 1$$
$$\left(\frac{7}{25}\right)^2 + \cos^2 y = 1$$
$$\cos^2 y = 1 - \frac{49}{625}$$
$$\cos^2 y = \frac{576}{625}$$
$$\cos y = \sqrt{\frac{576}{625}}$$
$$\cos y = \frac{24}{25}$$

Plug these back in!

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$= \left(\frac{9}{41}\right)\left(\frac{24}{25}\right) - \left(\frac{40}{41}\right)\left(\frac{7}{25}\right)$$
$$= -\frac{64}{1025}$$

Similar problems can also be done with the following:

Sum and Difference Identities for the Tangent Function

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

The methods would be the same, just using this identity when tangent is used.

You can use identities to verify other identities...

Ex. 3) Verify: $\csc\left(\frac{3\pi}{2} + A\right) = -\sec A$

The left side can be used with the sum identity for sin once flipped over.

$$\frac{1}{\sin\left(\frac{3\pi}{2} + A\right)} = -\sec A$$

$$\frac{1}{\sin\frac{3\pi}{2}\cos A + \cos\frac{3\pi}{2}\sin A} = -\sec A$$

$$\frac{1}{(-1)\cos A + (0)\sin A} = -\sec A$$

$$\frac{1}{-\cos A} = -\sec A \quad \checkmark$$

7.3 Additional Practice

p. 442-443, #15-39 odds