AP* Review Questions for Chapter 2

1. (no calculator)

Let $f(x) = (x^2 - 3)^4$.

(a) Write an equation of the line tangent to the graph of f at $x = 2$.

(b) Find the values of x for which the graph of f has a horizontal tangent.

(c) Find $f''(x)$.

2. (no calculator)

Let $f(x) = \sqrt{4x - 3}$ and $g(x) = \dfrac{f(x)}{x}$.

(a) What is the slope of the graph of f at $x = 3$? Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of g at $x = 3$.

(c) What is the slope of the line normal to the graph of g at $x = 3$?

3. (no calculator)

Evaluate each limit analytically.

(Note: Finding the answer should not involve a lengthy algebraic process.)

(a) $\lim_{h \to 0} \dfrac{\sin(x + h) - \sin x}{h}$

(b) $\lim_{h \to 0} \dfrac{\sqrt[3]{x + h} - \sqrt[3]{x}}{h}$

(c) $\lim_{h \to 0} \dfrac{\sqrt{16 + h} - 4}{h}$

(d) $\lim_{h \to 0} \dfrac{\frac{1}{5 + h} - \frac{1}{5}}{h}$

4. Given:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	-3	1	5	-2
5	4	7	-1	2

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(2)$.

(b) If $j(x) = f(g(x))$, find $j'(2)$.

(c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$.

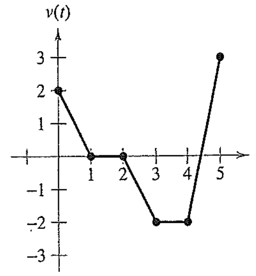
5. (no calculator)

Given: $f(x) = x^2$

(a) Find the slope of the normal line to the graph of f at $x = -3$.

(b) Two lines passing through the point (3, 8) will be tangent to the graph of f. Find an equation for each of these lines.

6. The accompanying diagram shows the graph of the velocity in $\dfrac{\text{ft}}{\text{sec}}$ for a particle moving along the line $x = 4$.



(a) During which time interval is the particle:

(i) moving upward?

(ii) moving downward?

(iii) at rest?

(b) State the acceleration of the particle at the specified times. Include units.

(i) $t = 0.75$

(ii) $t = 4.2$

7. (no calculator)

Given: $g(x) = f(x) \cdot \tan x + kx$, where k is a real number.

f is differentiable for all x; $f\left(\frac{\pi}{4}\right) = 4$; $f'\left(\frac{\pi}{4}\right) = -2$.

(a) For what values of x, if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.

(b) If $g'\left(\frac{\pi}{4}\right) = 6$, find the value of k.

8. The table provided below shows the position of a particle, S, at several times, t, as the particle moves along a straight line, where t is measured in seconds and S is measured in meters.

t	2.0	2.7	3.2	3.8
$S(t)$	5.2	7.8	10.6	12.2

Which of the following best estimates the velocity of the particle at $t = 3$?

(a) $9.2\,\frac{\text{m}}{\text{s}}$ (b) $7.8\,\frac{\text{m}}{\text{s}}$ (c) $5.6\,\frac{\text{m}}{\text{s}}$

9. (no calculator)

If $2y^3 - 3xy + x^2 = 4$, then $\dfrac{dy}{dx} =$

(A) $-\dfrac{2x}{6y^2 - 3}$ (B) $\dfrac{2x - 3y}{3x - 6y^2}$ (C) $\dfrac{2x - 3}{6y^2}$

(D) $-\dfrac{2x}{6y^2 - 3x}$ (E) $\dfrac{3y - 2x}{6y^2}$

AP2-2

10. (no calculator)

The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius and height of the cylinder are increasing at constant rates. The radius is expanding at $\frac{1}{3}\frac{\text{cm}}{\text{sec}}$ and the height is increasing at $\frac{1}{2}\frac{\text{cm}}{\text{sec}}$. At what rate, in cubic *cm* per second, is the volume of the cylinder increasing when its height is 9 cm and the radius is 4 cm?

(A) 32π

(B) 6π

(C) $\frac{8\pi}{3}$

(D) $\frac{4\pi}{3}$

(E) $\frac{\pi}{18}$

AP* Review Questions for Chapter 3

1. (no calculator)

Given: $g(x) = (2x + 4)^3(x - 6)$

(a) Find the critical numbers of g.

(b) For what values of x is g increasing? Justify your answer.

(c) Identify the x-coordinate of the critical points at which g has a relative minimum. Justify your answer.

2. (no calculator)

Let $f(x) = 2x + \cos(2x)$.

(a) Find the maximum value of f for $0 \le x \le \pi$. Justify your answer.

(b) Explain how the conditions of the Mean Value Theorem are satisfied by f for $0 \le x \le \pi$. Find the value of x, $0 \le x \le \pi$, whose existence is guaranteed by the Mean Value Theorem.

3. (no calculator)

Let $f(x) = \dfrac{1 - 4x^2}{x}$.

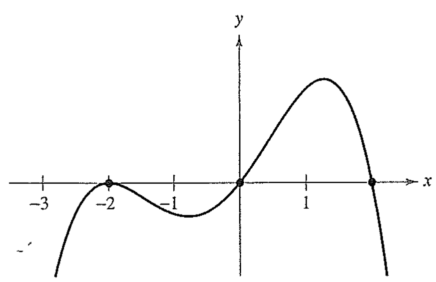
(a) State $f'(x)$ and identify the value(s) of x for which f' does not exist.

(b) For what values of x is f decreasing? Justify your answer.

(c) For what values of x is the graph of f concave downward? Show the work that leads to your answer.

(d) Does the graph of f contain an inflection point? Justify your answer.

4.



In the figure above, f', the *derivative* of function f, is shown. f is a twice differentiable function on $x \in (-\infty, \infty)$. $f''(-0.8) = 0$ and $f''(1.3) = 0$.

(a) Name the value(s) of x for which f has a relative minimum. Justify your answer.

(b) For what values of x is f increasing? Justify your answer.

(c) For what values of x is the graph of f concave downward? Justify your answer.

(d) Is $\dfrac{f(-0.5) - f(0)}{-0.5 - 0}$ positive or negative? Justify your answer.

5. The depth of the water at the end of a pier is shown in the table below and is modeled by differentiable function D for $t \ge 0$. Selected values of D are shown in the table below. D is expressed in meters, and t is the number of hours since midnight ($t = 0$).

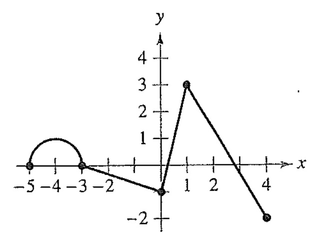
t (hours)	0	2	5	7	8	9	12
$D(t)$ (meters)	3.0	6.7	4.9	2.3	3.1	4.9	6.7

(a) Use the data in the table to estimate the rate at which the depth of the water is changing at 3:30 A.M. and 7:40 A.M. Include units.

(b) What is the least number of times in the interval $0 < t < 12$ for which $D'(t) = 0$? Justify your answer.

(c) Use the method of linear approximation to estimate the depth of the water at 2:30 A.M. ($t = 2.5$). Show the work that leads to your answer

6. (no calculator)



The graph of f', the derivative of f, is shown above. The function f is differentiable on the interval $-5 \le x \le 4$. $f'(-4) = 0$.

(a) Find $f'(-1)$.

(b) Find $f''(-1)$.

(c) Find the x-coordinate of each inflection point for the graph of f on the interval $-5 < x < 4$.

(d) If $g(x) = f(x) + \sin^2 x$, is g increasing or decreasing at $x = -\dfrac{\pi}{4}$? Justify your answer.

7. Given: f is continuous for $x \in (-\infty, \infty)$;

$f(2) = 4$; $\lim_{x \to \infty} f(x) = 0$

	$x < 4$	$x = 4$	$x > 4$
$f'(x)$	positive	does not exist	negative
$f''(x)$	negative	does not exist	positive

(A) For what values of x is f increasing?

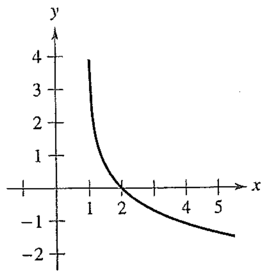
(B) Does f have a relative maximum at $x = 4$? Explain.

(C) If possible, name the x-coordinate of an inflection point on the graph of f. Justify your answer.

(D) Does the Mean Value Theorem apply over the interval [3, 5]? Justify your answer.

(E) Sketch a possible graph of f using the information from the table.

8.



Consider the graph of $y = f(x)$ shown above. If f is a function such that f' and f'' are defined in a region around $x = 2$, then which of the following must be true?

(A) $f''(2) < f(2)$

(B) $f''(2) < f'(2)$

(C) $f(2) = f'(2)$

(D) $f''(2) > f(2)$

(E) $f(2) = f''(2)$

9. (no calculator)

The position of an object along a vertical line is given by $s(t) = -t^3 + 3t^2 + 9t + 5$, where s is measured in feet and t in seconds. The maximum *velocity* of the object in the time interval $0 \leq t \leq 4$ is

(A) $32 \frac{\text{ft}}{\text{sec}}$

(B) $16 \frac{\text{ft}}{\text{sec}}$

(C) $12 \frac{\text{ft}}{\text{sec}}$

(D) $9 \frac{\text{ft}}{\text{sec}}$

(E) $-15 \frac{\text{ft}}{\text{sec}}$

10. (no calculator)

Which of the following is true for the graph of $f(x) = \dfrac{4 - x}{x - 2}$?

I. $x = 2$ is a vertical asymptote of the graph of f.

II. f is decreasing for $x \in (-\infty, \infty)$.

III. f is concave down for $x \in (-\infty, 2)$.

(A) None

(B) I and II only

(C) I and III only

(D) III only

(E) I, II and III

AP* Review Questions for Chapter 4

1. (no calculator)

For $0 \le t \le 9$, a particle moves along the x-axis. The velocity of the particle is given by $v(t) = \sin\left(\frac{\pi}{4}t\right)$. The particle is at position $x = -4$ when $t = 0$.

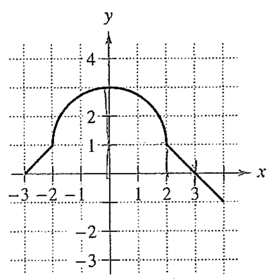
(a) For $0 \le t \le 9$, when is the particle moving to the right?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to $t = 9$.

(c) Find the acceleration of the particle at time $t = 3$. Is the particle speeding up, slowing down or neither at $t = 3$? Justify your response.

(d) Find the position of the particle at time $t = 3$.

2. (no calculator)



Let $F(x) = \int_3^x f(t)dt$. The graph of f on the interval $[-3, 4]$ consists of 2 line segments and a semicircle, as shown in the graph above.

(a) Find $F(0)$, $F'(0)$, and $F(4)$.

(b) Find all relative minimum values of $F(x)$ on the interval $[-3, 4]$. Justify your answer.

(c) Find the x-coordinates of the inflection points of $F(x)$ on the interval $[-3, 4]$. Justify your answer.

(d) Write the equation of the line tangent to the point where $x = 2$.

3. (calculator)

t (minutes)	0	3	5	7	8	12
$C(t)$ (degrees Celsius)	65	57	50	46	44	40

As a pot of coffee cools down, the temperature of the coffee is modeled by a differentiable function C, for $0 \le t \le 12$, where time is measured in minutes and the temperature $C(t)$ is measured in degrees Celsius. Selected values of time t are shown in the table above.

(a) Evaluate $\int_0^{12} C'(t)dt$. Using correct units, explain the meaning of your answer in the context of the problem.

(b) Using correct units, explain the meaning of $\frac{1}{12}\int_0^{12} C(t)dt$ in the context of the problem. Use a trapezoidal sum with 5 subintervals indicated by the table to approximate $\frac{1}{12}\int_0^{12} C(t)dt$.

(c) Use the data in the table to approximate the rate at which the temperature is changing at time $t = 4$. Use proper units and show the computations that lead to your answer.

(d) For $12 \le t \le 15$, the rate of cooling is modeled by $C'(t) = -2\cos(0.5t)$. Based on the model, what is the temperature of the coffee at time $t = 15$? Assume $C(t)$ is continuous at $t = 12$.

4. (calculator)

On a typical day, the snow on Pike Mountain melts at a rate modeled by the function

$M(t)$, given by $M(t) = \frac{\pi}{6}\sin\left(\frac{\pi t}{12}\right)$.

A snow maker adds snow at a rate modeled by the function S, given by

$S(t) = 0.006t^2 - 0.12t + 0.87$.

Both M and S have units in inches per hour and t is measured in hours for $0 \le t \le 6$.

At time $t = 0$, the mountain has 40 inches of snow.

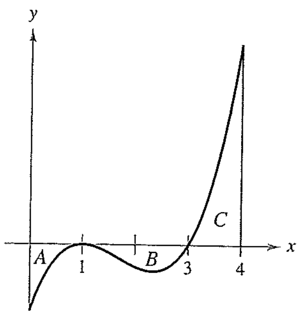
(a) How much snow will melt during the 6 hour period? Indicate units of measure.

(b) Write an expression for $I(t)$, the total number of inches of snow at any time t.

(c) Find the rate of change of the total amount of snow at time $t = 3$.

(d) For $0 \le t \le 6$, at what time t is the amount of snow a maximum? What is the maximum value? Justify your answer.

5. (no calculator)



The graph of a continuous function f is shown in the figure above. The three regions between the graph of f and the x-axis are marked A, B, and C and have unsigned areas 5.5, 6, and 15.5, respectively. Let $F(x)$ be an anti-derivative of f that is differentiable on $(0, 4)$ and with $F(1) = 9$.

(a) Find $F(0)$ and $F(4)$.

(b) What is the minimum number of times F equals 5 on the interval $[0, 4]$? Show the work that leads to your conclusion.

(c) Find all intervals where F is increasing. Justify your answer.

6. (calculator)

The velocity, $v(t)$, of a high speed rail train is positive over $0 \le t \le 60$ seconds. The velocity of the train is recorded for selected values of t, as shown in the table below.

t (seconds)	0	10	20	30
$v(t)$ (feet per second)	0	45	105	140

t (seconds)	40	50	60
$v(t)$ (feet per second)	165	195	210

(a) Estimate the acceleration of the train at $t = 25$. Indicate units of measure.

(b) Use a left Riemann sum with three subintervals of equal length to approximate $\int_{20}^{50} v(t)dt$. Using correct units, explain the meaning of $\int_{20}^{50} v(t)dt$ in the context of this problem.

(c) Evaluate $\int_{20}^{50} v'(t)dt$. Using correct units, explain the meaning of $\int_{20}^{50} v'(t)dt$ in the context of this problem.

(d) Estimate the average velocity of the high speed train over the 60 second period of time using a midpoint Riemann sum with 3 subintervals.

7. (no calculator)

Let g be the function defined by

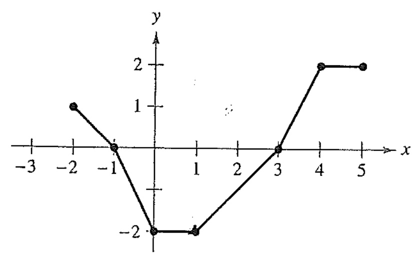
$$g(x) = \begin{cases} -x^2 + 4x & \text{for } 1 \le x \le 2 \\ \sqrt{2x - 4} & \text{for } 2 < x \le 4 \end{cases}$$

(a) Write the equation for the line tangent to the graph of g at $x = 3$.

(b) Is g continuous at $x = 2$? Use the definition of continuity to explain your answer.

(c) Find the average value of $f(x)$ from $x = 1$ to $x = 4$.

8. (no calculator)



The graph of a piecewise linear function f is shown above. If g is the function defined by $g(x) = \int_{4}^{x} f(t)dt$, find $g(-1)$.

(A) 4

(B) −4

(C) 0

(D) 6

(E) −6

9. (calculator)

The velocity of a particle is given by $v(t) = 4t^3 - 4t$ for the times $0 \le t \le 2$ in seconds. Find the average speed of the particle over that interval.

(A) 5

(B) −4

(C) 4

(D) 10

(E) 24

10. (calculator)

Let $f(x)$ be a continuous function such that $f(1) = 2$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

(A) 11.446

(B) 13.446

(C) 15.446

(D) 24.672

(E) 26.672

AP* Review Questions for Chapter 5

1. (calculator)

A particle travels along the x-axis for times $0 \le t \le 4$. The velocity of the particle is given by $v(t) = \sin\left(\frac{5\pi}{2}e^{-t/\pi}\right)$. At time $t = 0$, the particle is 2 units to the right of the origin.

(a) During what time intervals on $[0, 4]$ is the particle traveling to the left?

(b) Find the average velocity of the particle from $0 \le t \le 4$.

(c) Is the speed increasing or decreasing at time $t = 2$?

(d) For the time interval $0 \le t \le 4$, what is the farthest distance to the right that the particle travels from the origin?

2. (calculator)

Let f be a function defined by $f(x) = \frac{e^x + e^{-x}}{2}$.

(a) Find, if any, the x-coordinates of all relative minimum values of $f(x)$.

(b) Find the average value of $f(x)$ for $-1 \le x \le 1$.

(c) Find, if any, the x-coordinates of all inflection points of $f(x)$.

3. (calculator)

Water is flooding into a basement at a rate of $f(t) = -2\sqrt{t} + 5$. Water is being pumped out at a rate of $g(t) = 5(1 - e^{-0.5t})$. Both $f(t)$ and $g(t)$ are defined for $0 \le t \le 5$ hours. At time $t = 0$, there are 15 cubic feet of water in the basement.

(a) Calculate the total amount of water that enters the basement from $t = 0$ to $t = 2$ hours.

(b) How fast is the water level changing at time $t = 2$ hours? Use proper units.

(c) At what time t was the volume of water at a maximum? How much water was in the basement at the time?

4. (calculator)

The function f is defined by $f(x) = \frac{1}{\sqrt{4 - x^2}}$ for $-2 < x < 2$.

Let g be the function defined by

$$g(x) = \begin{cases} f(x) & \text{for } -2 \le x \le 0 \\ x + \frac{1}{2} & \text{for } 0 < x \le 2 \end{cases}.$$

(a) Is g continuous at $x = 0$? Use the definition of continuity to explain your answer.

(b) What is the area of the region bounded by f, the x-axis, the y-axis, and the line $x = 1$?

(c) Find the value of $\int_{-1}^{1} g(x)dx$.

5. Let $f(x) = \arccos(x)$ and let $g(x) = x^2$. Define $h(x) = f(g(x))$.

(a) At what values of x does $h(x)$ have a relative maximum?

(b) Write, but do not evaluate, an expression that can determine the area of the region bounded by the graphs of $h(x)$ and the horizontal line $y = \frac{\pi}{3}$.

(c) Evaluate $\frac{d}{dx}f^{-1}\left(\frac{\pi}{3}\right)$.

6. (no calculator)

Let $f(x) = e^x - x$.

(a) Find the critical values of f. Classify each of these values as a relative minimum, a relative maximum, or neither a minimum nor a maximum. Justify your conclusion.

(b) Write the equation of the line tangent to the graph of f at the point where $x = 1$.

(c) Given $\int_0^a f(x)dx = f'(a)$, find a.

7. (calculator)

Two towers of equal height are spaced 366 feet apart. A cable suspended between the two forms a catenary whose height above the ground is given by $f(x) = \frac{125}{2}(e^{x/250} + e^{-x/250})$, where $x = 0$ is at the point on the ground halfway between the two towers.

(a) What is the height of each tower (rounded to the nearest foot)? (Assume the cable is anchored to the *tops* of the towers.)

(b) Evaluate $f'(100)$. Explain what $f'(100)$ is in the context of the problem.

(c) What is the average height of the cable?

(d) What is the slope of the cable at the rightmost point where the height of the cable is equal to the average height of the cable?

8. (no calculator)

x	$f(x)$	$f'(x)$
1	3	1
2	-1	3
3	5	-2
4	-2	4
5	4	-2

The table above gives values of f and its derivative f' at selected values of x. The function f is twice differentiable at all x.

(a) Use the values in the table to estimate the value of $f'(1.5)$.

(b) Explain why there must be a value c for $1 < c < 3$, for which $f'(c) = 1$.

(c) Estimate the average value of $f(x)$ from $1 \le x \le 4$ using a trapezoidal sum with 3 subintervals, as indicated in the table.

(d) Assuming f is invertible, find the equation of the line tangent to the graph of $y = f^{-1}(x)$ at $x = 5$.

9. (no calculator)

Let f be a differentiable function such that $f(-2) = 3$, $f(3) = 5$, $f'(-2) = 4$, $f'(3) = 6$. Let g be a differentiable function, such that $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) 6

(E) 4

10. (no calculator)

If $\cos x = \ln y$, what is $\frac{dy}{dx}$ in terms of x?

(A) $-e^{\cos x} \sin x$

(B) $e^{\cos x} \sin x$

(C) $\frac{-1}{\sin x}$

(D) $\frac{1}{\sin x}$

(E) $\cos x \sin x$

AP* Review Questions for Chapter 6

1. (no calculator)

Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{xy}$ with $f(1) = 2$.

(a) Find $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.

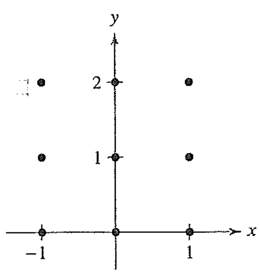
(b) Write an equation for the line tangent to the graph of f at $(1, 2)$ and use it to approximate $f(1.1)$. Is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the solution of the given differential equation that satisfies the initial condition $f(1) = 2$.

2. (no calculator)

Consider the differential equation $\frac{dy}{dx} = x^2(1 - y)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the 9 points indicated.



(b) While the slope field in part (a) is drawn only at 9 points, it is defined at every point on the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution in the form of $y = f(x)$ to the given differential equation with the initial condition $f(0) = 2$.

3. (BC only, no calculator)

Given the differential equation $y' = \frac{2x}{y}$ with a particular solution in the form of $y = f(x)$ that satisfies the initial condition $f(1) = 2$:

(a) Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $y(1.4)$. Show the work that leads to your answer.

(b) Find the particular solution to the given differential equation that passes through $(1, 2)$ and state its domain.

4. (BC only, no calculator)

Given the differential equation $\frac{dy}{dx} = xy$:

(a) Let $y = f(x)$ be the function that satisfies the differential equation with initial condition $f(1) = 1$. Use Euler's Method, starting at $x = 1$ with a step size of 0.1, to approximate $f(1.2)$. Show the work that leads to your answer.

(b) Find $\frac{d^2y}{dx^2}$. Determine whether the approximation found in part (a) is less than or greater than $f(1.2)$

(c) Find the particular solution of the differential equation that passes through $(1, 1)$.

5. (no calculator)

At any time, $t \geq 0$, in hours, the rate of growth of a population of bacteria is given by $\frac{dy}{dt} = \frac{1}{2}y$. Initially, there are 200 bacteria in the culture.

(a) Use separation of variables to solve for y, the number of bacteria present, at any time $t \geq 0$.

(b) Write, but do not evaluate an expression to find the average number of bacteria in the population for $0 \leq t \leq 10$.

(c) Write an expression to find the average rate of bacteria growth over the first 10 hours of growth. Indicate units of measure.

6. (BC only, no calculator)

At any time $t \geq 0$, the rate of the spread of an epidemic is modeled by a function y that satisfies the differential equation $\frac{dy}{dt} = \frac{1}{10}y\left(1 - \frac{y}{1000}\right)$.

In an isolated town of 1000 inhabitants, 100 people have a disease at the beginning of the week.

(a) Is the disease spreading faster when 100 people have the disease or when 200 people have the disease? Explain your reasoning.

(b) Use separation of variables to write a model for the population $y = f(t)$ at any time $t \geq 0$.

(c) For the function y found in part (b), what is $\lim_{t\to\infty} y(t)$?

7. (BC only, no calculator)

A population is modeled by a function $y(t)$ that satisfies the logistic differential equation

$$\frac{dy}{dt} = 2y\left(1 - \frac{y}{5}\right)$$

(a) What is $\lim_{t\to\infty} y(t)$?

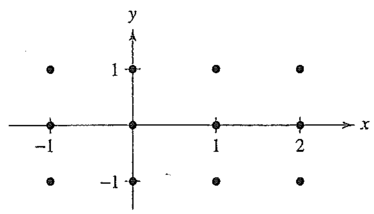
(b) For what value of y is the population growing the fastest?

(c) Find the particular solution satisfying $y(0) = 3$.

8. (no calculator)

Given the differential equation $\frac{dy}{dx} = \frac{x}{y^2}$:

(a) On the axes provided, sketch a slope field for the given differential equation at the 12 points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(c) Find the particular solution to the given differential equation that satisfies the initial condition $y(0) = 2$.

9. (no calculator)

If $\frac{dy}{dx} = 2xy^2$, and $y(-1) = 2$, find $y(2)$.

(A) $4e^3$

(B) $-4e^3$

(C) $-\frac{3}{2}$

(D) $-\frac{2}{5}$

(E) $-\frac{1}{4}$

10. (no calculator)

Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{3y}{x}$ with the initial condition $y(1) = -1$?

(A) $y = x^3$

(B) $y = -x^3$

(C) $y = 3x$

(D) $y = -3x$

(E) $y = -x^3 - 2$

AP* Review Questions for Chapter 7

1. (calculator)

The base of a solid is the region in the first quadrant bounded above by the line $y = 2$, below by $y = \sin^{-1} x$, and to the right by the line $x = 1$. For this solid, each cross-section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 1.429

(B) 2

(C) 2.184

(D) 0.766

(E) 4

2. (no calculator)

Find the area of the region bounded by the y-axis, the line $y = e$, and the graph of the function $y = e^{3x}$.

(A) $\frac{1}{3}$

(B) $e^{3e} - \frac{1}{3}$

(C) $1 - \frac{2}{3}e$

(D) $3 - \frac{8}{3}e$

(E) $\frac{1}{3} + e^2 - e^{3e}$

3. (no calculator)

A region in the xy-plane is bounded by the curves $y = 4x - x^2$ and $y = 2x - 3$.

(a) Find the points of intersection of the two curves.

(b) Sketch the region bounded by the curves. Label the bounding curves, show and label all points of intersection, and shade the bounded region.

(c) Find the area of the region, showing all work that leads to your answer.

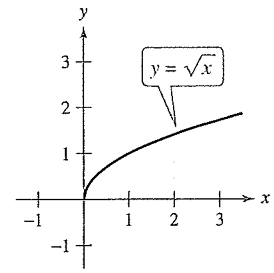
4. (no calculator)

A region in the xy-plane is bounded by $y = 2x + 2$, $x = \frac{y^2}{2} + 2$, $y = -2$, and $y = 2$.

(a) Sketch the bounded region on a Cartesian axis system. Label each boundary curve and shade the bounded region.

(b) Find the area of the bounded region, showing all work that leads to your answer.

5. (no calculator)



Consider the region shown above, which is bounded by $f(x) = \sqrt{x}$, $y = 0$, $x = 0$, and $x = 2$.

(a) Find the area of the region.

(b) Find the volume of the solid formed by rotating the region about the x-axis.

(c) The region pictured above is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.

6. (no calculator)

Consider the region R, bounded by the graphs of $y = x^3$, $y = 8$ and the y-axis. The region S is bounded by $y = x^3$, $x = 2$ and the x-axis.

(a) Find the area of region R.

(b) Find the volume of the solid formed by rotating region R about the y-axis.

(c) The region S is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a semi-circle with diameters extending from $y = x^3$ to the x-axis. Find the volume of this solid.

7. (BC only, calculator)

Consider the region bounded be the y-axis, $y = 10$, and $y = 1 + 6x^{3/2}$.

(a) Set up, but do not evaluate, an integral equation that will find the value of k so that $x = k$ cuts the region into 2 parts of equal area.

(b) Find the length of the curve $y = 1 + 6x^{3/2}$ on the interval $[0, 1]$.

(c) The region is the base of a solid. For this solid, the cross-sections perpendicular to the x-axis are rectangles with a height of 3 times that of its width. Find the volume of this solid.

8. (calculator)

Let R be the region bounded by the graph of $y = \ln x$ and the line $y = 2x - 3$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an expression involving one or more integrals that can be used to find the volume of the solid generated when R is revolved about the y-axis.

9. (calculator)

Let R be the region bounded by the graph of $y = x^2 - 1$ and the graph of $x = y^2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = 2$.

(c) Write, but do not evaluate, an expression involving one or more integrals to find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

10. (no calculator)

Consider the region bounded by the graphs of $f(x) = \sqrt{x}$, $y = 0$, and $x = 2$.

(a) Find the volume of the solid formed by rotating the region about the x-axis.

(b) Find the volume of the solid formed by rotating the region about the y-axis.

(c) Write, but do not evaluate, the volume integral of the solid formed by rotating the region about the line $y = -2$.