Rocket City Math League 2017-2018

Apollo Solutions Round 1

1. **Answer: 32**

To evaluate $f(g(36))$, first find $g(36)$: $\frac{\sqrt{36}}{3} = \frac{6}{3} = 2$. Next, evaluate $f(2)$: $9(2)^2 - 4 = 32$.

2. **Answer: $\frac{3y}{z}$**

1944 prime factorizes as $2^3 \cdot 3^5$, so $\sqrt{1944x^3y^2z} = 2 \cdot 3^2 \cdot xy\sqrt{2 \cdot 3 \cdot xz} = 18xy\sqrt{6xz}$. Hence, $a = 18xy$ and $b = 6xz$, so $\frac{a}{b} = \frac{3y}{z}$.

3. **Answer: 64**

The distance from the center of Xavier's ellipse to the vertex is 12, so $b = 12$. Similarly, the distance from the center to the co-vertex is 8, so $a = 8$. Therefore $5a + 2b = 40 + 24 = 64$.

4. **Answer: 56**

Let h equal the number of hours that Bill and Greg lasso. Then, the number of cows Bill lassoes is equal to $4h$, and the number of cows Greg lassoes is $3h + 7$. Setting these equal gives $h = 7$. In 7 hours, Bill lassoed $4(7) = 28$ cows, and Greg lassoed the same number ($3(7) + 7 = 28$ cows), so in total they lassoed 56 cows.

5. **Answer: 61**

Copy the first two columns of the matrix on the outside: $\begin{vmatrix} 9 & -2 & 5 \\ 7 & 13 & -3 \\ 11 & 3 & 4 \end{vmatrix} \begin{matrix} 9 & -2 \\ 7 & 13 \\ 11 & 3 \end{matrix}$.

Multiply down and add: $(9\times13\times4)+(-2\times-3\times11)+(5\times7\times3)=639$. Then, multiply up and subtract from 639:
$639-(11\times13\times5)-(3\times-3\times9)-(4\times7\times-2)=61$.

6. **Answer: 53**

First, $g(5) = 2(5) + 1 = 11$. Then, $f(11) = 2(11) + 1 + 3 = 26$. Finally, $g(26) = 2(26) + 1 = 53$.

7. **Answer: $\sqrt{13}$**

The midpoint of (6, 4) and (8, 2) is $\left(\frac{6+8}{2}, \frac{4+2}{2}\right)$, or (7, 3). Similarly, the midpoint of (11, 1) and (7, -1) is $\left(\frac{11+7}{2}, \frac{1-1}{2}\right)$, or (9, 0). The distance between these points is given by $\sqrt{(7-9)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$.

8. **Answer: $-\frac{85}{9}$**

$f^{-1}(241)$ can be found by substituting $f(x) = 241$ and solving for x:

$$241 = 3^x - 2$$
$$243 = 3^x$$
$$x = 5.$$

Next, $f(-2) = 3^{-2} - 2 = \frac{1}{9} - \frac{18}{9} = -\frac{17}{9}$. So, $5\left(-\frac{17}{9}\right) = -\frac{85}{9}$.

9. **Answer: 11**

Convert the equation to standard form. Shift the terms so x's and y's are on one side of the equation, and the constant is on the other: $x^2+16x+y^2+12y=21$. Complete the square by taking the x-term coefficient (16), multiplying it by one-half (8), squaring it (64), and adding it to both sides of the equation: $(x^2+16x+64)+y^2+12y=21+64$. Repeat with the y-term coefficient: $(x^2+16x+64)+(y^2+12y+36)=21+64+36$. Convert the left side of the equation to squared form, and simplify the right: $(x+8)^2+(y+6)^2=121$. The radius is $\sqrt{121}$, which is 11.

10. **Answer: -1, 3 (any order)**

$2^{2a+1} - 17 \cdot 2^a + 2^3 = 0$ can be rewritten as $(2^a)^2 \cdot 2 - 17 \cdot 2^a + 2^3$. Let $x = 2^a$. Then, $2x^2 - 17x + 8 = 0$. Factoring gives $(2x - 1)(x - 8) = 0$. So, $x = \frac{1}{2}, 8$. Hence, $2^a = \frac{1}{2}, 8$, and $a = -1, 3$.