C H A P T E R P
Preparation for Calculus

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CHAPTER P
Preparation for Calculus

Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

2. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

3. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

4. $y = x^3 - x$

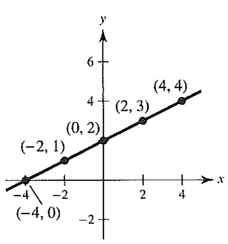
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

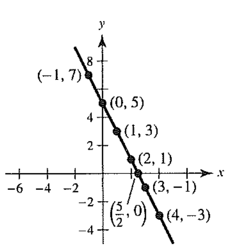
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



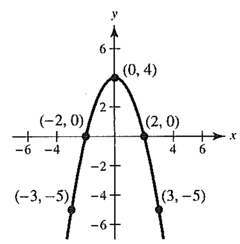
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



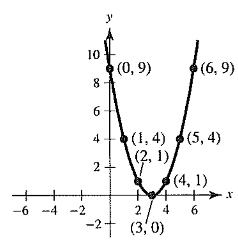
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



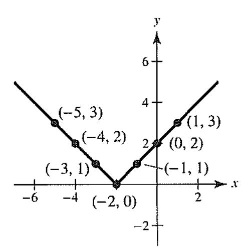
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



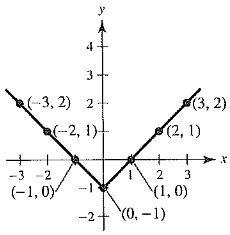
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



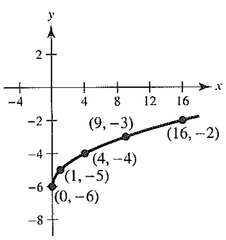
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



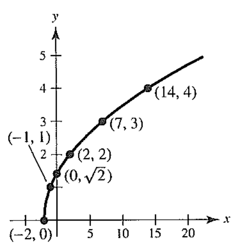
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



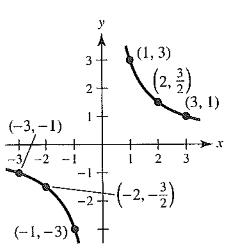
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



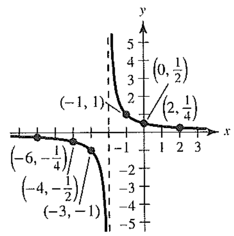
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1



14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



15.

Xmin = −5
Xmax = 4
Xscl = 1
Ymin = −5
Ymax = 8
Yscl = 1

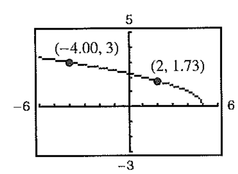
Note that $y = -3$ when $x = 0$ and $y = 0$ when $x = -1$.

16.

Xmin = −20
Xmax = 30
Xscl = 5
Ymin = −10
Ymax = 40
Yscl = 5

Note that $y = 16$ when $x = 0$ or 16.

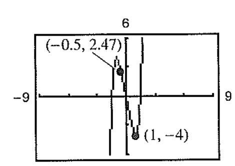
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5;\ (0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2};\ \left(\frac{5}{2}, 0\right)$

20. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3;\ (0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

22. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

24. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x}$

y-intercept: None. x cannot equal 0.

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

28. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

29. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

30. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

31. Symmetric with respect to the x-axis because
$(-y)^2 = y^2 = x^3 - 8x.$

32. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

33. Symmetric with respect to the origin because
$(-x)(-y) = xy = 4.$

34. Symmetric with respect to the x-axis because
$x(-y)^2 = xy^2 = -10.$

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

36. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

38. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis

because $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}.$

39. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis

because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

40. $|y| - x = 3$ is symmetric with respect to the x-axis

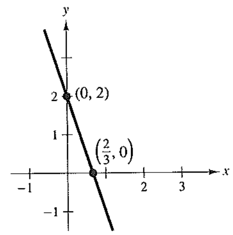
because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

41. $y = 2 - 3x$

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

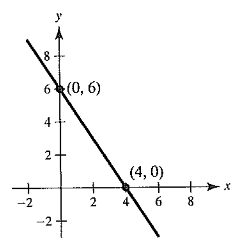
Symmetry: None



42. $y = -\frac{3}{2}x + 6$

Intercepts: $(0, 6), (4, 0)$

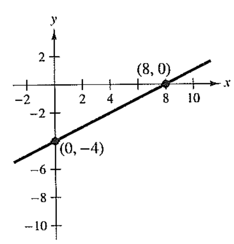
Symmetry: None



43. $y = \frac{1}{2}x - 4$

Intercepts: $(8, 0), (0, -4)$

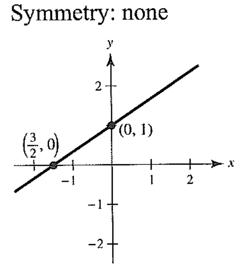
Symmetry: none



44. $y = \frac{2}{3}x + 1$

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

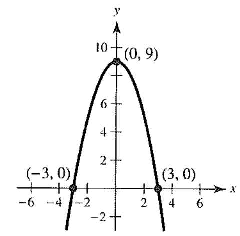
Symmetry: none



45. $y = 9 - x^2$

Intercepts: $(0, 9), (3, 0), (-3, 0)$

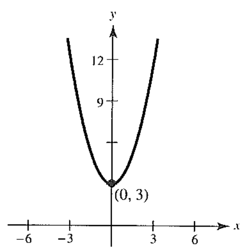
Symmetry: y-axis



46. $y = x^2 + 3$

Intercept: $(0, 3)$

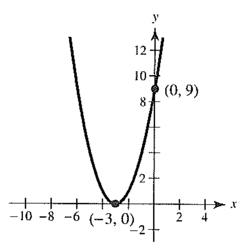
Symmetry: y-axis



47. $y = (x + 3)^2$

Intercepts: $(-3, 0), (0, 9)$

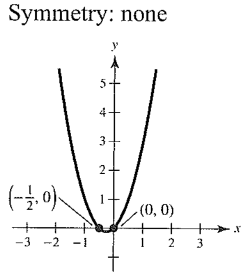
Symmetry: none



48. $y = 2x^2 + x = x(2x + 1)$

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

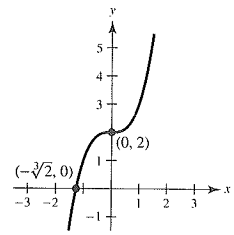
Symmetry: none



49. $y = x^3 + 2$

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

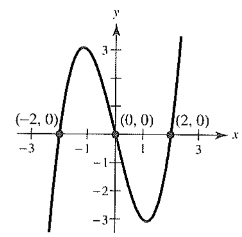
Symmetry: none



50. $y = x^3 - 4x$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

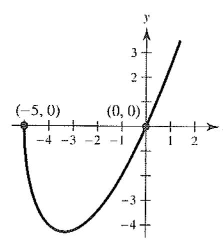
Symmetry: origin



51. $y = x\sqrt{x + 5}$

Intercepts: $(0, 0), (-5, 0)$

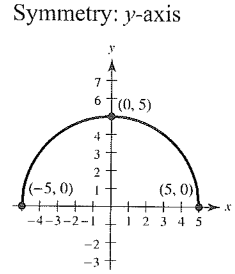
Symmetry: none



52. $y = \sqrt{25 - x^2}$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

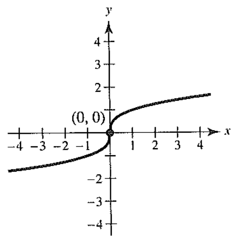
Symmetry: y-axis



53. $x = y^3$

Intercept: $(0, 0)$

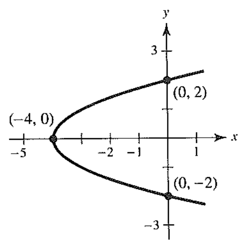
Symmetry: origin



54. $x = y^2 - 4$

Intercepts: $(0, 2), (0, -2), (-4, 0)$

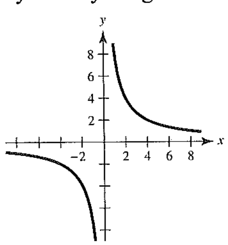
Symmetry: x-axis



55. $y = \dfrac{8}{x}$

Intercepts: none

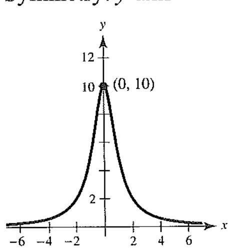
Symmetry: origin



56. $y = \dfrac{10}{x^2 + 1}$

Intercept: $(0, 10)$

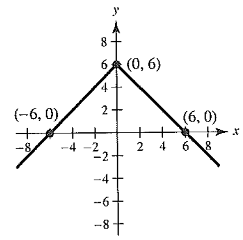
Symmetry: y-axis



57. $y = 6 - |x|$

Intercepts: $(0, 6), (-6, 0), (6, 0)$

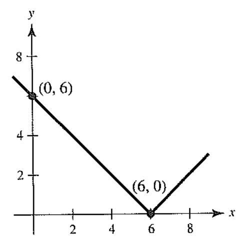
Symmetry: y-axis



58. $y = |6 - x|$

Intercepts: $(0, 6), (6, 0)$

Symmetry: none



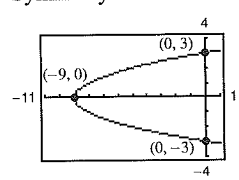
59. $y^2 - x = 9$

$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

Intercepts: $(0, 3), (0, -3), (-9, 0)$

Symmetry: x-axis

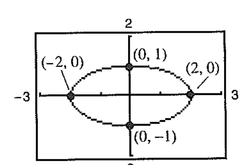


60. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

Symmetry: origin and both axes

Domain: $[-2, 2]$



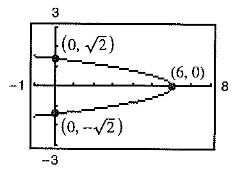
61. $x + 3y^2 = 6$

$3y^2 = 6 - x$

$y = \pm\sqrt{\dfrac{6 - x}{3}}$

Intercepts: $(6, 0), \left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$

Symmetry: x-axis



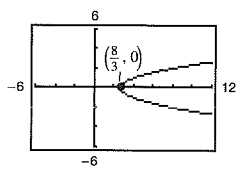
62. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\frac{3}{4}x - 2}$

Intercept: $\left(\frac{8}{3}, 0\right)$

Symmetry: x-axis



63. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

64. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$\dfrac{3x + 4}{2} = \dfrac{-4x - 10}{2}$

$3x + 4 = -4x - 10$

$7x = -14$

$x = -2$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

65. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$6 - x^2 = 4 - x$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

66. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

67. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

68. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

$25 - x^2 = (3x + 15)^2$

$25 - x^2 = 9x^2 + 90x + 225$

$0 = 10x^2 + 90x + 200$

$0 = x^2 + 9x + 20$

$0 = (x + 5)(x + 4)$

$x = -4$ or $x = -5$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

69.
$$y = x^3$$
$$y = x$$
$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x + 1)(x - 1) = 0$$
$$x = 0,\ x = -1, \text{ or } x = 1$$

The corresponding y-values are $y = 0$ (for $x = 0$), $y = -1$ (for $x = -1$), and $y = 1$ (for $x = 1$).

Points of intersection: $(0, 0)$, $(-1, -1)$, $(1, 1)$

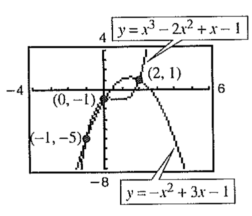
70.
$$y = x^3 - 4x$$
$$y = -(x + 2)$$
$$x^3 - 4x = -(x + 2)$$
$$x^3 - 3x + 2 = 0$$
$$(x - 1)^2(x + 2) = 0$$
$$x = 1 \text{ or } x = -2$$

The corresponding y-values are $y = -3$ (for $x = 1$) and $y = 0$ (for $x = -2$).

Points of intersection: $(1, -3)$, $(-2, 0)$

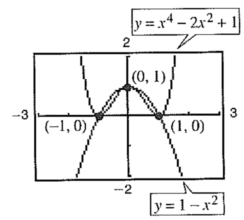
71. Analytically,
$$y = x^3 - 2x^2 + x - 1$$
$$y = -x^2 + 3x - 1$$
$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2.$$

Points of intersection: $(-1, -5)$, $(0, -1)$, $(2, 1)$



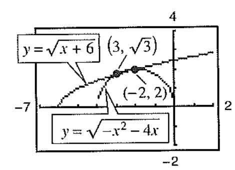
72. Analytically,
$$y = x^4 - 2x^2 + 1$$
$$y = 1 - x^2$$
$$1 - x^2 = x^4 - 2x^2 + 1$$
$$0 = x^4 - x^2$$
$$0 = x^2(x + 1)(x - 1)$$
$$x = -1, 0, 1.$$

Points of intersection: $(-1, 0)$, $(0, 1)$, $(1, 0)$



73. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$

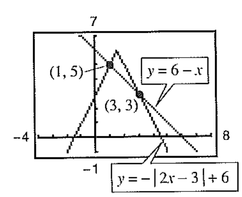


Points of intersection: $(-2, 2)$, $\left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically,
$$\sqrt{x + 6} = \sqrt{-x^2 - 4x}$$
$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$x = -3, -2.$$

74. $y = -|2x - 3| + 6$

$y = 6 - x$



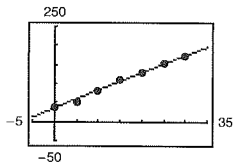
Points of intersection: $(3, 3)$, $(1, 5)$

Analytically, $-|2x - 3| + 6 = 6 - x$
$$|2x - 3| = x$$
$$2x - 3 = x \text{ or } 2x - 3 = -x$$
$$x = 3 \text{ or } \quad x = 1.$$

75. (a) Using a graphing utility, you obtain

$y = -0.027t^2 + 5.73t + 26.9.$

(b)



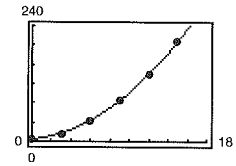
The model is a good fit for the data.

(c) For 2010, $t = 40$ and $y = 212.9.$

76. (a) Using a graphing utility, you obtain

$y = 0.77t^2 + 2.1t + 4$

(b)



The model is a good fit for the data.

(c) For 2015, $t = 25$ and $y \approx 538$ million subscribers.

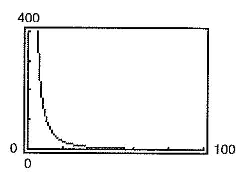
77.

$$\begin{aligned} C &= R \\ 5.5\sqrt{x} + 10{,}000 &= 3.29x \\ \left(5.5\sqrt{x}\right)^2 &= (3.29x - 10{,}000)^2 \\ 30.25x &= 10.8241x^2 - 65{,}800x + 100{,}000{,}000 \\ 0 &= 10.8241x^2 - 65{,}830.25x + 100{,}000{,}000 \quad \text{Use the Quadratic Formula.} \\ x &\approx 3133 \text{ units} \end{aligned}$$

The other root, $x \approx 2949$, does not satisfy the equation $R = C$.

This problem can also be solved by using a graphing utility and finding the intersection of the graphs of C and R.

78. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

79. Answers may vary. *Sample answer*:

$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

80. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

81. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

82. (a) v $\left[\text{Because } y = 3(-x)^2 + 3 = 3x^2 + 3\right]$

(b) i $\left[\text{Because } y = 3x^3 - 3x = 3x(x - 1)(x + 1) \text{ has } x\text{-intercepts } (0, 0), (1, 0), (-1, 0)\right]$

(c) None of the equations are symmetric with respect to the x-axis

(d) ii $\left[\text{Because } (-2 + 3)^2 = 1\right]$ and vi $\left[\text{Because } \sqrt{-2 + 3} = 1\right]$

(e) i $\left[\text{Because } 3(-x)^3 - 3(-x) = -3x^3 + 3x = -y\right]$ and iv $\left[\text{Because } \sqrt[3]{-x} = -\sqrt[3]{x} = -y\right]$

(f) i $\left[\text{Because } 3(0)^3 - 3(0) = 0\right]$ and iv $\left[\text{Because } \sqrt[3]{0} = 0\right]$

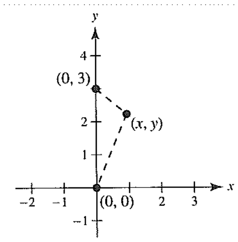
83. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

84. True. $f(4) = f(-4)$.

85. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

86. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

87.



$$2\sqrt{(x-0)^2 + (y-3)^2} = \sqrt{(x-0)^2 + (y-0)^2}$$
$$4\left[x^2 + (y-3)^2\right] = x^2 + y^2$$
$$4x^2 + 4y^2 - 24y + 36 = x^2 + y^2$$
$$3x^2 + 3y^2 - 24y + 36 = 0$$
$$x^2 + y^2 - 8y + 12 = 0$$
$$x^2 + (y-4)^2 = 4$$

Circle of radius 2 and center (0, 4).

88. Distance from the origin $= K \times$ Distance from $(2, 0)$

$$\sqrt{x^2 + y^2} = K\sqrt{(x-2)^2 + y^2},\ K \neq 1$$
$$x^2 + y^2 = K^2(x^2 - 4x + 4 + y^2)$$
$$(1 - K^2)x^2 + (1 - K^2)y^2 + 4K^2x - 4K^2 = 0$$

Note: This is the equation of a circle!

Section P.2 Linear Models and Rates of Change

1. $m = 1$

2. $m = 2$

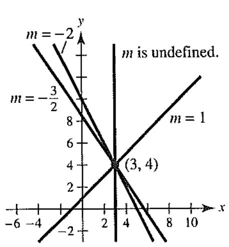
3. $m = 0$

4. $m = -1$

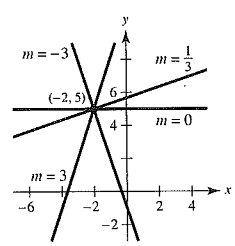
5. $m = -12$

6. $m = \frac{40}{3}$

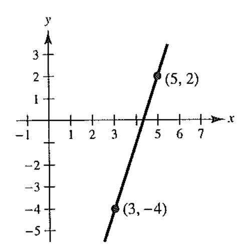
7.



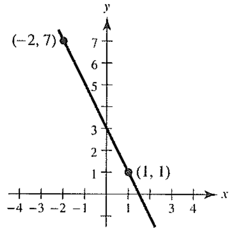
8.



9. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

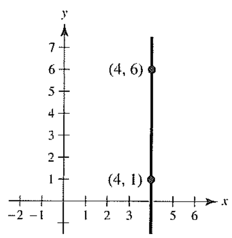


10. $m = \dfrac{7-1}{-2-1} = \dfrac{6}{-3} = -2$



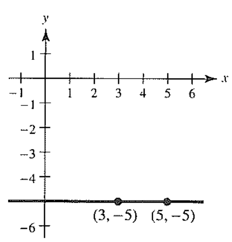
11. $m = \dfrac{1-6}{4-4} = \dfrac{-5}{0}$, undefined.

The line is vertical

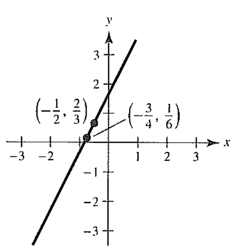


12. $m = \dfrac{-5-(-5)}{5-3} = \dfrac{0}{2} = 0$

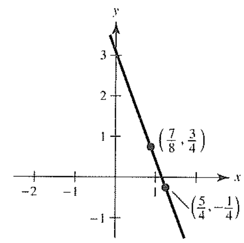
The line is horizontal



13. $m = \dfrac{\dfrac{2}{3} - \dfrac{1}{6}}{-\dfrac{1}{2} - \left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



14. $m = \dfrac{\left(\dfrac{3}{4}\right) - \left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right) - \left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



15. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2), (1, 2), (5, 2)$.

16. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0), (-4, 1), (-4, 2)$.

17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

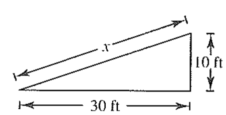
18. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

19. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

(b)



By the Pythagorean Theorem,

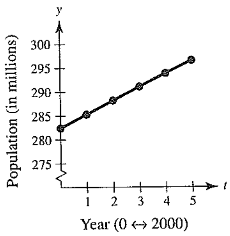
$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

20. (a) $m = 800$ indicates that the revenues increase by 800 in one day.

(b) $m = 250$ indicates that the revenues increase by 250 in one day.

(c) $m = 0$ indicates that the revenues do not change from one day to the next.

21. (a)

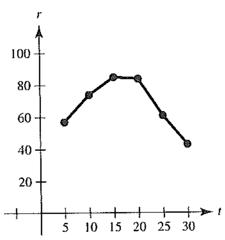


(b) The slopes are:

$$\frac{285.3 - 282.4}{1 - 0} = 2.9$$
$$\frac{288.2 - 285.3}{2 - 1} = 2.9$$
$$\frac{291.1 - 288.2}{3 - 2} = 2.9$$
$$\frac{293.9 - 291.1}{4 - 3} = 2.8$$
$$\frac{296.6 - 293.9}{5 - 4} = 2.7$$

The population increased least rapidly from 2004 to 2005.

22. (a)



(b) The slopes are:

$$\frac{74 - 57}{10 - 5} = 3.4$$
$$\frac{85 - 74}{15 - 10} = 2.2$$
$$\frac{84 - 85}{20 - 15} = -0.2$$
$$\frac{61 - 84}{25 - 20} = -4.6$$
$$\frac{43 - 61}{30 - 25} = -3.6$$

The rate changed most rapidly between 20 and 25 seconds. The change is -4.6 mi/h/sec.

23. $y = 4x - 3$

the slope is $m = 4$ and the y-intercept is $(0, -3)$.

24. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is $(0, 1)$.

25. $x + 5y = 20$

$y = -\frac{1}{5}x + 4$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

26. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

27. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

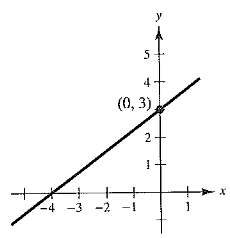
28. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

29. $y = \frac{3}{4}x + 3$

$4y = 3x + 12$

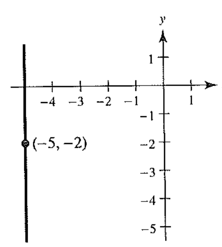
$0 = 3x - 4y + 12$



30. The slope is undefined so the line is vertical.

$x = -5$

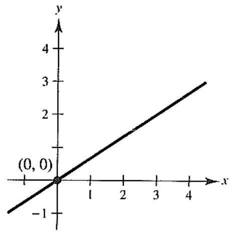
$x + 5 = 0$



31. $y = \frac{2}{3}x$

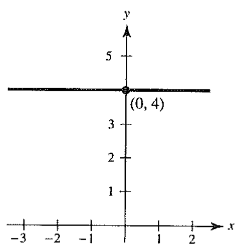
$3y = 2x$

$0 = 2x - 3y$



32. $y = 4$

$y - 4 = 0$

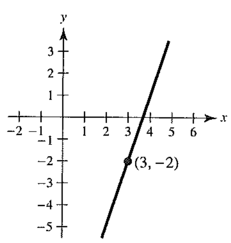


33. $y + 2 = 3(x - 3)$

$y + 2 = 3x - 9$

$y = 3x - 11$

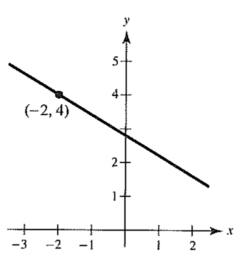
$0 = 3x - y - 11$



34. $y - 4 = -\frac{3}{5}(x + 2)$

$5y - 20 = -3x - 6$

$3x + 5y - 14 = 0$

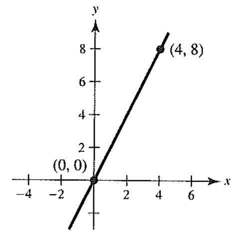


35. $m = \dfrac{8 - 0}{4 - 0} = 2$

$y - 0 = 2(x - 0)$

$y = 2x$

$0 = 2x - y$

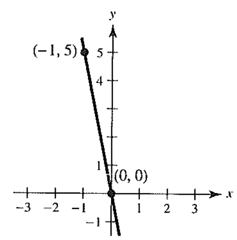


36. $m = \dfrac{5 - 0}{-1 - 0} = -5$

$y - 0 = -5(x - 0)$

$y = -5x$

$5x + y = 0$

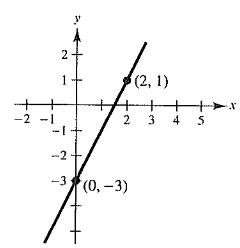


37. $m = \dfrac{1 - (-3)}{2 - 0} = 2$

$y - 1 = 2(x - 2)$

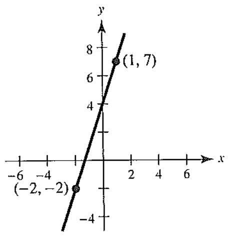
$y - 1 = 2x - 4$

$0 = 2x - y - 3$



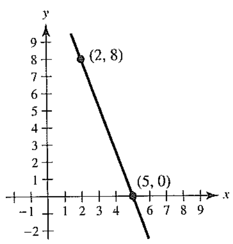
38. $m = \dfrac{7-(-2)}{1-(-2)} = \dfrac{9}{3} = 3$

$$y-(-2) = 3(x-(-2))$$
$$y+2 = 3(x+2)$$
$$y = 3x+4$$
$$0 = 3x-y+4$$



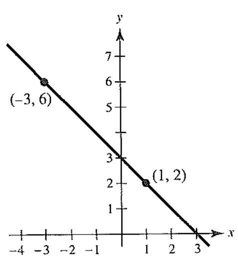
39. $m = \dfrac{8-0}{2-5} = -\dfrac{8}{3}$

$$y-0 = -\frac{8}{3}(x-5)$$
$$y = -\frac{8}{3}x+\frac{40}{3}$$
$$8x+3y-40 = 0$$



40. $m = \dfrac{6-2}{-3-1} = \dfrac{4}{-4} = -1$

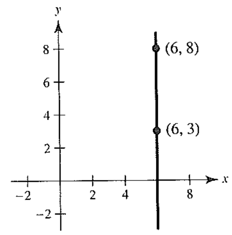
$$y-2 = -1(x-1)$$
$$y-2 = -x+1$$
$$x+y-3 = 0$$



41. $m = \dfrac{8-3}{6-6} = \dfrac{5}{0}$, undefined

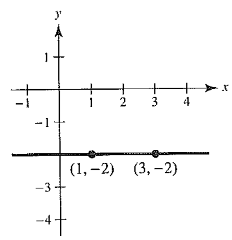
The line is horizontal.

$$x = 6$$
$$x-6 = 0$$



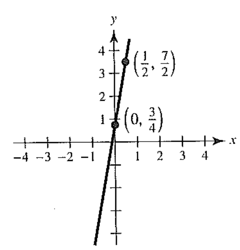
42. $m = \dfrac{-2-(-2)}{3-1} = \dfrac{0}{2} = 0$

$$y = -2$$
$$y+2 = 0$$



43. $m = \dfrac{\frac{7}{2}-\frac{3}{4}}{\frac{1}{2}-0} = \dfrac{\frac{11}{4}}{\frac{1}{2}} = \dfrac{11}{2}$

$$y-\frac{3}{4} = \frac{11}{2}(x-0)$$
$$y = \frac{11}{2}x+\frac{3}{4}$$
$$0 = 22x-4y+3$$

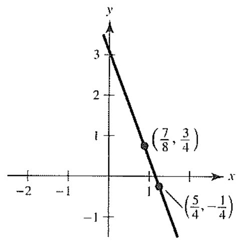


44. $m = \dfrac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \dfrac{1}{-\frac{3}{8}} = -\dfrac{8}{3}$

$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$

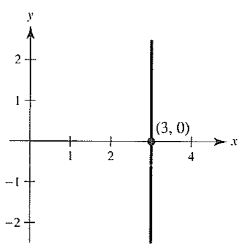
$$12y + 3 = -32x + 40$$

$$32x + 12y - 37 = 0$$



45. $x = 3$

$x - 3 = 0$

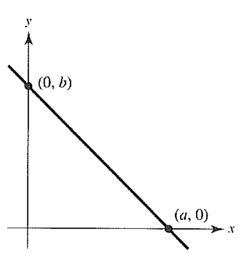


46. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



47. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

48. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$

$$3x + y = -2$$

$$3x + y + 2 = 0$$

49. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{1}{a} + \frac{2}{a} = 1$$

$$\frac{3}{a} = 1$$

$$a = 3 \Rightarrow x + y = 3$$

$$x + y - 3 = 0$$

50. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

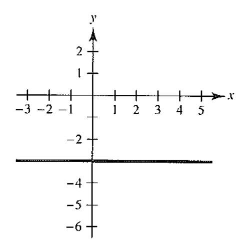
$$\frac{-3}{a} + \frac{4}{a} = 1$$

$$\frac{1}{a} = 1$$

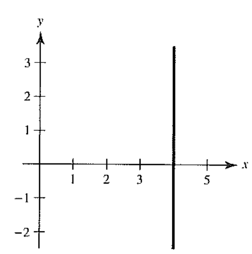
$$a = 1 \Rightarrow x + y = 1$$

$$x + y - 1 = 0$$

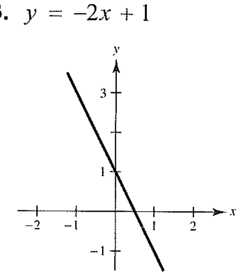
51. $y = -3$



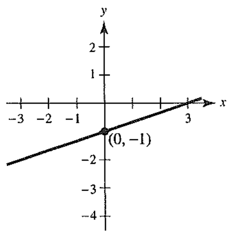
52. $x = 4$



53. $y = -2x + 1$

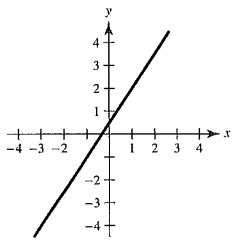


54. $y = \frac{1}{3}x - 1$



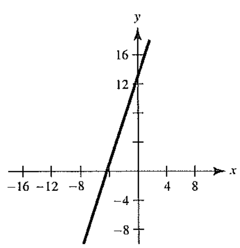
55. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



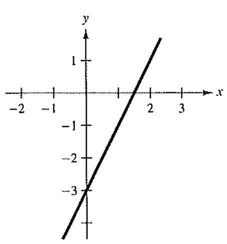
56. $y - 1 = 3(x + 4)$

$y = 3x + 13$



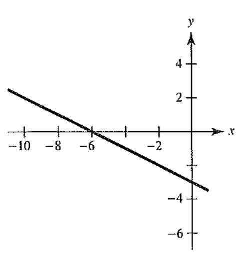
57. $2x - y - 3 = 0$

$y = 2x - 3$

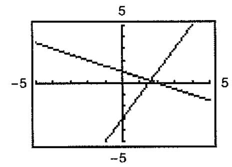


58. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

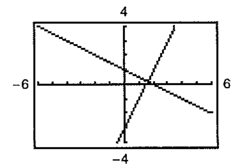


59. (a)



The lines do not appear perpendicular.

(b)



The lines appear perpendicular.

The lines are perpendicular because their slopes 2 and $-\frac{1}{2}$ are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular. Answers depend on calculator used.

60. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$-5x + 8y = 4$

$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$5x + 2y = 4$

$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$

(e) $a = \frac{5}{2}$ and $b = 3$.

$\frac{5}{2}x + 3y = 4$

$5x + 6y = 8$

61. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

62. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

63. $4x - 2y = 3$

$$y = 2x - \frac{3}{2}$$
$$m = 2$$

(a) $y - 1 = 2(x - 2)$

$$y - 1 = 2x - 4$$
$$0 = 2x - y - 3$$

(b) $y - 1 = -\frac{1}{2}(x - 2)$

$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$

64. $x + y = 7$

$$y = -x + 7$$
$$m = -1$$

(a) $y - 2 = -1(x + 3)$

$$y - 2 = -x - 3$$
$$x + y + 1 = 0$$

(b) $y - 2 = 1(x + 3)$

$$y - 2 = x + 3$$
$$0 = x - y + 5$$

65. $5x - 3y = 0$

$$y = \frac{5}{3}x$$
$$m = \frac{5}{3}$$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$$24y - 21 = 40x - 30$$
$$0 = 40x - 24y - 9$$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$$40y - 35 = -24x + 18$$
$$24x + 40y - 53 = 0$$

66. $3x + 4y = 7$

$$4y = -3x + 7$$
$$y = -\frac{3}{4}x + \frac{7}{4}$$
$$m = -\frac{3}{4}$$

(a) $y - (-5) = -\frac{3}{4}(x - 4)$

$$y + 5 = -\frac{3}{4}x + 3$$
$$4y + 20 = -3x + 12$$
$$3x + 4y + 8 = 0$$

(b) $y - (-5) = \frac{4}{3}(x - 4)$

$$y + 5 = \frac{4}{3}x - \frac{16}{3}$$
$$3y + 15 = 4x - 16$$
$$0 = 4x - 3y - 31$$

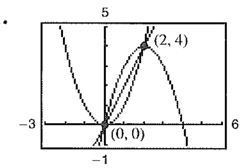
67. The slope is 250. $V = 1850$ when $t = 8$.
$V = 250(t - 8) + 1850 = 250t - 150$.

68. The slope is 4.5. $V = 156$ when $t = 4$.
$V = 4.5(t - 4) + 156 = 4.5t + 138$

69. The slope is -1600. $V = 17{,}200$ when $t = 8$.
$V = -1600(t - 8) + 17{,}200 = -1600t + 30{,}000$

70. The slope is -5600. $V = 245{,}000$ when $t = 4$.
$V = -5600(t - 4) + 245{,}000 = -5600t + 267{,}400$

71.



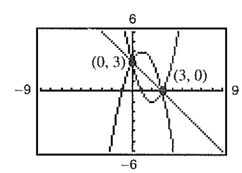
You can use the graphing utility to determine that the points of intersection are (0, 0) and (2, 4). Analytically,

$$x^2 = 4x - x^2$$
$$2x^2 - 4x = 0$$
$$2x(x - 2) = 0$$
$$x = 0 \Rightarrow y = 0 \Rightarrow (0, 0)$$
$$x = 2 \Rightarrow y = 4 \Rightarrow (2, 4).$$

The slope of the line joining (0, 0) and (2, 4) is $m = (4 - 0)/(2 - 0) = 2$. So, an equation of the line is

$$y - 0 = 2(x - 0)$$
$$y = 2x.$$

72. $y = x^2 - 4x + 3,\ y = -x^2 + 2x + 3$



You can use the graphing utility to determine that the points of intersection are (0, 3) and (3, 0). Analytically,

$$x^2 - 4x + 3 = -x^2 + 2x + 3$$
$$2x^2 - 6x = 0$$
$$2x(x - 3) = 0$$
$$x = 0 \Rightarrow y = 3 \Rightarrow (0, 3)$$
$$x = 3 \Rightarrow y = 0 \Rightarrow (3, 0).$$

The slope of the line joining (0, 3) and (3, 0) is $m = (0 - 3)/(3 - 0) = -1$. So, an equation of the line is

$$y - 3 = -1(x - 0)$$
$$y = -x + 3.$$

73. $m_1 = \dfrac{1-0}{-2-(-1)} = -1$

$m_2 = \dfrac{-2-0}{2-(-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

74. $m_1 = \dfrac{-6-4}{7-0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11-4}{-5-0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

75. Equations of perpendicular bisectors:

$$y - \frac{c}{2} = \frac{a-b}{c}\left(x - \frac{a+b}{2}\right)$$

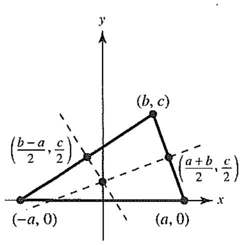
$$y - \frac{c}{2} = \frac{a+b}{-c}\left(x - \frac{b-a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2+b^2+c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.



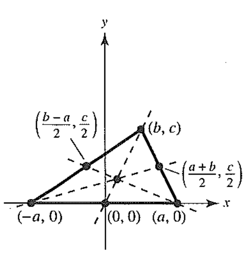
76. Equations of medians:

$$y = \frac{c}{b}x$$

$$y = \frac{c}{3a+b}(x+a)$$

$$y = \frac{c}{-3a+b}(x-a)$$

Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.



77. Equations of altitudes:

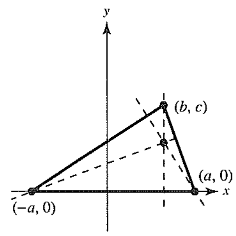
$$y = \frac{a-b}{c}(x+a)$$

$$x = b$$

$$y = -\frac{a+b}{c}(x-a)$$

Solving simultaneously, the point of intersection is

$$\left(b, \frac{a^2-b^2}{c}\right).$$



78. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2-b^2}{c}\right)$ is:

$$m_1 = \frac{\left[(a^2-b^2)/c\right] - (c/3)}{b - (b/3)}$$

$$= \frac{(3a^2-3b^2-c^2)/(3c)}{(2b)/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$ is:

$$m_2 = \frac{\left[(-a^2+b^2+c^2)/(2c)\right] - (c/3)}{0 - (b/3)}$$

$$= \frac{(-3a^2+3b^2+3c^2-2c^2)/(6c)}{-b/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

$m_1 = m_2$

Therefore, the points are collinear.

79. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \tfrac{180}{100} = \tfrac{9}{5}$$

$$F - 32 = \tfrac{9}{5}(C - 0)$$

$$F = \tfrac{9}{5}C + 32$$

or

$$C = \tfrac{1}{9}(5F - 160)$$

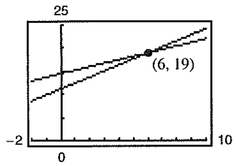
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

80. $C = 0.48x + 175$

For $x = 137$, $C = 0.48(137) + 175 = \$240.76$.

81. (a) $W_1 = 0.75x + 14.50$
$W_2 = 1.30x + 11.20$

(b) 

Using a graphing utility, the point of intersection is (6, 19)

Analytically,

$$W_1 = W_2$$
$$0.75x + 14.50 = 1.30x + 11.20$$
$$3.3 = 0.55x$$
$$6 = x$$
$$y = 1.30(6) + 11.20 = 19.$$

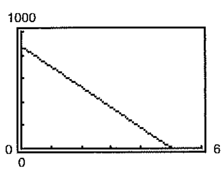
(c) When six units are produced, the wage for both options is $19.00 per hour. Choose option 1 if you think you will produce less than six units per hour, and choose option 2 if you think you will produce more than six.

82. (a) Depreciation per year:

$\frac{875}{5} = \$175$

$y = 875 - 175x$

where $0 \le x \le 5$.



(b) $y = 875 - 175(2) = \$525$

(c) $200 = 875 - 175x$
$175x = 675$
$x \approx 3.86$ years

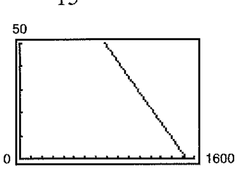
83. (a) Two points are (50, 780) and (47, 825).

The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$
$$p - 780 = -15(x - 50)$$
$$p = -15x + 750 + 780 = -15x + 1530$$

or

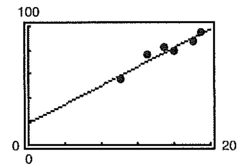
$$x = \frac{1}{15}(1530 - p)$$

(b) 

If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units.

84. (a) $y = 18.91 + 3.97x$
(x = quiz score, y = test score)

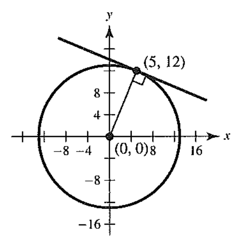
(b) 

(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

85. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).

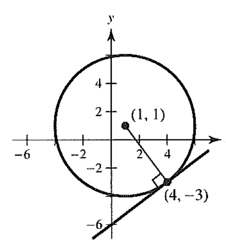


Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$
$$5x + 12y - 169 = 0.$$

86. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is $\frac{1 + 3}{1 - 4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$
$$0 = 3x - 4y - 24$$

87. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}}$

$$= \frac{10}{5} = 2$$

88. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

89. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}}$

$$= \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

90. $x + 1 = 0 \Rightarrow d = \dfrac{|1(6) + (0)(2) + 1|}{\sqrt{1^2 + 0^2}} = 7$

91. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

92. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

93. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$
$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$
$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$\left(\dfrac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \dfrac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right)$ point of intersection

The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$

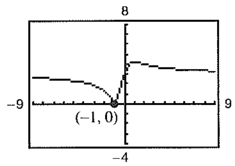
$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$

$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

94. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

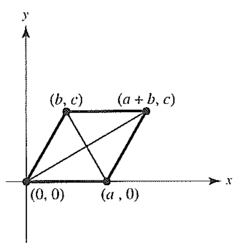
The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).



95. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure. The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$ and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



96. For simplicity, let the vertices of the quadrilateral be (0, 0), $(a, 0)$, (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

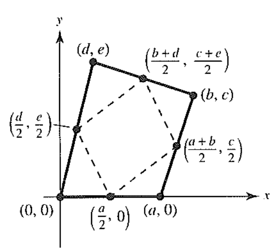
$$\left(\frac{a}{2}, 0\right), \left(\frac{a + b}{2}, \frac{c}{2}\right), \left(\frac{b + d}{2}, \frac{c + e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a + b}{2} - \frac{a}{2}} = \frac{\frac{c + e}{2} - \frac{e}{2}}{\frac{b + d}{2} - \frac{d}{2}} = \frac{c}{b}$$

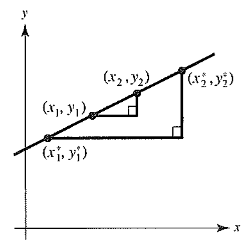
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c + e}{2}}{\frac{a + b}{2} - \frac{b + d}{2}} = -\frac{e}{a - d}$$

Therefore, the figure is a parallelogram.



97. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



98. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

99. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

100. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

Section P.3 Functions and Their Graphs

1. (a) Domain of f: $-4 \le x \le 4 \Rightarrow [-4, 4]$

Range of f: $-3 \le y \le 5 \Rightarrow [-3, 5]$

Domain of g: $-3 \le x \le 3 \Rightarrow [-3, 3]$

Range of g: $-4 \le y \le 4 \Rightarrow [-4, 4]$

(b) $f(-2) = -1$

$g(3) = -4$

(c) $f(x) = g(x)$ for $x = -1$

(d) $f(x) = 2$ for $x = 1$

(e) $g(x) = 0$ for $x = -1, 1$ and 2

2. (a) Domain of f: $-5 \le x \le 5 \Rightarrow [-5, 5]$

Range of f: $-4 \le y \le 4 \Rightarrow [-4, 4]$

Domain of g: $-4 \le x \le 5 \Rightarrow [-4, 5]$

Range of g: $-4 \le y \le 2 \Rightarrow [-4, 2]$

(b) $f(-2) = -2$

$g(3) = 2$

(c) $f(x) = g(x)$ for $x = -2$ and $x = 4$

(d) $f(x) = 2$ for $x = -4, 4$

(e) $g(x) = 0$ for $x = -1$

3. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

4. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(-8) = \sqrt{-8 + 5} = \sqrt{-3}$, undefined

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

5. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

6. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

7. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

8. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

9. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \ \Delta x \ne 0$$

10. $$\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3, \ x \ne 1$$

11. $$\frac{f(x) - f(2)}{x - 2} = \frac{\left(1/\sqrt{x - 1} - 1\right)}{x - 2}$$

$$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)} = \frac{-1}{\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)}, \ x \ne 2$$

12. $$\frac{f(x) - f(1)}{x - 1} = \frac{x^3 - x - 0}{x - 1} = \frac{x(x + 1)(x - 1)}{x - 1} = x(x + 1), \ x \ne 1$$

13. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

14. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$

$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

17. $f(t) = \sec\frac{\pi t}{4}$

$\frac{\pi t}{4} \neq \frac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n an integer

Range: $(-\infty, -1] \cup [1, \infty)$

18. $h(t) = \cot t$

Domain: all $t = n\pi$, n an integer

Range: $(-\infty, \infty)$

19. $f(x) = \frac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

20. $g(x) = \frac{2}{x - 1}$

Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

21. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

22. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x - 2)(x - 1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

23. $g(x) = \frac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

24. $h(x) = \frac{1}{\sin x - (1/2)}$

$\sin x - \frac{1}{2} \neq 0$

$\sin x \neq \frac{1}{2}$

Domain: all $x \neq \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$, n integer

25. $f(x) = \frac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

26. $g(x) = \frac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x - 2)(x + 2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

28. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \ge 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \le 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

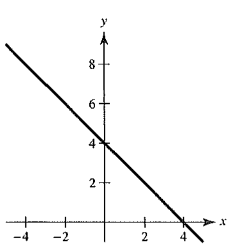
Domain: $[-4, \infty)$

Range: $[0, \infty)$

31. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

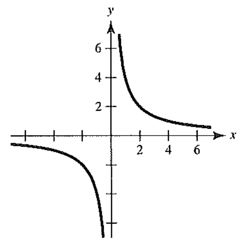
Range: $(-\infty, \infty)$



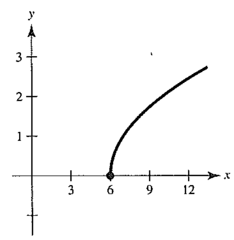
32. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



33. $h(x) = \sqrt{x - 6}$

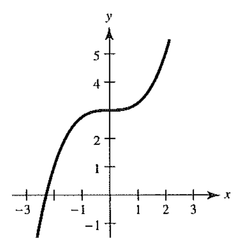


Domain: $x - 6 \ge 0$

$x \ge 6 \Rightarrow [6, \infty)$

Range: $[0, \infty)$

34. $f(x) = \frac{1}{4}x^3 + 3$



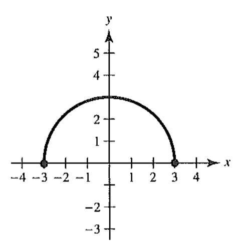
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

35. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



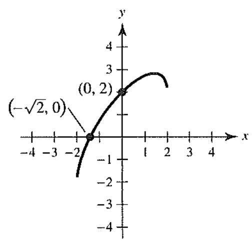
36. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

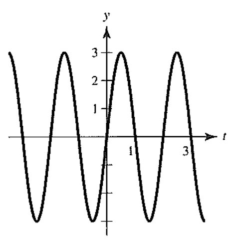
Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $\left(-\sqrt{2}, 0\right)$



37. $g(t) = 3 \sin \pi t$



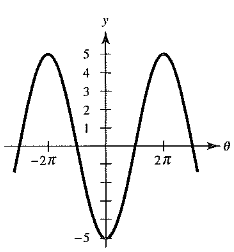
Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

38. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

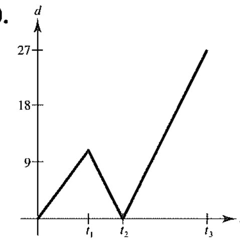
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



39. The student travels $\dfrac{2 - 0}{4 - 0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the next 2 minutes. Finally, the student travels $\dfrac{6 - 2}{10 - 6} = 1$ mi/min during the final 4 minutes.

40.



41. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

42. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

43. y is a function of x. Vertical lines intersect the graph at most once.

44. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

45. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

46. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

47. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

48. $x^2 y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

49. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

50. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

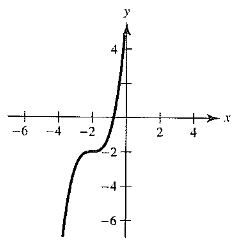
51. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

52. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

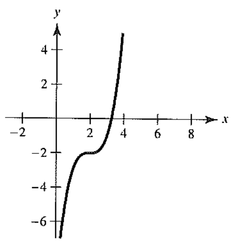
53. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

54. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

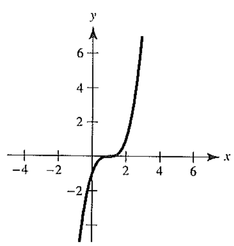
55. (a) the graph is shifted 3 units to the left.



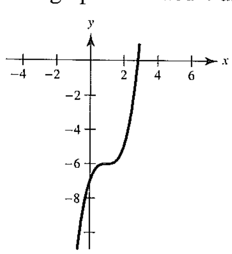
(b) The graph is shifted 1 unit to the right.



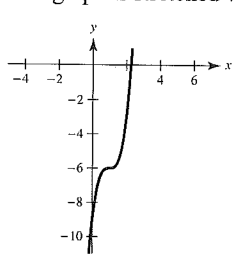
(c) The graph is shifted 2 units upward.



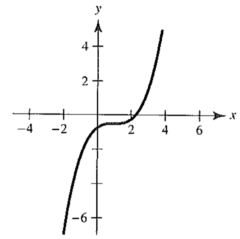
(d) The graph is shifted 4 units downward.



(e) The graph is stretched vertically by a factor of 3.



(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.

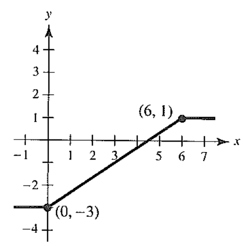


56. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

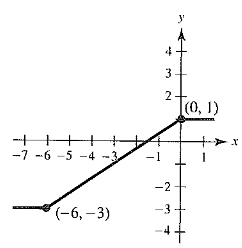
$g(0) = f(-4) = -3$

Shift f right 4 units



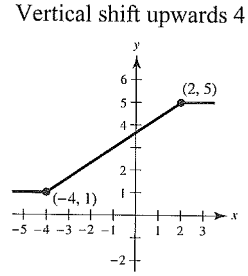
(b) $g(x) = f(x + 2)$

Shift f left 2 units



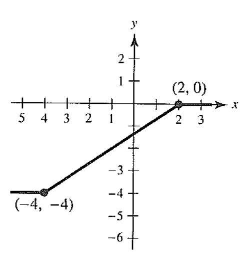
(c) $g(x) = f(x) + 4$

Vertical shift upwards 4 units



(d) $g(x) = f(x) - 1$

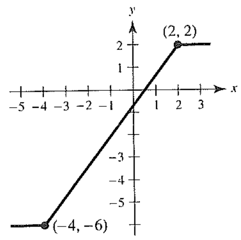
Vertical shift down 1 unit



(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

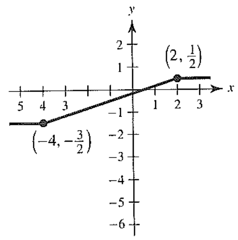
$g(-4) = 2f(-4) = -6$



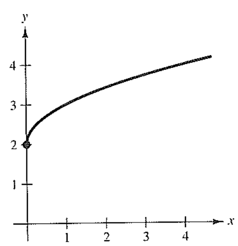
(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

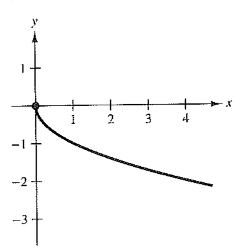


57. (a) $y = \sqrt{x} + 2$



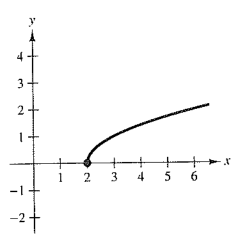
Vertical shift 2 units upward

(b) $y = -\sqrt{x}$



Reflection about the x-axis

(c) $y = \sqrt{x - 2}$



Horizontal shift 2 units to the right

58. (a) $h(x) = \sin(x + (\pi/2)) + 1$ is a horizontal shift $\pi/2$ units to the left, followed by a vertical shift 1 unit upwards.

(b) $h(x) = -\sin(x - 1)$ is a horizontal shift 1 unit to the right followed by a reflection about the x-axis.

59. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

60. $f(x) = \sin x, \ g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\frac{\pi}{4}\right)\right) = g\left(\sin\left(\frac{\pi}{4}\right)\right)$

$= g\left(\frac{\sqrt{2}}{2}\right) = \pi\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

61. $f(x) = x^2, \ g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x, \ x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

62. $f(x) = x^2 - 1,\ g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

63. $f(x) = \dfrac{3}{x},\ g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

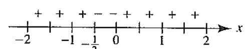
No, $f \circ g \neq g \circ f$.

64. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

65. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

66. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

67. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x,\ g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

68. $F(x) = -4 \sin(1 - x)$

Let $f(x) = -4x,\ g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4 \sin(1 - x) = F(x)$.

[Other answers possible]

69. $f(-x) = (-x)^2\left(4 - (-x)^2\right) = x^2(4 - x^2) = f(x)$

Even

70. $f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)$

Odd

71. $f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

Odd

72. $f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

Even

73. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

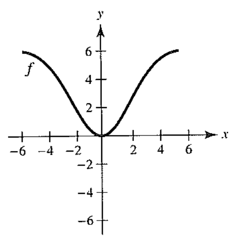
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

74. (a) If f is even, then $(-4, 9)$ is on the graph.

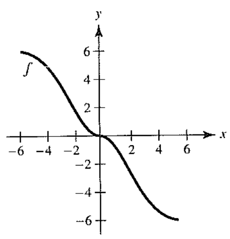
(b) If f is odd, then $(-4, -9)$ is on the graph.

75. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

76. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.

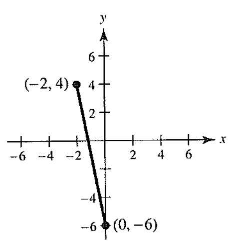


77. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

$$y - 4 = -5(x - (-2))$$
$$y - 4 = -5x - 10$$
$$y = -5x - 6$$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, -2 \le x \le 0$

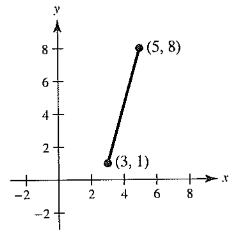


78. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

$$y - 1 = \frac{7}{2}(x - 3)$$
$$y - 1 = \frac{7}{2}x - \frac{21}{2}$$
$$y = \frac{7}{2}x - \frac{19}{2}$$

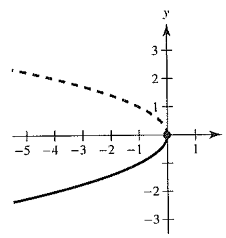
For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, 3 \le x \le 5$



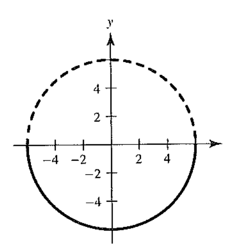
79. $x + y^2 = 0$

$$y^2 = -x$$
$$y = -\sqrt{-x}$$
$$f(x) = -\sqrt{-x}, x \le 0$$

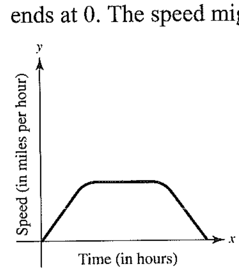


80. $x^2 + y^2 = 36$

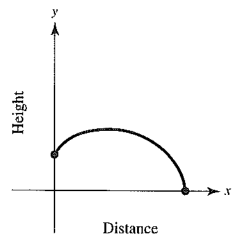
$$y^2 = 36 - x^2$$
$$y = -\sqrt{36 - x^2}, -6 \le x \le 6$$



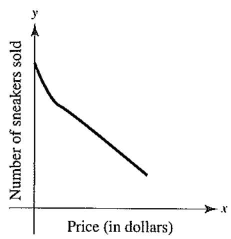
81. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:



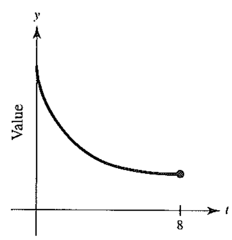
82. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



83. Answers will vary. *Sample answer*: In general , as the price decreases, the store will sell more.



84. Answers will vary. *Sample answer*: As time goes on, the value of the car will decrease



85.

$$y = \sqrt{c - x^2}$$
$$y^2 = c - x^2$$
$$x^2 + y^2 = c, \text{ a circle.}$$

For the domain to be $[-5, 5]$, $c = 25$.

86. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \neq 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$
$$9c^2 < 24$$
$$c^2 < \tfrac{8}{3}$$
$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$
$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

87. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

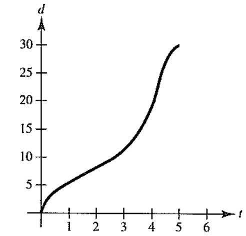
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

88. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

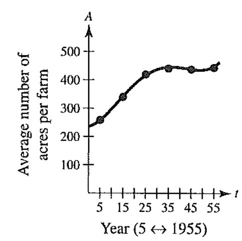
Range: $0 \le d \le 30$

(c)



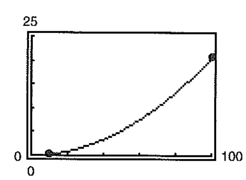
(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

89. (a)



(b) $A(20) \approx 384$ acres/farm

90. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.002\left(\dfrac{x}{1.6}\right)^2 + 0.005\left(\dfrac{x}{1.6}\right) - 0.029$

$= 0.00078125x^2 + 0.003125x - 0.029$

91. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x \le 0 \\ 2, & 0 < x < 2. \\ 2x - 2, & x \ge 2 \end{cases}$$

92. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

93.
$$\begin{aligned} f(-x) &= a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) \\ &= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right] \\ &= -f(x) \end{aligned}$$

Odd

94.
$$\begin{aligned} f(-x) &= a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 \\ &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\ &= f(x) \end{aligned}$$

Even

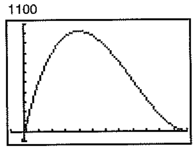
95. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.
So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then
$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.
So, $F(x)$ is even.

96. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then
$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.
So, $F(x)$ is odd.

97. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

(c)

x	length and width	volume
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

98. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

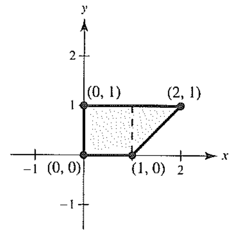
99. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

100. True

101. True. The function is even.

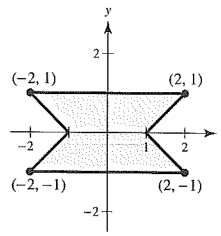
102. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

103. First consider the portion of R in the first quadrant: $x \geq 0,\ 0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

104. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Fitting Models to Data

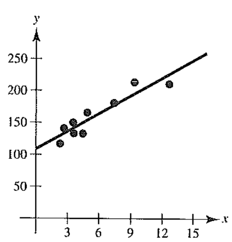
1. Trigonometric function

2. Quadratic function

3. No relationship

4. Linear function

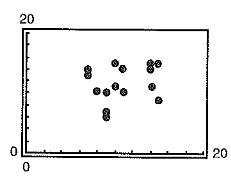
5. (a), (b)



Yes. The cancer mortality increases linearly with increased exposure to the carcinogenic substance.

(c) If $x = 3$, then $y \approx 136$.

6. (a)

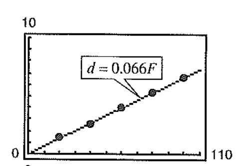


No, the relationship does not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, etc. These variables may change from one quiz to the next.

7. (a) $d = 0.066F$

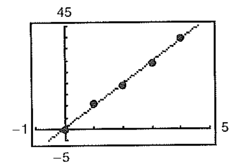
(b)



The model fits the data well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

8. (a) $s = 9.7t + 0.4$

(b)


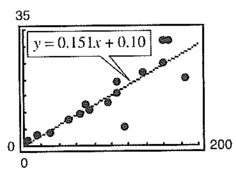
The model fits the data well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

9. (a) Using a graphing utility,

$y = 0.151x + 0.10$

The correlation coefficient is $r \approx 0.880$.

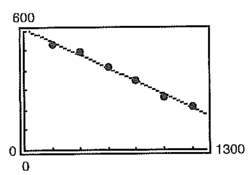
(b)


(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national product. The four countries that differ most from the linear model are Venezuela, South Korea, Hong Kong and United Kingdom.

(d) Using a graphing utility,

$y = 0.155x + 0.22$ and $r \approx 0.984$.

10. (a) Linear model: $H = -0.3323t + 612.9333$

(b)


The model fits the data well.

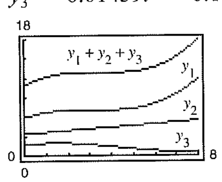
(c) When $t = 500$,

$H = -0.3323(500) + 612.9333 \approx 446.78$.

11. (a) $y_1 = 0.04040t^3 - 0.3695t^2 + 1.123t + 5.88$

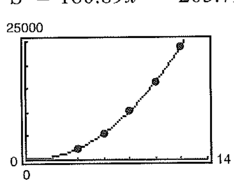
$y_2 = 0.264t + 3.35$

$y_3 = 0.01439t^3 - 0.1886t^2 + 0.476t + 1.59$

(b)


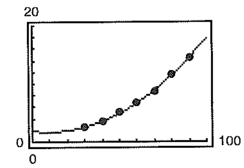
For year 12, $y_1 + y_2 + y_3 \approx 47.5$ cents/mile.

12. (a) $S = 180.89x^2 - 205.79x + 272$

(b)


(c) When $x = 2$, $S \approx 583.98$ pounds.

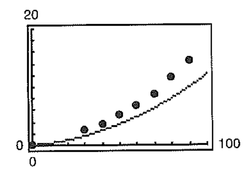
13. (a) $t = 0.002s^2 - 0.04s + 1.9$

(b)


(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same.

(d) Adding (0, 0) to the data produces

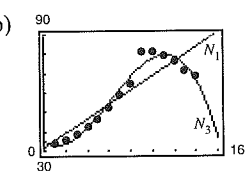
$t = 0.002s^2 + 0.02s + 0.1$



(e) No. From the graph in part (b), you can see that the model from part (a) follows the data more closely than the model from part (d).

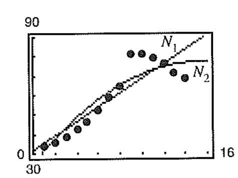
14. (a) $N_1 = 3.72t + 31.6$

$N_3 = -0.0932t^3 + 1.735t^2 - 3.77t + 35.1$

(b)


(c) The cubic model is better.

(d) $N_2 = -0.221t^2 + 6.81t + 24.9$

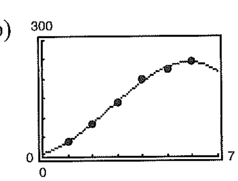


The model does not fit the data well.

(e) For 2007, $t = 17$, and $N_1 \approx 94.8$ million and $N_3 \approx 14.5$ million. Neither seem accurate. The linear model's estimate is too high and the cubic model's estimate is too low.

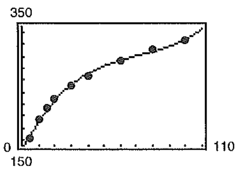
(f) Answers will vary

15. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b)


(c) If $x = 4.5$, $y \approx 214$ horsepower.

16. (a) $T = 2.9856 \times 10^{-4}p^3 - 0.0641p^2 + 5.282p + 143.1$

(b) 

(c) For $T = 300°F$, $p \approx 68.29$ lb/in.^{2}.

(d) The model is based on data up to 100 pounds per square inch.

17. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

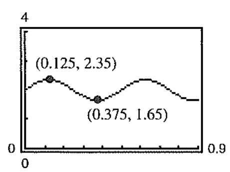
(b) The amplitude is approximately

$(2.35 - 1.65)/2 = 0.35.$

The period is approximately

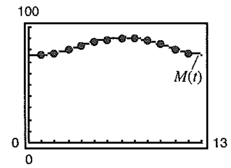
$2(0.375 - 0.125) = 0.5.$

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

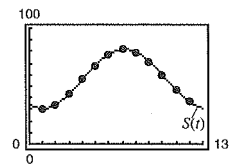
(d) 

The model appears to fit the data.

18. (a) $S(t) = 56.37 + 25.47 \sin(0.5080t - 2.07)$

(b) 

The model is a good fit.

(c) 

The model is a good fit.

(d) The average is the constant term in each model. 83.70°F for Miami and 56.37°F for Syracuse.

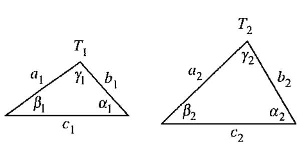
(e) The period for Miami is $2\pi/0.4912 \approx 12.8$. The period for Syracuse is $2\pi/0.5080 \approx 12.4$. In both cases the period is approximately 12, or one year.

(f) Syracuse has greater variability because $25.47 > 7.46$.

19. Answers will vary.

20. Answers will vary.

21. Yes, $A_1 \le A_2$. To see this, consider the two triangles of areas A_1 and A_2:



For $i = 1, 2$, the angles satisfy $\alpha i + \beta i + \gamma i = \pi$. At least one of $\alpha_1 \le \alpha_2$, $\beta_1 \le \beta_2$, $\gamma_1 \le \gamma_2$ must hold. Assume $\alpha_1 \le \alpha_2$. Because $\alpha_2 \le \pi/2$ (acute triangle), and the sine function increases on $[0, \pi/2]$, you have

$$\begin{aligned} A_1 &= \tfrac{1}{2}b_1c_1 \sin \alpha_1 \le \tfrac{1}{2} b_2c_2 \sin \alpha_1 \\ &\le \tfrac{1}{2}b_2c_2 \sin \alpha_2 = A_2 \end{aligned}$$

Review Exercises for Chapter P

1. $y = 5x - 8$

$x = 0$: $y = 5(0) - 8 = -8 \Rightarrow (0, -8)$ y-intercept

$y = 0$: $0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$ x-intercept

2. $y = (x - 2)(x - 6)$

$x = 0$: $y = (0 - 2)(0 - 6) = 12 \Rightarrow (0, 12)$ y-intercept

$y = 0$: $0 = (x - 2)(x - 6) \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$ x-intercepts

3. $y = \dfrac{x-3}{x-4}$

$x = 0\text{: } y = \dfrac{0-3}{0-4} = \dfrac{3}{4} \Rightarrow \left(0, \dfrac{3}{4}\right)$ y-intercept

$y = 0\text{: } 0 = \dfrac{x-3}{x-4} \Rightarrow x = 3 \Rightarrow (3, 0)$ x-intercept.

4. $xy = 4$

$x = 0$ and $y = 0$ are both impossible. No intercepts.

5. Symmetric with respect to y-axis because

$$(-x)^2 y - (-x)^2 + 4y = 0$$
$$x^2 y - x^2 + 4y = 0.$$

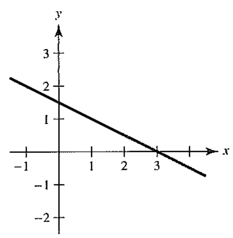
6. Symmetric with respect to y-axis because

$$y = (-x)^4 - (-x)^2 + 3$$
$$y = x^4 - x^2 + 3.$$

7. $y = -\frac{1}{2}x + \frac{3}{2}$

Slope: $-\frac{1}{2}$

y-intercept: $\frac{3}{2}$

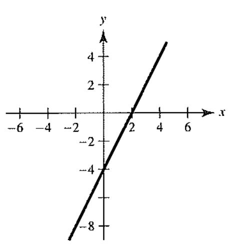


8. $6x - 3y = 12$

$$-3y = -6x + 12$$
$$y = 2x - 4$$

Slope: 2

y-intercept: $(0, -4)$

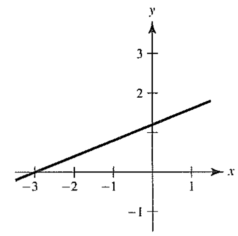


9. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$$-\tfrac{2}{5}x + y = \tfrac{6}{5}$$
$$y = \tfrac{2}{5}x + \tfrac{6}{5}$$

Slope: $\frac{2}{5}$

y-intercept: $\frac{6}{5}$

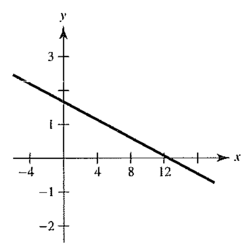


10. $0.02x + 0.15y = 0.25$

$$2x + 15y = 25$$
$$y = -\tfrac{2}{15}x + \tfrac{5}{3}$$

Slope: $-\frac{2}{15}$

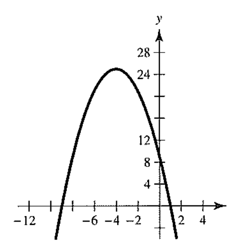
y-intercept: $\left(0, \frac{5}{3}\right)$



11. $y = 9 - 8x - x^2 = -(x-1)(x+9)$

y-intercept: $(0, 9)$

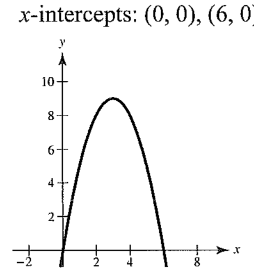
x-intercepts: $(1, 0), (-9, 0)$



12. $y = x(6 - x)$

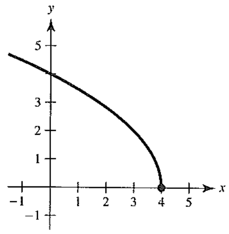
y-intercept: $(0, 0)$

x-intercepts: $(0, 0), (6, 0)$

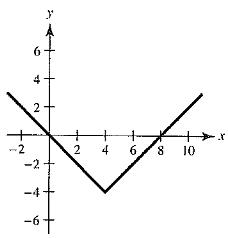


13. $y = 2\sqrt{4 - x}$

Domain: $(-\infty, 4]$



14. $y = |x - 4| - 4$



15. $y = 4x^2 - 25$

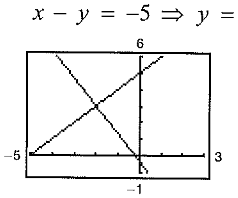
Xmin = −5
Xmax = 5
Xscl = 1
Ymin = −30
Ymax = 10
Yscl = 5

16. $y = 8\sqrt[3]{x - 6}$

Xmin = −40
Xmax = 40
Xscl = 10
Ymin = −40
Ymax = 40
Yscl = 10

17. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$



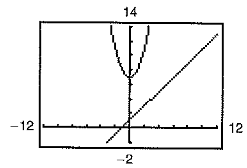
Using a graphing utility, the lines intersect at $(-2, 3)$. Analytically,

$$\frac{1}{3}(-5x - 1) = x + 5$$
$$-5x - 1 = 3x + 15$$
$$-16 = 8x$$
$$-2 = x.$$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

18. $x - y + 1 = 0 \Rightarrow y = x + 1$

$y - x^2 = 7 \Rightarrow y = x^2 + 7$



$$y = x + 1$$
$$(x + 1) - x^2 = 7$$
$$0 = x^2 - x + 6$$

No real solution.

No points of intersection.

The graphs of $y = x + 1$ and $y = x^2 + 7$ do not intersect.

19. Answers will vary. *Sample answer*:

You need factors $(x + 4)$ and $(x - 4)$.

Multiply by x to obtain origin symmetry.

$y = x(x + 4)(x - 4) = x^3 - 16x$

20. $y = kx^3$

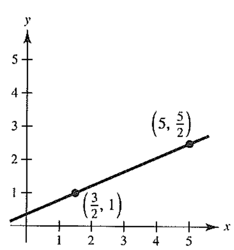
(a) $4 = k(1)^3 \Rightarrow k = 4$ and $y = 4x^3$

(b) $1 = k(-2)^3 \Rightarrow k = -\frac{1}{8}$ and $y = -\frac{1}{8}x^3$

(c) $0 = k(0)^3 \Rightarrow$ and k will do!

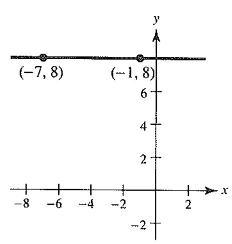
(d) $-1 = k(-1)^3 \Rightarrow k = 1 \Rightarrow y = x^3$

21.



$$\text{Slope} = \frac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$$

22. The line is horizontal and has slope 0.



23. $\dfrac{t-5}{0-(-8)} = \dfrac{-1-5}{2-(-8)}$

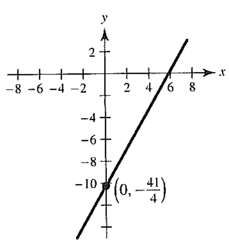
$$\frac{t-5}{8} = \frac{-6}{10}$$
$$\frac{t-5}{8} = -\frac{3}{5}$$
$$5t - 25 = -24$$
$$5t = 1$$
$$t = \frac{1}{5}$$

24. $\dfrac{3-(-1)}{-3-t} = \dfrac{3-6}{-3-8}$

$$\frac{4}{-3-t} = \frac{-3}{-11}$$
$$-44 = 9 + 3t$$
$$-53 = 3t$$
$$t = -\frac{53}{3}$$

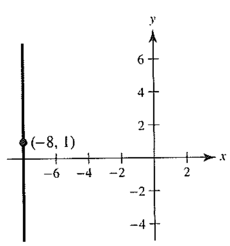
25. $y - (-5) = \frac{7}{4}(x - 3)$

$$y + 5 = \tfrac{7}{4}x - \tfrac{21}{4}$$
$$4y + 20 = 7x - 21$$
$$0 = 7x - 4y - 41$$



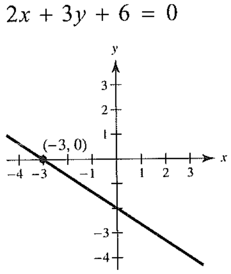
26. Because m is undefined the line is vertical.

$x = -8$ or $x + 8 = 0$



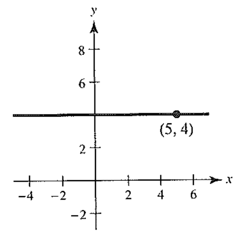
27. $y - 0 = -\frac{2}{3}(x - (-3))$

$$y = -\tfrac{2}{3}x - 2$$
$$2x + 3y + 6 = 0$$



28. Because $m = 0$, the line is horizontal.

$$y - 4 = 0(x - 5)$$
$$y = 4 \text{ or } y - 4 = 0$$



29. (a) $y - 5 = \frac{7}{16}(x + 3)$

$$16y - 80 = 7x + 21$$
$$0 = 7x - 16y + 101$$

(b) $5x - 3y = 3$ has slope $\frac{5}{3}$.

$$y - 5 = \tfrac{5}{3}(x + 3)$$
$$3y - 15 = 5x + 15$$
$$0 = 5x - 3y + 30$$

(c) $m = \dfrac{5-0}{-3-0} = -\dfrac{5}{3}$

$$y - 5 = -\tfrac{5}{3}(x + 3)$$
$$3y - 15 = -5x - 15$$
$$5x + 3y = 0$$

(d) Slope is undefined so the line is vertical.

$$x = -3$$
$$x + 3 = 0$$

30. (a) $y - 4 = -\frac{2}{3}(x - 2)$

$$3y - 12 = -2x + 4$$
$$2x + 3y - 16 = 0$$

(b) $x + y = 0$ has slope -1. Slope of the perpendicular line is 1.

$$y - 4 = 1(x - 2)$$
$$y = x + 2$$
$$0 = x - y + 2$$

(c) $m = \dfrac{4-1}{2-6} = -\dfrac{3}{4}$

$$y - 4 = -\frac{3}{4}(x - 2)$$
$$4y - 16 = -3x + 6$$
$$3x + 4y - 22 = 0$$

(d) Because the line is horizontal the slope is 0.

$$y = 4$$
$$y - 4 = 0$$

31. The slope is -850.

$V = -850t + 12{,}500.$

$V(3) = -850(3) + 12{,}500 = \$9950$

32. (a) $C = 9.25t + 13.50t + 36{,}500 = 22.75t + 36{,}500$

(b) $R = 30t$

(c) $30t = 22.75t + 36{,}500$

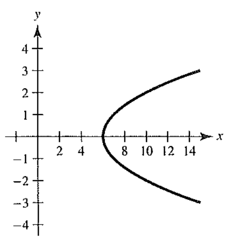
$7.25t = 36{,}500$

$t \approx 5034.48$ hours to break even

33. $x - y^2 = 6$

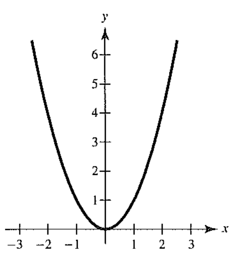
$y = \pm\sqrt{x - 6}$

Not a function because there are two values of y for some x.



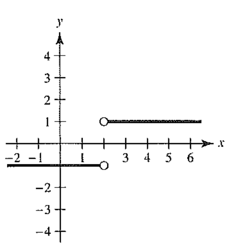
34. $x^2 - y = 0$

Function of x because there is one value for y for each x.



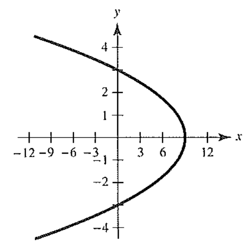
35. $y = \dfrac{|x - 2|}{x - 2}$

y is a function of x because there is one value of y for each x.



36. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



37. $f(x) = \dfrac{1}{x}$

(a) $f(0)$ does not exist.

(b) $$\frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{\frac{1}{1 + \Delta x} - \frac{1}{1}}{\Delta x} = \frac{1 - 1 - \Delta x}{(1 + \Delta x)\Delta x}$$

$$= \frac{-1}{1 + \Delta x}, \ \Delta x \neq -1, 0$$

38. $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ |x - 2|, & x \geq 0 \end{cases}$

(a) $f(-4) = (-4)^2 + 2 = 18$ (because $-4 < 0$)

(b) $f(0) = |0 - 2| = 2$

(c) $f(1) = |1 - 2| = 1$

39. (a) Domain: $36 - x^2 \geq 0 \Rightarrow -6 \leq x \leq 6$ or $[-6, 6]$

Range: $[0, 6]$

(b) Domain: all $x \neq 5$ or $(-\infty, 5) \cup (5, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

(c) Domain: all x or $(-\infty, \infty)$

Range: all y or $(-\infty, \infty)$

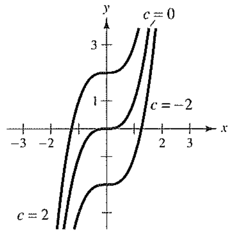
40. $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(a) $f(x) - g(x) = (1 - x^2) - (2x + 1) = -x^2 - 2x$

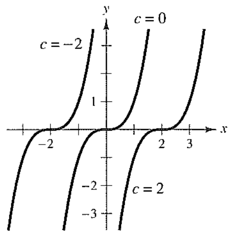
(b) $f(x)g(x) = (1 - x^2)(2x + 1) = -2x^3 - x^2 + 2x + 1$

(c) $g(f(x)) = g(1 - x^2) = 2(1 - x^2) + 1 = 3 - 2x^2$

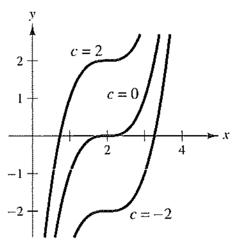
41. (a) $f(x) = x^3 + c,\ c = -2, 0, 2$



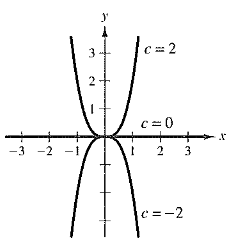
(b) $f(x) = (x - c)^3,\ c = -2, 0, 2$



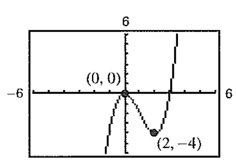
(c) $f(x) = (x - 2)^3 + c,\ c = -2, 0, 2$



(d) $f(x) = cx^3,\ c = -2, 0, 2$



42. $f(x) = x^3 - 3x^2$



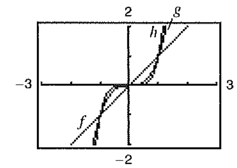
(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$$g(x) = -[f(x) - 1] = -x^3 + 3x^2 + 1$$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

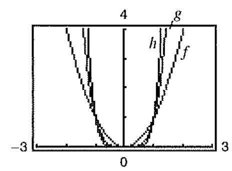
$$g(x) = f(x - 2) + 1 = (x - 2)^3 - 3(x - 2)^2 + 1$$

43. (a) Odd powers: $f(x) = x,\ g(x) = x^3,\ h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points (0, 0), (1, 1), and $(-1, -1)$ and are symmetric with respect to the origin.

Even powers: $f(x) = x^2,\ g(x) = x^4,\ h(x) = x^6$



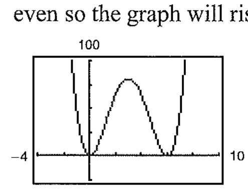
The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$ and are symmetric with respect to the y-axis.

All of the graphs, even and odd, pass through the origin. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply. $y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

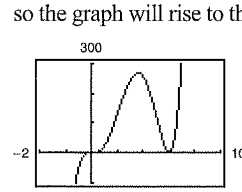
44. (a) $f(x) = x^2(x - 6)^2$

The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.



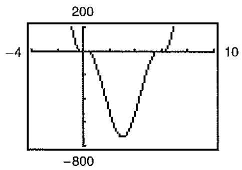
(b) $g(x) = x^3(x - 6)^2$

The leading coefficient is positive and the degree is odd so the graph will rise to the right and fall to the left.

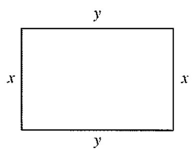


(c) $h(x) = x^3(x - 6)^3$

The leading coefficient is positive and the degree is even so the graph will rise to the left and to the right.

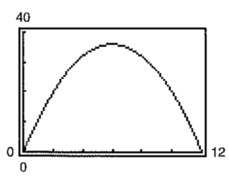


45. (a)



$$2x + 2y = 24$$
$$y = 12 - x$$
$$A = xy = x(12 - x)$$

(b) Domain: $0 < x < 12$ or $(0, 12)$



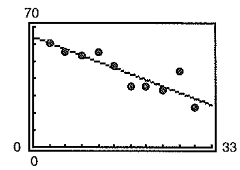
(c) Maximum area is $A = 36$ in.^{2}. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

46. For company (a) the profit rose rapidly for the first year, and then leveled off. For the second company (b), the profit dropped, and then rose again later.

47. (a) 3 (cubic), negative leading coefficient
(b) 4 (quartic), positive leading coefficient
(c) 2 (quadratic), negative leading coefficient
(d) 5, positive leading coefficient

48. (a) $y = -1.204x + 64.2667$

(b)



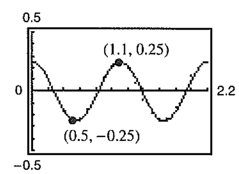
(c) The data point (27, 44) is probably an error. Without this point, the new model is
$y = -1.4344x + 66.4387$.

49. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $(0.25 - (-0.25))/2 = 0.25$. The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)



The model appears to fit the data.

Problem Solving for Chapter P

1. (a)
$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4); Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\frac{4}{3}$. Slope of tangent line is $-\frac{3}{4}$. So, $y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x$ Tangent line

(c) Slope of line from (6, 0) to (3, 4) is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. So, $y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2}$ Tangent line

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point (0, 1). Because the center of the circle is at $(0, -1)$ and the radius is 1 you have the following.

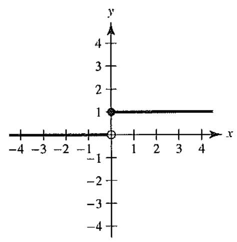
$$x^2 + (y+1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

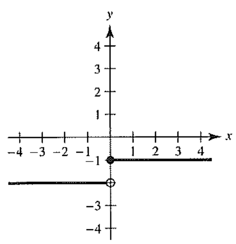
$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

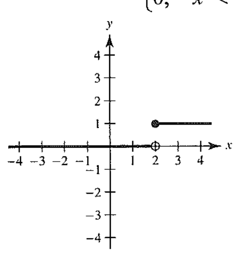
3. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



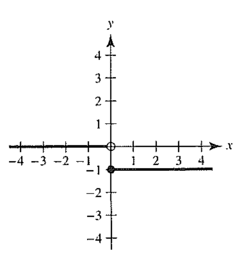
(a) $H(x) - 2 = \begin{cases} -1, & x \ge 0 \\ -2, & x < 0 \end{cases}$



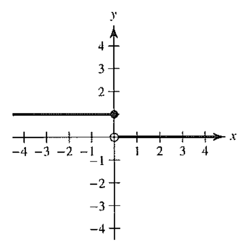
(b) $H(x - 2) = \begin{cases} 1, & x \ge 2 \\ 0, & x < 2 \end{cases}$



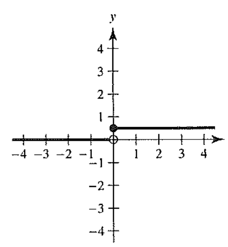
(c) $-H(x) = \begin{cases} -1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



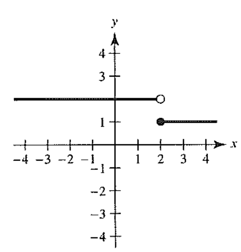
(d) $H(-x) = \begin{cases} 1, & x \le 0 \\ 0, & x > 0 \end{cases}$



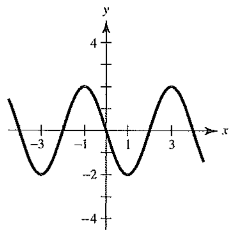
(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \ge 0 \\ 0, & x < 0 \end{cases}$



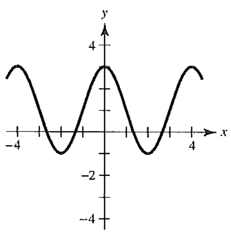
(f) $-H(x - 2) + 2 = \begin{cases} 1, & x \ge 2 \\ 2, & x < 2 \end{cases}$



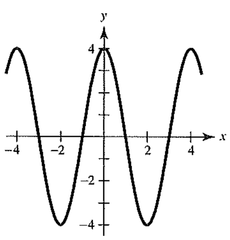
4. (a) $f(x+1)$



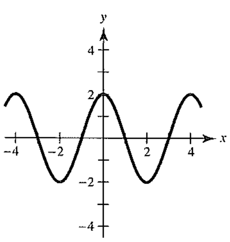
(b) $f(x) + 1$



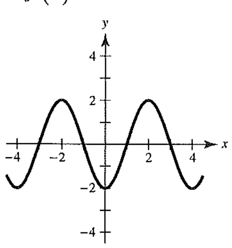
(c) $2f(x)$



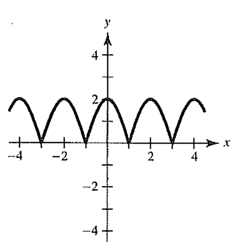
(d) $f(-x)$



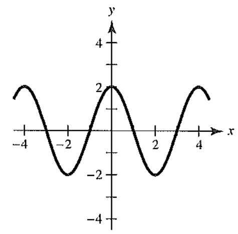
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

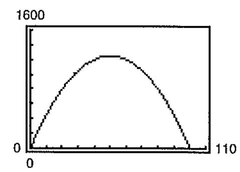


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$ or $(0, 100)$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\frac{1}{2}(x^2 - 100x)$

$= -\frac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250\ \text{m}^2$ is the maximum.

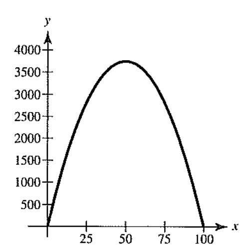
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$$A(x) = x(2y) = x\left(\frac{300 - 3x}{2}\right) = \frac{-3x^2 + 300x}{2}$$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c) $A(x) = -\frac{3}{2}(x^2 - 100x)$

$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$

$= -\frac{3}{2}(x - 50)^2 + 3750$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. So, the total time is $T = \dfrac{\sqrt{4 + x^2}}{2} + \dfrac{\sqrt{1 + (3 - x)^2}}{4}$ hours.

8. Let d be the distance from the starting point to the beach.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{2d}{\dfrac{d}{120} + \dfrac{d}{60}} = \frac{2}{\dfrac{1}{120} + \dfrac{1}{60}} = 80 \text{ km/h}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

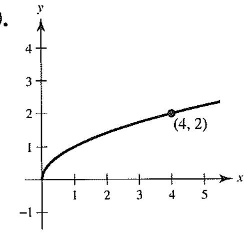
(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d)
$$\text{Slope} = \frac{f(2 + h) - f(2)}{(2 + h) - 2} = \frac{(2 + h)^2 - 4}{h} = \frac{4h + h^2}{h} = 4 + h,\ h \neq 0$$

(e) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at (2, 4) is 4.

10.



(a) Slope $= \dfrac{3 - 2}{9 - 4} = \dfrac{1}{5}$. Slope of tangent line is greater than $\dfrac{1}{5}$.

(b) Slope $= \dfrac{2 - 1}{4 - 1} = \dfrac{1}{3}$. Slope of tangent line is less than $\dfrac{1}{3}$.

(c) Slope $= \dfrac{2.1 - 2}{4.41 - 4} = \dfrac{10}{41}$. Slope of tangent line is greater than $\dfrac{10}{41}$.

(d) Slope $= \dfrac{f(4 + h) - f(4)}{(4 + h) - 4} = \dfrac{\sqrt{4 + h} - 2}{h}$

(e)
$$\frac{\sqrt{4 + h} - 2}{h} = \frac{\sqrt{4 + h} - 2}{h} \cdot \frac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} = \frac{(4 + h) - 4}{h\left(\sqrt{4 + h} + 2\right)} = \frac{1}{\sqrt{4 + h} + 2},\ h \neq 0$$

As h gets closer to 0, the slope gets closer to $\dfrac{1}{4}$. The slope is $\dfrac{1}{4}$ at the point (4, 2).

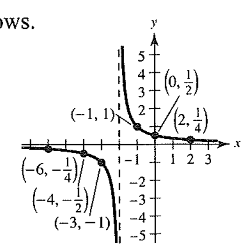
11. Using the definition of absolute value, you can rewrite the equation.

$$y + |y| = x + |x|$$

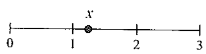
$$\begin{cases} 2y, & y > 0 \\ 0, & y \le 0 \end{cases} = \begin{cases} 2x, & x > 0 \\ 0, & x \le 0 \end{cases}.$$

For $x > 0$ and $y > 0$, you have $2y = 2x \Rightarrow y = x$.

For any $x \le 0$, y is any $y \le 0$. So, the graph of $y + |y| = x + |x|$ is as follows.



12. (a) $\dfrac{I}{x^2} = \dfrac{2I}{(x-3)^2}$



$x^2 - 6x + 9 = 2x^2$

$x^2 + 6x - 9 = 0$

$x = \dfrac{-6 \pm \sqrt{36+36}}{2} = -3 \pm \sqrt{18} \approx 1.2426, -7.2426$

(b) $\dfrac{I}{x^2 + y^2} = \dfrac{2I}{(x-3)^2 + y^2}$

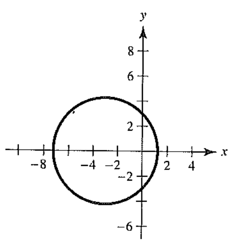
$(x-3)^2 + y^2 = 2(x^2 + y^2)$

$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$

$x^2 + y^2 + 6x - 9 = 0$

$(x+3)^2 + y^2 = 18$

Circle of radius $\sqrt{18}$ and center $(-3, 0)$.



13. (a) $\dfrac{I}{x^2 + y^2} = \dfrac{kI}{(x-4)^2 + y^2}$

$(x-4)^2 + y^2 = k(x^2 + y^2)$

$(k-1)x^2 + 8x + (k-1)y^2 = 16$

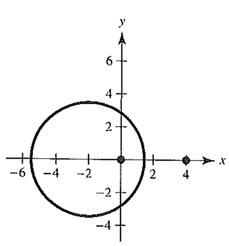
If $k = 1$, then $x = 2$ is a vertical line. Assume $k \neq 1$.

$x^2 + \dfrac{8x}{k-1} + y^2 = \dfrac{16}{k-1}$

$x^2 + \dfrac{8x}{k-1} + \dfrac{16}{(k-1)^2} + y^2 = \dfrac{16}{k-1} + \dfrac{16}{(k-1)^2}$

$\left(x + \dfrac{4}{k-1}\right)^2 + y^2 = \dfrac{16k}{(k-1)^2}$, Circle

(b) If $k = 3$, $(x+2)^2 + y^2 = 12$

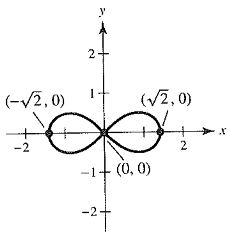


(c) As k becomes very large, $\dfrac{4}{k-1} \to 0$ and $\dfrac{16k}{(k-1)^2} \to 0$.

The center of the circle gets closer to (0, 0), and its radius approaches 0.

14.

$$\begin{aligned}
d_1 d_2 &= 1\\
\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] &= 1\\
(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 &= 1\\
\left(x^2-1\right)^2 + y^2\left[2x^2 + 2\right] + y^4 &= 1\\
x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 &= 1\\
\left(x^4 + 2x^2y^2 + y^4\right) - 2x^2 + 2y^2 &= 0\\
\left(x^2 + y^2\right)^2 &= 2\left(x^2 - y^2\right)
\end{aligned}$$



Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$.

So, $(0, 0)$, $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$ are on the curve.

15. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1 - \left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(d) The graph is not a line. it has holes at $(0, 0)$ and $(1, 1)$.