

8. Not continuous at $x = b$. $\lim_{x \rightarrow b} f(x) \neq f(b)$
9. Not continuous at $x = c$ since $\lim_{x \rightarrow c} f(x)$ does not exist.
10. Continuous at $x = d$
11. $\lim_{x \rightarrow -2} x^2(x+1) = (-2)^2(-2+1) = 4(-1) = -4$
12. $\lim_{x \rightarrow -3} (x+2)(x-5) = 5(-2) = -10$ 13. $\lim_{x \rightarrow 3} \frac{x-3}{x^2} = \frac{3-3}{3^2} = \frac{0}{9} = 0$
14. $\lim_{x \rightarrow -1} \frac{x^2+1}{3x^2-2x+5} = \frac{2}{3+2+5} = \frac{1}{5}$
15. $\lim_{x \rightarrow -2} \left(\frac{x}{x+1}\right)\left(\frac{3x+5}{x^2+x}\right) = \left(\frac{-2}{-1}\right)\left(\frac{-1}{4-2}\right) = -1$
16. $\lim_{x \rightarrow 1} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right) = \frac{1}{2} \frac{7}{1} \frac{2}{7} = 1$ 17. $\lim_{x \rightarrow 4} \sqrt{1-2x}$ does not exist.
18. $\lim_{x \rightarrow 5} \sqrt[4]{9-x^2}$ does not exist. 19. $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1} x+1 = 2$
20. $\lim_{x \rightarrow -5} \frac{x^2+3x-10}{x+5} = \lim_{x \rightarrow -5} \frac{(x+5)(x-2)}{x+5} = \lim_{x \rightarrow -5} x-2 = -7$
21. $\lim_{x \rightarrow 2} \frac{x-2}{x^2+x-6} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x+3)} = \lim_{x \rightarrow 2} \frac{1}{x+3} = \frac{1}{5}$
22. $\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x^3-2x^2+x} = \lim_{x \rightarrow 1} \frac{1}{x} = 1$
23. $\lim_{x \rightarrow 0} \frac{(1+x)(2+x)-2}{x} = \lim_{x \rightarrow 0} \frac{3x+x^2}{x} = \lim_{x \rightarrow 0} 3+x = 3$
24. $\lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \lim_{x \rightarrow 0} \frac{2-(2+x)}{2(2+x)} \frac{1}{x} = \lim_{x \rightarrow 0} \frac{-1}{2(2+x)} = -\frac{1}{4}$
25. $\lim_{x \rightarrow \infty} \frac{2x+3}{5x+7} = \lim_{x \rightarrow \infty} \frac{2+\frac{3}{x}}{5+\frac{7}{x}} = \frac{2+0}{5+0} = \frac{2}{5}$
26. $\lim_{x \rightarrow \infty} \frac{2x^2+3}{5x^2+7} = \lim_{x \rightarrow \infty} \frac{2+\frac{3}{x^2}}{5+\frac{7}{x^2}} = \frac{2}{5}$
27. $\lim_{x \rightarrow -\infty} \frac{x^2-4x+8}{3x^3} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x} - \frac{4}{x^2} + \frac{8}{x^3}}{3} = 0$
28. $\lim_{x \rightarrow \infty} \frac{1}{x^2-7x+1} = \lim_{x \rightarrow \infty} \frac{1}{1-\frac{7}{x}+\frac{1}{x^2}} = \frac{0}{1-0+0} = 0$
29. $\lim_{x \rightarrow -\infty} \frac{x^2-7x}{x+1} = \lim_{x \rightarrow -\infty} \frac{x(x-7)}{x+1} = \lim_{x \rightarrow -\infty} \frac{x(1-\frac{7}{x})}{1+\frac{1}{x}} = -\infty$
because $\lim_{x \rightarrow -\infty} \frac{1-\frac{7}{x}}{1+\frac{1}{x}} = 1$.
30. $\lim_{x \rightarrow \infty} \frac{x^4+x^3}{12x^3+128} = \lim_{x \rightarrow \infty} \frac{x(x^3+x^2)}{12x^3+128} = \lim_{x \rightarrow \infty} x\left(\frac{1+\frac{1}{x}}{12+\frac{128}{x^3}}\right) = \infty$

31. $\lim_{x \rightarrow 3^+} \frac{1}{x-3} = \infty$

32. $\lim_{x \rightarrow 3^-} \frac{1}{x-3} = -\infty$

33. $\lim_{x \rightarrow 0^+} \frac{1}{x^2} = \infty$

34. $\lim_{x \rightarrow 0^-} \frac{1}{|x|} = \infty$

35. $\lim_{x \rightarrow 0} \frac{\sin 2x}{4x} = \lim_{x \rightarrow 0} \frac{1}{2} \frac{\sin 2x}{2x} = \frac{1}{2} \cdot 1 = \frac{1}{2}$

36. $\lim_{x \rightarrow 0} \frac{x + \sin x}{x} = \lim_{x \rightarrow 0} \left(1 + \frac{\sin x}{x}\right) = 1 + 1 = 2$

37. $\lim_{x \rightarrow 0} \frac{\sin^3 2x}{x^3} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x}\right)^3 = \lim_{x \rightarrow 0} \left(2 \frac{\sin 2x}{2x}\right)^3 = (2 \cdot 1)^3 = 8$

38. $\lim_{x \rightarrow 0} \frac{2 \csc 5x}{\csc 3x} = \lim_{x \rightarrow 0} \frac{2 \sin 3x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{2 \sin 3x}{3x} \frac{5x}{\sin 5x} \frac{3x}{5x} = 2 \cdot 1 \cdot 1 \cdot \frac{3}{5} = \frac{6}{5}$

39. a) the y -intercept is approximately 0.78.

b) The values are all close to 0.78

c) f appears to have a minimal value at $x = 0$.

$$\begin{aligned} \text{d) } \lim_{x \rightarrow 0} \frac{\sec 2x \csc 9x}{\cot 7x} &= \lim_{x \rightarrow 0} \frac{1}{\cos 2x} \frac{\sin 7x}{\cos 7x} \frac{1}{\sin 9x} \\ &= \lim_{x \rightarrow 0} \frac{1}{\cos 2x} \frac{1}{\cos 7x} \frac{\sin 7x}{7x} \frac{1}{\frac{\sin 9x}{9x}} \frac{7x}{9x} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{7}{9} = \frac{7}{9} \end{aligned}$$

40. a) 1.60 b) quite close for x in the range $[-0.1, 0.1]$ c) f appears to have a minimal value at $x = 0$.

$$\text{d) } \lim_{x \rightarrow 0} \frac{\sin 8x \cos 3x}{\sin 5x \cos 8x} = \lim_{x \rightarrow 0} \cos 3x \frac{1}{\cos 8x} \frac{\sin 8x}{8x} \frac{5x}{\sin 5x} \frac{8x}{5x} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{8}{5} = 8/5$$

41. a) $\lim_{x \rightarrow -2^+} \frac{x+3}{x+2} = \infty$ b) $\lim_{x \rightarrow -2^-} \frac{x+3}{x+2} = -\infty$. As $x \rightarrow -2$, $x+3$ is positive and $x+2$ is positive in a) but negative in b). The answer may be confirmed by graphing $y = \frac{x+3}{x+2}$.

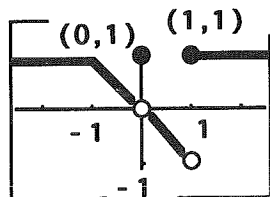
42. a) $\lim_{x \rightarrow 2^+} \frac{x-1}{x^2(x-2)} = \infty$

b) $\lim_{x \rightarrow 2^-} \frac{x-1}{x^2(x-2)} = -\infty$

c) $\lim_{x \rightarrow 0^+} \frac{x-1}{x^2(x-2)} = \infty$

d) $\lim_{x \rightarrow 0^-} \frac{x-1}{x^2(x-2)} = \infty$

43. a)

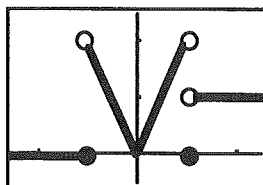


b) $\lim_{x \rightarrow -1^+} f(x) = 1$, $\lim_{x \rightarrow -1^-} f(x) = 1$, $\lim_{x \rightarrow 0^+} f(x) = 0$, $\lim_{x \rightarrow 0^-} f(x) = 0$, $\lim_{x \rightarrow 1^+} f(x) = 1$, $\lim_{x \rightarrow 1^-} f(x) = -1$

c) $\lim_{x \rightarrow -1} f(x) = 1$, $\lim_{x \rightarrow 0} f(x) = 0$ but $\lim_{x \rightarrow 1} f(x)$ does not exist because the right-hand and left-hand limits of f at 1 are not equal.

d) Only at $x = -1$

44. a)

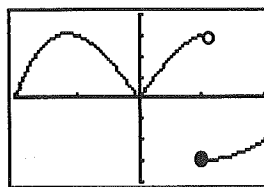


b) $\lim_{x \rightarrow -1^-} f(x) = 0$, $\lim_{x \rightarrow -1^+} f(x) = 2$, $\lim_{x \rightarrow 0^-} f(x) = 0$, $\lim_{x \rightarrow 0^+} f(x) = 0$, $\lim_{x \rightarrow 1^-} f(x) = 2$, $\lim_{x \rightarrow 1^+} f(x) = 1$

c) $\lim_{x \rightarrow 0} f(x) = 0$ but $\lim_{x \rightarrow -1} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ do not exist because the left-hand and right-hand limits at $x = -1$ are not equal and the same is true at $x = 1$.

d) Only at $x = 0$.

45. a) A graph of f may be obtained by graphing the functions $y = \text{abs}(x^3 - 4x) + 0\sqrt{1-x}$ and $y = x^2 - 2x - 2 + 0\sqrt{x-1}$ in the viewing rectangle $[-5, 7]$ by $[-4, 10]$.



$[-2, 2]$ by $[-4, 4]$

b) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 - 2x - 2) = -3$.

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} |x^3 - 4x| = 3$

c) f does not have a limit at $x = 1$ because the right-hand and left-hand limits at $x = 1$ are not equal.

d) $x^3 - 4x$ is continuous by 2.2 Example 5 and $|x|$ is continuous by 2.2 Example 8. Thus $|x^3 - 4x|$ is continuous by Theorem 5 and so f is continuous for $x < 1$. For $x > 1$, $f(x) = x^2 - 2x - 2$, a polynomial, is continuous. Thus $f(x)$ is continuous at all points except $x = 1$.

- e) f is not continuous at $x = 1$ because the two limits in b) are not equal and so $\lim_{x \rightarrow 1} f(x)$ does not exist.
46. a) A good idea of the graph is obtained by graphing $y = 1 - \sqrt{3 - 2x} + 0\sqrt{1.5 - x}$ and $y = 1 + \sqrt{2x - 3} + 0\sqrt{x - 1.5}$ in the rectangle $[-2, 4]$ by $[-3, 5]$.
- b) Both limits are equal to 1
- c) $\lim_{x \rightarrow 3/2} f(x) = 1$
- d) f is continuous at all points because $f(c) = \lim_{x \rightarrow c} f(x)$ for all points c .
- e) There are no points of discontinuity because of d)
47. a) A graph of f is obtained by graphing $y = -x + 0\sqrt{1 - x}$ and $y = x - 1 + 0\sqrt{x - 1}$ in the rectangle $[-2, 4]$ by $[-2, 4]$.
- b) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x - 1) = 0$. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} -x = -1$.
- c) No value assigned to $f(1)$ makes f continuous at $x = 1$.
48. a) The idea of a complete graph of f can be obtained by graphing $y = 3x^2 + 0\sqrt{1 - x}$ and $y = 4 - x^2 + 0\sqrt{x - 1}$ in the viewing rectangle $[-2, 4]$ by $[-2, 4]$.
- b) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = 3$
- c) b) shows that $\lim_{x \rightarrow 1} f(x) = 3$ so define $f(1) = 3$.
49. f is not defined at $x = \pm 2$ so is not continuous at these points. By 2.2 Example 6 f is continuous at all other points.
50. $3x + 2$ and $\sqrt[3]{x}$ are continuous by Examples 5 and 7 of 2.2. Hence the composite, f , is continuous by Theorem 5. There are no discontinuities.
51. $\lim_{x \rightarrow \pm\infty} \frac{2x+1}{x^2-2x+1} = \lim_{x \rightarrow \pm\infty} \frac{\frac{2}{x} + \frac{1}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = \frac{0+0}{1-0+0} = 0$.
52. $\lim_{x \rightarrow \pm\infty} \frac{2x^2+5x-1}{x^2+2x} = \lim_{x \rightarrow \pm\infty} \frac{2 + \frac{5}{x} - \frac{1}{x^2}}{1 + \frac{2}{x}} = \frac{2+0-0}{1+0} = 2$. $y = 2$ is the end behavior asymptote.
53. By long division $h(x) = x^2 - x + \frac{3}{x-3}$. Thus $y = x^2 - x$ is the end behavior asymptote of h .
54. $T(x) = x - \frac{2x^2+1}{x^3-x+1}$. Thus $y = x$ is the end behavior asymptote of T .

55. a) $\lim_{x \rightarrow c} 3f(x) = 3 \lim_{x \rightarrow c} f(x) = 3(-7) = -21$
 b) $\lim_{x \rightarrow c} (f(x))^2 = \lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} f(x) = (-7)(-7) = 49$
 c) $\lim_{x \rightarrow c} f(x) \cdot g(x) = \lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} g(x) = (-7)(0) = 0$
 d) $\lim_{x \rightarrow c} \frac{f(x)}{g(x)-7} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)-7} = \frac{-7}{0-7} = 1$
 e) $\lim_{x \rightarrow c} \cos(g(x)) = \cos[\lim_{x \rightarrow c} g(x)] = \cos 0 = 1.$
 f) $\lim_{x \rightarrow c} |f(x)| = |\lim_{x \rightarrow c} f(x)| = |-7| = 7.$
56. a) $\lim_{x \rightarrow 0} -g(x) = -\sqrt{2}$ b) $\lim_{x \rightarrow 0} g(x) \cdot f(x) = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$
 c) $\lim_{x \rightarrow 0} [f(x) + g(x)] = \frac{1}{2} + \sqrt{2}$ d) $\lim_{x \rightarrow 0} [1/f(x)] = 2$
 e) $\lim_{x \rightarrow 0} [x + f(x)] = 0 + \frac{1}{2} = \frac{1}{2}$ f) $\lim_{x \rightarrow 0} \frac{f(x) \cdot \sin x}{x} = \frac{1}{2} \cdot 1 = \frac{1}{2}$
57. Both 0 and \sqrt{x} approach 0 as $x \rightarrow 0$. By the Sandwich Theorem $|\sqrt{x} \sin \frac{1}{x}| \rightarrow 0$ and hence $\sqrt{x} \sin \frac{1}{x} \rightarrow 0$ as $x \rightarrow 0$.
58. Both 0 and x^2 approach 0 as $x \rightarrow 0$. By the Sandwich Theorem $|x^2 \sin \frac{1}{x}| \rightarrow 0$ and hence $x^2 \sin \frac{1}{x} \rightarrow 0$ as $x \rightarrow 0$.
59. $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x} = \lim_{x \rightarrow \infty} [1 + \frac{\sin x}{x}] = 1 + 0 = 1$
60. $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x} = \lim_{x \rightarrow \infty} \frac{1 + \frac{\sin x}{x}}{1 + \frac{\cos x}{x}} = \frac{1+0}{1+0} = 1$
61. $-1 \leq \sin x \leq 1$. Hence $-\frac{1}{\sqrt{x}} \leq \frac{\sin x}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$ for $x > 0$. Both $-\frac{1}{\sqrt{x}}$ and $\frac{1}{\sqrt{x}}$ approach 0 as $x \rightarrow \infty$. By the Sandwich Theorem $\lim_{x \rightarrow \infty} \frac{\sin x}{\sqrt{x}} = 0$.
62. $-\frac{1}{\sqrt{x}} \leq \frac{\cos x}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$. Since $\lim_{x \rightarrow \infty} \pm \frac{1}{\sqrt{x}} = 0$, $\lim_{x \rightarrow \infty} \frac{\cos x}{\sqrt{x}} = 0$.
63. $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{x-3} = \lim_{x \rightarrow 3} (x+5) = 8$. In order to have $\lim_{x \rightarrow 3} f(x) = f(3) = k$, we should set $k = 8$.
64. $\lim_{x \rightarrow 0^+} x^x = 1$. The limit must be approached from the right-hand side only because x^x is defined only for $x > 0$.
65. a) $\lim_{x \rightarrow 0^-} f(x) = 0$ b) $\lim_{x \rightarrow 0^+} f(x) = \infty$
 c) The limit does not exist. To exist both one-sided limits must exist and be equal.
66. $\lim_{x \rightarrow 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{1}{2}$. Thus the value $\frac{1}{2}$ should be assigned to k .
67. This is not a contradiction because $0 < x < 1$ is not a *closed* interval.

68. Let $y = f(x) = |x|$ on $-1 \leq x < 1$. This function attains its minimum ($f(0) = 0$) and its maximum ($f(-1) = 1$). This is not a contradiction of the theorem which is only a sufficient (not necessary) condition for the attainment of extreme values.
69. True because $0 = f(1) < 2.5 < f(2) = 3$ and so by Theorem 7, $2.5 = f(c)$ for some c in $[1, 2]$.
70. Let $f(x) = x + \cos x$. Then $f(x)$ is continuous. $f(-\pi/2) = -\frac{\pi}{2}$ and $f(0) = 1$. Thus $f(-\pi/2) < 0 < f(0)$. By Theorem 7, $f(c) = 0$ for some c in $[-\pi/2, 0]$.
71. Let $f(x) = x + \log x$. $f(\frac{1}{10}) = \frac{1}{10} - 1 < 0$. $f(1) = 1 > 0$. By the Intermediate Value Theorem there is a number c , $\frac{1}{10} < c < 1$ such that $f(c) = 0$. This means that c is a solution of the given equation.
72. $\lim_{x \rightarrow 1} f(x) = 3$ means given any radius $\varepsilon > 0$ about 3 there exists a radius $\delta > 0$ about 1 such that for all x $0 < |x - 1| < \delta$ implies $|f(x) - 3| < \varepsilon$.
73. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ means given any radius $\varepsilon > 0$ about 1 there exists a radius $\delta > 0$ about 0 such that, for all x , $0 < |x - 0| < \delta$ implies $|f(x) - 1| < \varepsilon$.
74. Let $f(x) = x^2$. $f(x)$ gets closer to -1 as x approaches 0 but -1 is not equal to $\lim_{x \rightarrow 0} f(x)$.
75. This "definition" puts a requirement on $f(x)$ but not necessarily for x near x_0 . Thus for any given $\varepsilon > 0$ there is an x such that $|x^2 - 0| < \varepsilon$ but this does not imply $\lim_{x \rightarrow x_0} x^2 = 0$ in general.
76. $-1 < \sqrt{x+2} - 4 < 1$ leads successively to $3 < \sqrt{x+2} < 5$, $9 < x+2 < 25$, $7 < x < 23$, the last interval being the solution of the first part of the problem. The midpoint of this interval is $\frac{7+23}{2} = 15$. The interval can be described by $|x - 15| < 8$.
77. $-\frac{1}{2} < \sqrt{\frac{x+1}{2}} - 1 < \frac{1}{2}$ is equivalent to each of $\frac{1}{2} < \sqrt{\frac{x+1}{2}} < \frac{3}{2}$, $\frac{1}{4} < \frac{x+1}{2} < \frac{9}{4}$, $\frac{1}{2} < x+1 < \frac{9}{2}$, $-\frac{1}{2} < x < \frac{7}{2}$. The midpoint of the latter interval is $(-\frac{1}{2} + \frac{7}{2})/2 = 3/2$ so that it can be written as $|x - 3/2| < 2$.
78. Let $f(x) = \frac{x-1}{x-3}$. Following the method of 2.5 Example 6, we solve the equations $f(x) = 1.9$ and $f(x) = 2.1$, obtaining 5.22 and 4.82. Thus $1.9 < f(x) < 2.1$ if $4.82 < x < 5.22$. Letting $x_0 = 5$, we have $|x - 5| < 0.18$ implies $|f(x) - 2| < 0.1$.

79. Let $f(x) = \frac{x-1}{x-3}$. Following the method of 2.5 Example 6, we solve the equations $f(x) = -2.1$ and $f(x) = -1.9$ obtaining 2.35 and 2.31. Thus $f(x)$ is within 0.1 unit of -2 if $2.31 < x < 2.35$ or $|x - 7/3| < 0.02$.
80. Let $f(x) = x^3 - 4x$. We graph $f(x)$, $y = 3.9$ and $y = 4.1$ and we use zoom-in to find the x -coordinates of the two points of intersection obtaining 2.38 and 2.39. Thus $f(x)$ is within 0.1 unit of 4 if $2.38 < x < 2.39$. If we take $x_0 = 2.383$ (near the root of $f(x) = 4$), we can say $|x - 2.383| < 0.003$ implies $|f(x) - 4| < 0.1$.
81. Let $f(x) = x^3 - 4x$. We graph $f(x)$, $y = 0.9$ and $y = 1.1$ and use zoom-in to find the x -coordinates of the points of intersection ($-1 < x < 0$) obtaining -0.28 and -0.228 . Thus $f(x)$ is within 0.1 unit of 1 if $-0.280 < x < -0.228$ (rounding appropriately) or if $|x + 0.254| < 0.026$.
82. $|f(x) - 1| = |2x - 4| = 2|x - 2| < \varepsilon$ is equivalent to $|x - 2| < \varepsilon/2$. Thus we must have $0 < \delta \leq \varepsilon/2$.
83. $|f(x) - 0| + \|x\| = |x| < \varepsilon$ is equivalent to $|x - 0| < \varepsilon$. Thus we must have $0 < \delta \leq \varepsilon$.
84. $\lim_{x \rightarrow \infty} \frac{1-2x}{3x-1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}-2}{3-\frac{1}{x}} = \frac{-2}{3}$. To confirm this using the definition of this type of limit, we follow Example 6. Let $\varepsilon > 0$ be given. $|\frac{1-2x}{3x-1} + \frac{2}{3}| < \varepsilon$ is equivalent to each of the following. $|\frac{3-6x+6x-2}{3(3x-1)}| < \varepsilon$, $\frac{1}{3(3x-1)} < \varepsilon$ (assuming x is large and positive), $\frac{1}{3\varepsilon} < 3x - 1$, $\frac{1}{3\varepsilon} + 1 < 3x$, $x > \frac{1}{9\varepsilon} + \frac{1}{3} = \frac{3\varepsilon+1}{9\varepsilon}$. So we choose $N = \frac{3\varepsilon+1}{9\varepsilon}$. Then $x > N$ implies $|\frac{1-2x}{3x-1} + \frac{2}{3}| < \varepsilon$. This confirms the limit.
85. As $x \rightarrow \frac{1}{3}^+$, $1 - 2x \rightarrow \frac{1}{3}$ and $3x - 1 \rightarrow 0$ through positive values. Hence $\lim_{x \rightarrow \frac{1}{3}^+} \frac{1-2x}{3x-1} = \infty$. With $x > \frac{1}{3}$, all of the following are equivalent. $\frac{1-2x}{3x-1} > N$, $1 - 2x > 3xN - N$, $1 + N > x(3N + 2)$, $x < \frac{N+1}{3N+2} = \frac{1}{3} + \frac{1}{3(3N+2)}$, $x - \frac{1}{3} < \frac{1}{3(3N+2)}$. So we choose $\delta = \frac{1}{3(3N+2)}$. Then $0 < x - \frac{1}{3} < \delta$ implies $\frac{1-2x}{3x-1} > N$. Since this is true for any $N > 0$, this confirms the limit.
86. $L = \lim_{x \rightarrow 3} (5x - 10) = 5 \cdot 3 - 10 = 5$. $|f(x) - 5| = |5x - 15| = 5|x - 3| < \varepsilon = 0.05$ is equivalent to $|x - 3| < 0.01$. Let $\delta = 0.01$. Then for all x $0 < |x - 3| < \delta \Rightarrow |f(x) - 5| < \varepsilon$.
87. $L = \lim_{x \rightarrow 2} (5x - 10) = 0$. $|f(x) - L| = |5x - 10| = 5|x - 2| < \varepsilon$ is equivalent to $|x - 2| < \varepsilon/5$. Thus $\delta = \varepsilon/5 = 0.01$.

88. $L = \lim_{x \rightarrow 9} \sqrt{x-5} = \sqrt{4} = 2$. $|f(x) - L| = |\sqrt{x-5} - 2| < \varepsilon = 1$ is equivalent to $-1 < \sqrt{x-5} - 2 < 1$, $1 < \sqrt{x-5} < 3$, $1 < x-5 < 9$, $6 < x < 14$, $-3 < x-9 < 5$. Thus $|x-9| < 3$ guarantees $|f(x) - L| < \varepsilon$ and so we take $\delta = 3$.
89. $L = \lim_{x \rightarrow 2} \sqrt{2x-3} = 1$. $1/2 < \sqrt{2x-3} < 3/2$ yields $\frac{1}{4} < 2x-3 < \frac{9}{4}$, $-\frac{3}{4} < 2x-4 < \frac{5}{4}$, $-\frac{3}{8} < x-2 < \frac{5}{8}$. Thus $|x-2| < \frac{3}{8} \Rightarrow |f(x) - L| < \varepsilon$ so $\delta = 3/8$.
90. $L = \lim_{x \rightarrow 5} \frac{x^2-x}{x+2} = \frac{20}{7} = 2.86$. We use the method of 2.6. In the viewing rectangle $[4.98, 5.02]$ by $[2.85, 2.87]$, we find the coordinates of two points on the graph of $f(x)$ and use these to determine the slope m . We find $m = 0.88$ approximately and $\delta = \varepsilon/|m| = \varepsilon/0.88 = 1.13\varepsilon$ rounding down to be safe.
91. $L = \lim_{x \rightarrow -5} \frac{x-1}{x^2+3x} = \frac{-6}{10} = -0.6$. In the viewing rectangle $[-5.1, -4.9]$ by $[-0.61, -0.59]$ we graph $f(x)$ and using two points on the graph we find $m = -0.32$. Hence we take $\delta = \varepsilon/|m| = 3.12\varepsilon$ rounding down to be safe.
92. $0.8 < \frac{\sqrt{x}}{2} < 1.2$ is equivalent to $1.6 < \sqrt{x} < 2.4$, $1.6^2 < x < 2.4^2$ or $2.56 < x < 5.76$. $0.9 < \frac{\sqrt{x}}{2} < 1.1$ is equivalent to $1.8 < \sqrt{x} < 2.2$ or $3.24 < x < 4.84$.
93. $9.9995 < 10 + (t-70) \times 10^{-4} < 10.0005$ is equivalent to $-0.0005 < (t-70)10^{-4} < 0.0005$, $-5 < t-70 < 5$ or $65^\circ < t < 75^\circ$.
94. $1000x^2 + x - 10^{-15} = 0$. If $x = 0$, the left-hand side becomes $-10^{-15} \neq 0$, so $x = 0$ is not a solution. By the quadratic formula $x = \frac{-1 \pm \sqrt{1+4(10^{-12})}}{2000}$. On a calculator the positive solution reads 10^{-15} . Because of the limits of calculator precision the exact solution cannot be found. We can be sure the error is less than 10^{-14} .