CHAPTER 2
Differentiation

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CHAPTER 2
Differentiation

Section 2.1 The Derivative and the Tangent Line Problem

1. (a) At (x_1, y_1), slope $= 0$.

At (x_2, y_2), slope $= \frac{5}{2}$.

(b) At (x_1, y_1), slope $= -\frac{5}{2}$.

At (x_2, y_2), slope $= 2$.

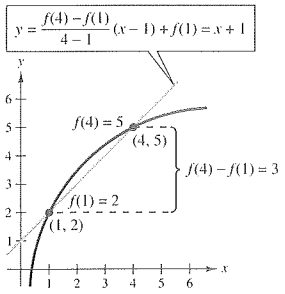
2. (a) At (x_1, y_1), slope $= \frac{2}{3}$.

At (x_2, y_2), slope $= -\frac{2}{5}$.

(b) At (x_1, y_1), slope $= \frac{5}{4}$.

At (x_2, y_2), slope $= \frac{5}{4}$.

3. (a), (b)



(c) $y = \dfrac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$

$= \dfrac{3}{3}(x - 1) + 2$

$= 1(x - 1) + 2$

$= x + 1$

4. (a) $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{5 - 2}{3} = 1$

$\dfrac{f(4) - f(3)}{4 - 3} \approx \dfrac{5 - 4.75}{1} = 0.25$

So, $\dfrac{f(4) - f(1)}{4 - 1} > \dfrac{f(4) - f(3)}{4 - 3}$.

(b) The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. So,

$\dfrac{f(4) - f(1)}{4 - 1} < f'(1)$.

5. $f(x) = 3 - 5x$ is a line. Slope $= -5$

6. $g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

7. Slope at $(2, -5) = \lim\limits_{\Delta x \to 0} \dfrac{g(2 + \Delta x) - g(2)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{(2 + \Delta x)^2 - 9 - (-5)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{4 + 4(\Delta x) + (\Delta x)^2 - 4}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} (4 + \Delta x) = 4$

8. Slope at $(1, 5) = \lim\limits_{\Delta x \to 0} \dfrac{g(1 + \Delta x) - g(1)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{6 - (1 + \Delta x)^2 - 5}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{6 - 1 - 2(\Delta x) - (\Delta x)^2 - 5}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} (-2 - \Delta x) = -2$

9. Slope at $(0, 0) = \lim\limits_{\Delta t \to 0} \dfrac{f(0 + \Delta t) - f(0)}{\Delta t}$

$= \lim\limits_{\Delta t \to 0} \dfrac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t}$

$= \lim\limits_{\Delta t \to 0} (3 - \Delta t) = 3$

10. Slope at $(-2, 7) = \lim\limits_{\Delta t \to 0} \dfrac{h(-2 + \Delta t) - h(-2)}{\Delta t}$

$= \lim\limits_{\Delta t \to 0} \dfrac{(-2 + \Delta t)^2 + 3 - 7}{\Delta t}$

$= \lim\limits_{\Delta t \to 0} \dfrac{4 - 4(\Delta t) + (\Delta t)^2 - 4}{\Delta t}$

$= \lim\limits_{\Delta t \to 0} (-4 + \Delta t) = -4$

11. $f(x) = 7$

$f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{7 - 7}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} 0 = 0$

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12. $g(x) = -3$

$$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-3 - (-3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

13. $f(x) = -10x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-10(x + \Delta x) - (-10x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-10\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (-10) = -10$$

14. $f(x) = 3x + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[3(x + \Delta x) + 2] - [3x + 2]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 3 = 3$$

15. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

16. $f(x) = 8 - \frac{1}{5}x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8 - \frac{1}{5}(x + \Delta x) - \left(8 - \frac{1}{5}x\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{5}(\Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(-\frac{1}{5}\right) = -\frac{1}{5}$$

17. $f(x) = x^2 + x - 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 1) = 2x + 1$$

18. $f(x) = 2 - x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2 - (x + \Delta x)^2 - (2 - x^2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2 - x^2 - 2x(\Delta x) - (\Delta x)^2 - 2 + x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-2x(\Delta x) - (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} [-2x - \Delta x] = -2x$$

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19. $f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12\right) = 3x^2 - 12$$

20. $f(x) = x^3 + x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + (x + \Delta x)^2\right] - \left[x^3 + x^2\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + x^2 + 2x\,\Delta x + (\Delta x)^2 - x^3 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x\,\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 + 2x + (\Delta x)\right) = 3x^2 + 2x$$

21. $f(x) = \dfrac{1}{x - 1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 1} - \dfrac{1}{x - 1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x - 1) - (x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 1)(x - 1)}$$

$$= -\frac{1}{(x - 1)^2}$$

22. $f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x + \Delta x)^2} - \dfrac{1}{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{\Delta x(x + \Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x(x + \Delta x)^2 x^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x + \Delta x)^2 x^2}$$

$$= \frac{-2x}{x^4}$$

$$= -\frac{2}{x^3}$$

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23. $f(x) = \sqrt{x+4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+4} - \sqrt{x+4}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x+4} + \sqrt{x+4}}{\sqrt{x+\Delta x+4} + \sqrt{x+4}} \right)$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x+4) - (x+4)}{\Delta x\left[\sqrt{x+\Delta x+4} + \sqrt{x+4}\right]}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+4} + \sqrt{x+4}} = \frac{1}{\sqrt{x+4} + \sqrt{x+4}} = \frac{1}{2\sqrt{x+4}}$$

24. $f(x) = \dfrac{4}{\sqrt{x}}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{4}{\sqrt{x+\Delta x}} - \dfrac{4}{\sqrt{x}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4\sqrt{x} - 4\sqrt{x+\Delta x}}{\Delta x\sqrt{x}\sqrt{x+\Delta x}} \cdot \left(\frac{\sqrt{x} + \sqrt{x+\Delta x}}{\sqrt{x} + \sqrt{x+\Delta x}} \right)$$

$$= \lim_{\Delta x \to 0} \frac{4x - 4(x+\Delta x)}{\Delta x\sqrt{x}\sqrt{x+\Delta x}\left(\sqrt{x} + \sqrt{x+\Delta x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{-4}{\sqrt{x}\sqrt{x+\Delta x}\left(\sqrt{x} + \sqrt{x+\Delta x}\right)}$$

$$= \frac{-4}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-2}{x\sqrt{x}}$$

25. (a) $f(x) = x^2 + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2 + 3\right] - \left[x^2 + 3\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\,\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

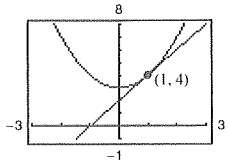
At $(1, 4)$, the slope of the tangent line is $m = 2(1) = 2$. The equation of the tangent line is

$$y - 4 = 2(x - 1)$$

$$y - 4 = 2x - 2$$

$$y = 2x + 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $(1, 4)$.

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26. (a) $f(x) = x^2 + 3x + 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 3(x + \Delta x) + 4 - (x^2 + 3x + 4)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + 3x + 3\Delta x + 4 - x^2 - 3x - 4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + 3\Delta x}{\Delta x}$$

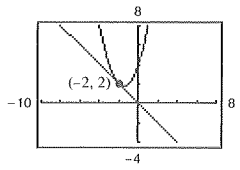
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 3) = 2x + 3$$

At $(-2, 2)$, the slope of the tangent line is $m = 2(-2) + 3 = -1$. The equation of the tangent line is

$$y - 2 = -1(x + 2)$$

$$y = -x$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = -1$ at $(-2, 2)$.

27. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

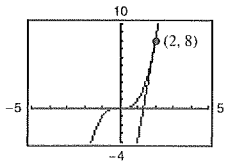
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$. The equation of the tangent line is

$$y - 8 = 12(x - 2)$$

$$y = 12x - 16.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 12$ at $(2, 8)$.

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28. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + 1\right] - (x^3 + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$

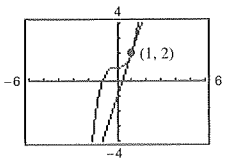
$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] = 3x^2$$

At $(1, 2)$, the slope of the tangent is $m = 3(1)^2 = 3$. The equation of the tangent line is

$$y - 2 = 3(x - 1)$$

$$y = 3x - 1.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 3$ at $(1, 2)$.

29. (a) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

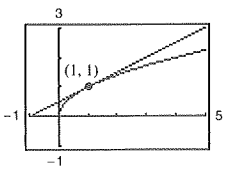
At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

The equation of the tangent line is

$$y - 1 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x + \frac{1}{2}.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{1}{2}$ at $(1, 1)$.

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30. (a) $f(x) = \sqrt{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x-1} + \sqrt{x-1}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} \right)$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x-1) - (x-1)}{\Delta x\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

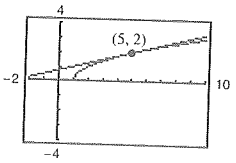
At $(5, 2)$, the slope of the tangent line is $m = \dfrac{1}{2\sqrt{5-1}} = \dfrac{1}{4}$

The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x-5)$$

$$y = \frac{1}{4}x + \frac{3}{4}$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{1}{4}$ at $(5, 2)$.

31. (a) $f(x) = x + \dfrac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x) + \dfrac{4}{x+\Delta x} - \left(x + \dfrac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x+\Delta x)(x+\Delta x) + 4x - x^2(x+\Delta x) - 4(x+\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x+\Delta x)}$$

$$= \frac{x^2-4}{x^2} = 1 - \frac{4}{x^2}$$

At $(4, 5)$, the slope of the tangent line is $m = 1 - \dfrac{4}{16} = \dfrac{3}{4}$.

The equation of the tangent line is

$$y - 5 = \frac{3}{4}(x-4)$$

$$y = \frac{3}{4}x + 2.$$

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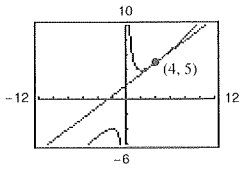
(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{3}{4}$ at $(4, 5)$.

32. (a) $f(x) = \frac{1}{x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+1} - \frac{1}{x+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+1) - (x+\Delta x+1)}{\Delta x(x+\Delta x+1)(x+1)}$$

$$= \lim_{\Delta x \to 0} -\frac{1}{(x+\Delta x+1)(x+1)}$$

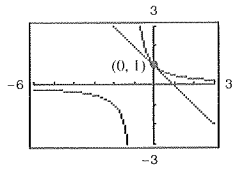
$$= -\frac{1}{(x+1)^2}$$

At $(0, 1)$, the slope of the tangent line is

$$m = \frac{-1}{(0+1)^2} = -1.$$

The equation of the tangent line is $y = -x + 1$.

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = -1$ at $(0, 1)$.

33. Using the limit definition of derivative, $f'(x) = 2x$. Because the slope of the given line is 2, you have

$2x = 2$

$x = 1$

At the point $(1, 1)$ the tangent line is parallel to $2x - y + 1 = 0$. The equation of this line is

$y - 1 = 2(x - 1)$

$y = 2x - 1.$

34. Using the limit definition of derivative, $f'(x) = 4x$. Because the slope of the given line is -4, you have

$4x = -4$

$x = -1.$

At the point $(-1, 2)$ the tangent line is parallel to $4x + y + 3 = 0$. The equation of this line is

$y - 2 = -4(x + 1)$

$y = -4x - 2.$

35. From Exercise 27 we know that $f'(x) = 3x^2$. Because the slope of the given line is 3, you have

$3x^2 = 3$

$x = \pm 1.$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$. These lines have equations

$y - 1 = 3(x - 1)$ and $y + 1 = 3(x + 1)$

$y = 3x - 2$ $\qquad$ $y = 3x + 2.$

36. Using the limit definition of derivative, $f'(x) = 3x^2$. Because the slope of the given line is 3, you have

$3x^2 = 3$

$x^2 = 1 \Rightarrow x = \pm 1.$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$y - 3 = 3(x - 1)$ and $y - 1 = 3(x + 1)$

$y = 3x$ $\qquad$ $y = 3x + 4.$

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37. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2x\sqrt{x}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2}$$

$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}.$$

38. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x-1)^{3/2}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$

$$1 = (x-1)^{3/2}$$

$$1 = x - 1 \Rightarrow x = 2.$$

At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 2.$$

39. $f(x) = x \Rightarrow f'(x) = 1$ Matches (b).

40. $f(x) = x^2 \Rightarrow f'(x) = 2x$ Matches (d).

41. $f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$ Matches (a).

(decreasing slope as $x \to \infty$)

42. f' does not exist at $x = 0$. Matches (c).

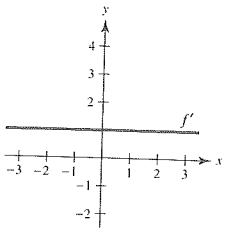
43. $g(4) = 5$ because the tangent line passes through $(4, 5)$.

$$g'(4) = \frac{5 - 0}{4 - 7} = -\frac{5}{3}$$

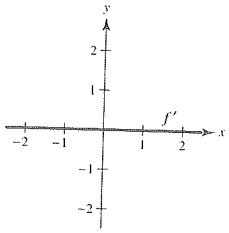
44. $h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

$$h'(-1) = \frac{6 - 4}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

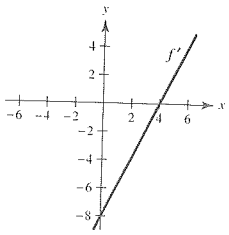
45. The slope of the graph of f is 1 for all x-values.



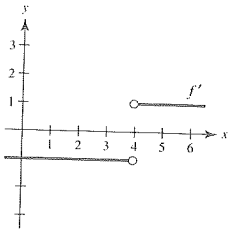
46. The slope of the graph of f is 0 for all x-values.



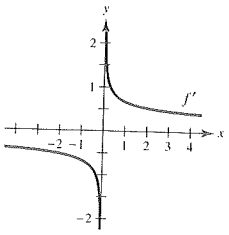
47. The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



48. The slope of the graph of f is -1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.



49. The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.



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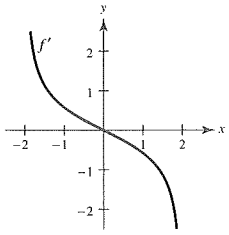
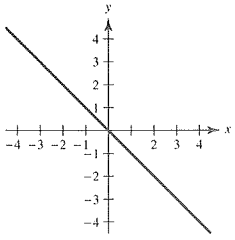
50. The slope is positive for $-2 < x < 0$ and negative for $0 < x < 2$. The slope is undefined at $x = \pm 2$, and 0 at $x = 0$.



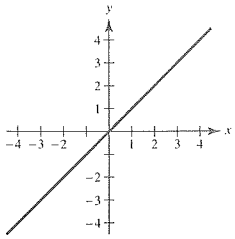
51. Answers will vary.

Sample answer: $y = -x$



52. Answers will vary.

Sample answer: $y = x$



53. $f(x) = 5 - 3x$ and $c = 1$

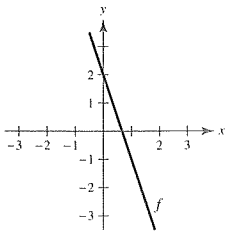
54. $f(x) = x^3$ and $c = -2$

55. $f(x) = -x^2$ and $c = 6$

56. $f(x) = 2\sqrt{x}$ and $c = 9$

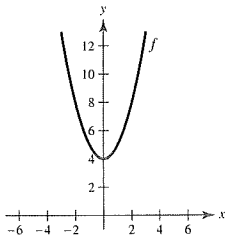
57. $f(0) = 2$ and $f'(x) = -3, -\infty < x < \infty$

$f(x) = -3x + 2$



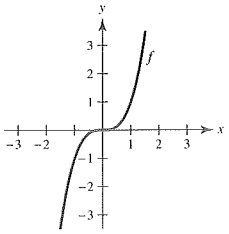
58. $f(0) = 4$, $f'(0) = 0$; $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$

Answers will vary: *Sample answer:* $f(x) = x^2 + 4$



59. $f(0) = 0$; $f'(0) = 0$; $f'(x) > 0$ if $x \neq 0$

Answers will vary: *Sample answer:* $f(x) = x^3$



60. (a) If $f'(c) = 3$ and f is odd, then $f'(-c) = f'(c) = 3$.

(b) If $f'(c) = 3$ and f is even, then $f'(-c) = -f'(c) = -3$.

61. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 4 - 2x$. The slope of the line through $(2, 5)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{5 - y_0}{2 - x_0} = 4 - 2x_0$$

$$5 - y_0 = (2 - x_0)(4 - 2x_0)$$

$$5 - (4x_0 - x_0^2) = 8 - 8x_0 + 2x_0^2$$

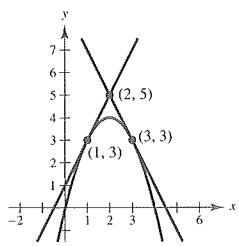
$$0 = x_0^2 - 4x_0 + 3$$

$$0 = (x_0 - 1)(x_0 - 3) \Rightarrow x_0 = 1, 3$$

Therefore, the points of tangency are $(1, 3)$ and $(3, 3)$, and the corresponding slopes are 2 and –2. The equations of the tangent lines are:

$y - 5 = 2(x - 2)$ $\quad$ $y - 5 = -2(x - 2)$

$y = 2x + 1$ $\quad$ $y = -2x + 9$



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62. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 2x$. The slope of the line through $(1, -3)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{-3 - y_0}{1 - x_0} = 2x_0$$

$$-3 - y_0 = (1 - x_0)2x_0$$

$$-3 - x_0^2 = 2x_0 - 2x_0^2$$

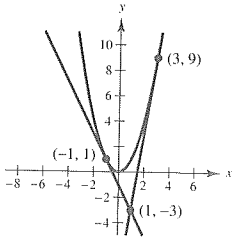
$$x_0^2 - 2x_0 - 3 = 0$$

$$(x_0 - 3)(x_0 + 1) = 0 \Rightarrow x_0 = 3, -1$$

Therefore, the points of tangency are $(3, 9)$ and $(-1, 1)$, and the corresponding slopes are 6 and -2. The equations of the tangent lines are:

$y + 3 = 6(x - 1)$ $\quad$ $y + 3 = -2(x - 1)$

$y = 6x - 9$ $\quad$ $y = -2x - 1$



63. (a) $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

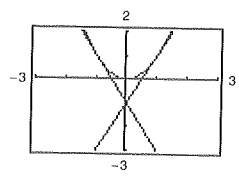
At $x = -1$, $f'(-1) = -2$ and the tangent line is

$y - 1 = -2(x + 1)$ $\quad$ or $\quad$ $y = -2x - 1$.

At $x = 0$, $f'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $f'(1) = 2$ and the tangent line is

$y = 2x - 1$.



For this function, the slopes of the tangent lines are always distinct for different values of x.

(b) $g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right) = 3x^2$$

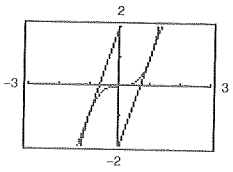
At $x = -1$, $g'(-1) = 3$ and the tangent line is

$y + 1 = 3(x + 1)$ $\quad$ or $\quad$ $y = 3x + 2$.

At $x = 0$, $g'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $g'(1) = 3$ and the tangent line is

$y - 1 = 3(x - 1)$ $\quad$ or $\quad$ $y = 3x - 2$.



For this function, the slopes of the tangent lines are sometimes the same.

64. (a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$, and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

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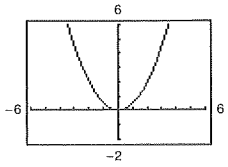
65. $f(x) = \frac{1}{2}x^2$

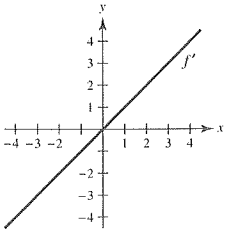
(a)



$f'(0) = 0,\ f'(1/2) = 1/2,\ f'(1) = 1,\ f'(2) = 2$

(b) By symmetry: $f'(-1/2) = -1/2,\ f'(-1) = -1,\ f'(-2) = -2$

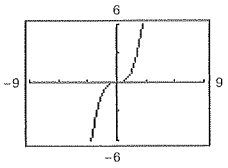
(c)



(d) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0}\left(x + \frac{\Delta x}{2}\right) = x$$

66. $f(x) = \frac{1}{3}x^3$

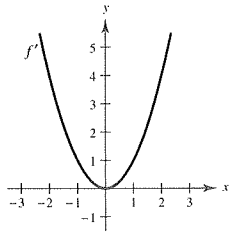
(a)



$f'(0) = 0,\ f'(1/2) = 1/4,\ f'(1) = 1,\ f'(2) = 4,\ f'(3) = 9$

(b) By symmetry: $f'(-1/2) = 1/4,\ f'(-1) = 1,\ f'(-2) = 4,\ f'(-3) = 9$

(c)



(d) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}(x + \Delta x)^3 - \frac{1}{3}x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}\left(x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3\right) - \frac{1}{3}x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\left[x^2 + x(\Delta x) + \frac{1}{3}(\Delta x)^2\right] = x^2$$

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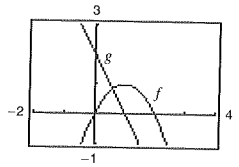
67. $g(x) = \dfrac{f(x + 0.01) - f(x)}{0.01}$

$= \left[2(x + 0.01) - (x + 0.01)^2 - 2x + x^2\right]100$

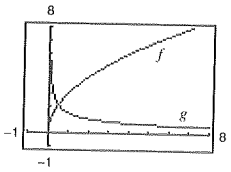
$= 2 - 2x - 0.01$



The graph of $g(x)$ is approximately the graph of $f'(x) = 2 - 2x$.

68. $g(x) = \dfrac{f(x + 0.01) - f(x)}{0.01}$

$= \left(3\sqrt{x + 0.01} - 3\sqrt{x}\right)100$



The graph of $g(x)$ is approximately the graph of $f'(x) = \dfrac{3}{2\sqrt{x}}$.

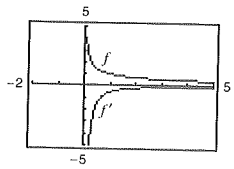
69. $f(2) = 2(4 - 2) = 4, f(2.1) = 2.1(4 - 2.1) = 3.99$

$f'(2) \approx \dfrac{3.99 - 4}{2.1 - 2} = -0.1 \quad [\text{Exact: } f'(2) = 0]$

70. $f(2) = \dfrac{1}{4}(2^3) = 2, f(2.1) = 2.31525$

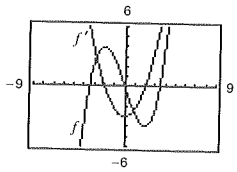
$f'(2) \approx \dfrac{2.31525 - 2}{2.1 - 2} = 3.1525 [\text{Exact: } f'(2) = 3]$

71. $f(x) = \dfrac{1}{\sqrt{x}}$ and $f'(x) = \dfrac{-1}{2x^{3/2}}$.



As $x \to \infty$, f is nearly horizontal and thus $f' \approx 0$.

72. $f(x) = \dfrac{x^3}{4} - 3x$ and $f'(x) = \dfrac{3}{4}x^2 - 3$



When f is increasing, $f' > 0$.

When f is decreasing, $f' < 0$.

73. $f(x) = x^2 - 5, c = 3$

$f'(3) = \lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$

$= \lim\limits_{x \to 3} \dfrac{x^2 - 5 - (9 - 5)}{x - 3}$

$= \lim\limits_{x \to 3} \dfrac{(x - 3)(x + 3)}{x - 3}$

$= \lim\limits_{x \to 3} (x + 3) = 6$

74. $g(x) = x(x - 1) = x^2 - x, c = 1$

$g'(1) = \lim\limits_{x \to 1} \dfrac{g(x) - g(1)}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{x^2 - x - 0}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{x(x - 1)}{x - 1} = \lim\limits_{x \to 1} x = 1$

75. $f(x) = x^3 + 2x^2 + 1, c = -2$

$f'(-2) = \lim\limits_{x \to -2} \dfrac{f(x) - f(-2)}{x + 2}$

$= \lim\limits_{x \to -2} \dfrac{(x^3 + 2x^2 + 1) - 1}{x + 2}$

$= \lim\limits_{x \to -2} \dfrac{x^2(x + 2)}{x + 2} = \lim\limits_{x \to -2} x^2 = 4$

76. $f(x) = x^3 + 6x, c = 2$

$f'(2) = \lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$

$= \lim\limits_{x \to 2} \dfrac{(x^3 + 6x) - 20}{x - 2}$

$= \lim\limits_{x \to 2} \dfrac{(x - 2)(x^2 + 2x + 10)}{x - 2}$

$= \lim\limits_{x \to 2} (x^2 + 2x + 10) = 18$

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77. $g(x) = \sqrt{|x|}, c = 0$

$$g'(0) = \lim_{x\to 0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0}\frac{\sqrt{|x|}}{x}.$$ Does not exist.

As $x \to 0^-$, $\dfrac{\sqrt{|x|}}{x} = \dfrac{-1}{\sqrt{|x|}} \to -\infty.$

As $x \to 0^+$, $\dfrac{\sqrt{|x|}}{x} = \dfrac{1}{\sqrt{x}} \to \infty.$

Therefore $g(x)$ is not differentiable at $x = 0$.

78. $f(x) = \dfrac{2}{x}, c = 5$

$$f'(5) = \lim_{x\to 5}\frac{f(x) - f(5)}{x - 5}$$

$$= \lim_{x\to 5}\frac{\frac{2}{x} - \frac{2}{5}}{x - 5}$$

$$= \lim_{x\to 5}\frac{2(5 - x)}{5x(x - 5)}$$

$$= \lim_{x\to 5} -\frac{2}{5x} = -\frac{2}{25}$$

79. $f(x) = (x - 6)^{2/3}, c = 6$

$$f'(6) = \lim_{x\to 6}\frac{f(x) - f(6)}{x - 6}$$

$$= \lim_{x\to 6}\frac{(x - 6)^{2/3} - 0}{x - 6} = \lim_{x\to 6}\frac{1}{(x - 6)^{1/3}}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

80. $g(x) = (x + 3)^{1/3}, c = -3$

$$g'(-3) = \lim_{x\to -3}\frac{g(x) - g(-3)}{x - (-3)}$$

$$= \lim_{x\to -3}\frac{(x + 3)^{1/3} - 0}{x + 3} = \lim_{x\to -3}\frac{1}{(x + 3)^{2/3}}.$$

Does not exist.

Therefore $g(x)$ is not differentiable at $x = -3$.

81. $h(x) = |x + 7|, c = -7$

$$h'(-7) = \lim_{x\to -7}\frac{h(x) - h(-7)}{x - (-7)}$$

$$= \lim_{x\to -7}\frac{|x + 7| - 0}{x + 7} = \lim_{x\to -7}\frac{|x + 7|}{x + 7}.$$

Does not exist.

Therefore $h(x)$ is not differentiable at $x = -7$.

82. $f(x) = |x - 6|, c = 6$

$$f'(6) = \lim_{x\to 6}\frac{f(x) - f(6)}{x - 6}$$

$$= \lim_{x\to 6}\frac{|x - 6| - 0}{x - 6} = \lim_{x\to 6}\frac{|x - 6|}{x - 6}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

83. $f(x)$ is differentiable everywhere except at $x = 3$. (Discontinuity)

84. $f(x)$ is differentiable everywhere except at $x = \pm 3$. (Sharp turns in the graph)

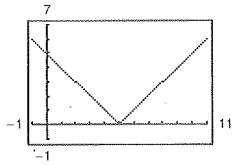
85. $f(x)$ is differentiable everywhere except at $x = -4$. (Sharp turn in the graph)

86. $f(x)$ is differentiable everywhere except at $x = \pm 2$. (Discontinuities)

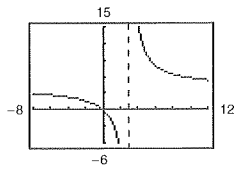
87. $f(x)$ is differentiable on the interval $(1, \infty)$. (At $x = 1$ the tangent line is vertical.)

88. $f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

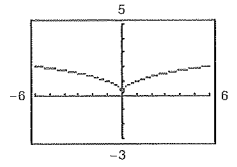
89. $f(x) = |x - 5|$ is differentiable everywhere except at $x = -5$. There is a sharp corner at $x = 5$.



90. $f(x) = \dfrac{4x}{x - 3}$ is differentiable everywhere except at $x = 3$. f is not defined at $x = 3$. (Vertical asymptote)



91. $f(x) = x^{2/5}$ is differentiable for all $x \neq 0$. There is a sharp corner at $x = 0$.



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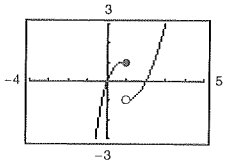
92. f is differentiable for all $x \neq 1$.

f is not continuous at $x = 1$.



93. $f(x) = |x - 1|$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{|x - 1| - 0}{x - 1} = -1.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{|x - 1| - 0}{x - 1} = 1.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

94. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1}$$

$$= \lim_{x \to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}}$$

$$= \lim_{x \to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty.$$

(Vertical tangent)

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

95. $f(x) = \begin{cases} (x - 1)^3, & x \leq 1 \\ (x - 1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(x - 1)^3 - 0}{x - 1}$$

$$= \lim_{x \to 1^-} (x - 1)^2 = 0.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)^2 - 0}{x - 1}$$

$$= \lim_{x \to 1^+} (x - 1) = 0.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $(f'(1) = 0)$

96. $f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{x - 1}{x - 1} = \lim_{x \to 1^-} 1 = 1.$$

The derivative from the right is

$$\lim_{x \to 1^+} = \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} (x + 1) = 2.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

97. Note that f is continuous at $x = 2$.

$$f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{(x^2 + 1) - 5}{x - 2}$$

$$= \lim_{x \to 2^-} (x + 2) = 4.$$

The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x \to 2^+} 4 = 4.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $(f'(2) = 4)$

98. Note that f is continuous at $x = 2$.

$$f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{\left(\frac{1}{2}x + 1\right) - 2}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{\frac{1}{2}(x - 2)}{x - 2} = \frac{1}{2}.$$

The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{\sqrt{2x} - 2}{x - 2} \cdot \frac{\sqrt{2x} + 2}{\sqrt{2x} + 2}$$

$$= \lim_{x \to 2^+} \frac{2x - 4}{(x - 2)\left(\sqrt{2x} + 2\right)}$$

$$= \lim_{x \to 2^+} \frac{2(x - 2)}{(x - 2)\left(\sqrt{2x} + 2\right)}$$

$$= \lim_{x \to 2^+} \frac{2}{\sqrt{2x} + 2} = \frac{1}{2}.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = \frac{1}{2}\right)$

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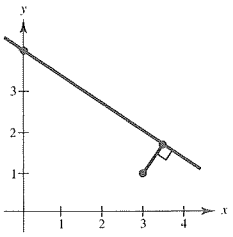
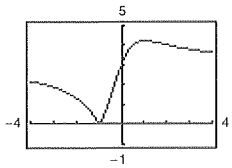
99. (a) The distance from $(3, 1)$ to the line $mx - y + 4 = 0$ is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \dfrac{|m(3) - 1(1) + 4|}{\sqrt{m^2 + 1}} = \dfrac{|3m + 3|}{\sqrt{m^2 + 1}}$

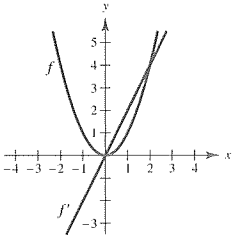


(b)

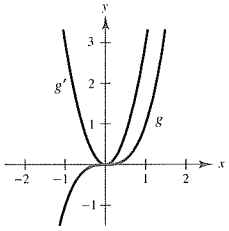


The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point $(3, 1)$.

100. (a) $f(x) = x^2$ and $f'(x) = 2x$



(b) $g(x) = x^3$ and $g'(x) = 3x^2$



(c) The derivative is a polynomial of degree 1 less than the original function. If $h(x) = x^n$, then $h'(x) = nx^{n-1}$.

(d) If $f(x) = x^4$, then

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - x^4}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x\left(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3\right)}{\Delta x} = \lim_{\Delta x \to 0}\left(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3\right) = 4x^3. \end{aligned}$$

So, if $f(x) = x^4$, then $f'(x) = 4x^3$ which is consistent with the conjecture. However, this is not a proof because you must verify the conjecture for all integer values of n, $n \geq 2$.

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101. False. The slope is $\lim_{\Delta x \to 0} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

102. False. $y = |x - 2|$ is continuous at $x = 2$, but is not differentiable at $x = 2$. (Sharp turn in the graph)

103. False. If the derivative from the left of a point does not equal the derivative from the right of a point, then the derivative does not exist at that point. For example, if $f(x) = |x|$, then the derivative from the left at $x = 0$ is -1 and the derivative from the right at $x = 0$ is 1. At $x = 0$, the derivative does not exist.

104. True—see Theorem 2.1.

105. $f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, you have $-|x| \leq x \sin(1/x) \leq |x|$, $x \neq 0$. So, $\lim_{x \to 0} x \sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative, you have

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x \sin(1/x) - 0}{x - 0} = \lim_{x \to 0} \left(\sin \frac{1}{x} \right).$$

Because this limit does not exist ($\sin(1/x)$ oscillates between -1 and 1), the function is not differentiable at $x = 0$.

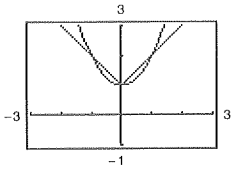
$$g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Using the Squeeze Theorem again, you have $-x^2 \leq x^2 \sin(1/x) \leq x^2$, $x \neq 0$. So, $\lim_{x \to 0} x^2 \sin(1/x) = 0 = g(0)$ and g is continuous at $x = 0$. Using the alternative form of the derivative again, you have

$$\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin(1/x) - 0}{x - 0} = \lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.

106.



As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph of $y_2 = |x| + 1$ always has a sharp corner at $(0, 1)$. y_2 is not differentiable at $(0, 1)$.

Section 2.2 Basic Differentiation Rules and Rates of Change

1. (a) $y = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$y'(1) = \frac{1}{2}$

(b) $y = x^3$

$y' = 3x^2$

$y'(1) = 3$

2. (a) $y = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$y'(1) = -\frac{1}{2}$

(b) $y = x^{-1}$

$y' = -x^{-2}$

$y'(1) = -1$

3. $y = 12$

$y' = 0$

4. $f(x) = -9$

$f'(x) = 0$

5. $y = x^7$

$y' = 7x^6$

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6. $y = x^{16}$

$y' = 16x^{15}$

7. $y = \dfrac{1}{x^5} = x^{-5}$

$y' = -5x^{-6} = -\dfrac{5}{x^6}$

8. $y = \dfrac{1}{x^8} = x^{-8}$

$y' = -8x^{-9} = -\dfrac{8}{x^9}$

9. $y = \sqrt[5]{x} = x^{1/5}$

$y' = \dfrac{1}{5}x^{-4/5} = \dfrac{1}{5x^{4/5}}$

10. $y = \sqrt[4]{x} = x^{1/4}$

$y' = \dfrac{1}{4}x^{-3/4} = \dfrac{1}{4x^{3/4}}$

11. $f(x) = x + 11$

$f'(x) = 1$

12. $g(x) = 3x - 1$

$g'(x) = 3$

13. $f(t) = -2t^2 + 3t - 6$

$f'(t) = -4t + 3$

14. $y = t^2 + 2t - 3$

$y' = 2t + 2$

15. $g(x) = x^2 + 4x^3$

$g'(x) = 2x + 12x^2$

16. $y = 8 - x^3$

$y' = -3x^2$

17. $s(t) = t^3 + 5t^2 - 3t + 8$

$s'(t) = 3t^2 + 10t - 3$

18. $f(x) = 2x^3 - x^2 + 3x$

$f'(x) = 6x^2 - 2x + 3$

19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$

$y' = \dfrac{\pi}{2}\cos\theta + \sin\theta$

20. $g(t) = \pi\cos t$

$g'(t) = -\pi\sin t$

21. $y = x^2 - \frac{1}{2}\cos x$

$y' = 2x + \frac{1}{2}\sin x$

22. $y = 7 + \sin x$

$y' = \cos x$

23. $y = \dfrac{1}{x} - 3\sin x$

$y' = -\dfrac{1}{x^2} - 3\cos x$

24. $y = \dfrac{5}{(2x)^3} + 2\cos x = \dfrac{5}{8}x^{-3} + 2\cos x$

$y' = \dfrac{5}{8}(-3)x^{-4} - 2\sin x = -\dfrac{15}{8x^4} - 2\sin x$

	Function	*Rewrite*	*Differentiate*	*Simplify*
25.	$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\dfrac{5}{x^3}$
26.	$y = \dfrac{2}{3x^2}$	$y = \dfrac{2}{3}x^{-2}$	$y' = -\dfrac{4}{3}x^{-3}$	$y' = -\dfrac{4}{3x^3}$
27.	$y = \dfrac{6}{(5x)^3}$	$y = \dfrac{6}{125}x^{-3}$	$y' = -\dfrac{18}{125}x^{-4}$	$y' = -\dfrac{18}{125x^4}$
28.	$y = \dfrac{\pi}{(3x)^2}$	$y = \dfrac{\pi}{9}x^{-2}$	$y' = -\dfrac{2\pi}{9}x^{-3}$	$y' = -\dfrac{2\pi}{9x^3}$
29.	$y = \dfrac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\dfrac{1}{2}x^{-3/2}$	$y' = -\dfrac{1}{2x^{3/2}}$
30.	$y = \dfrac{4}{x^{-3}}$	$y = 4x^3$	$y' = 12x^2$	$y' = 12x^2$

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31. $f(x) = \dfrac{8}{x^2} = 8x^{-2}, (2, 2)$

$f'(x) = -16x^{-3} = -\dfrac{16}{x^3}$

$f'(2) = -2$

32. $f(t) = 3 - \dfrac{3}{5t}, \left(\dfrac{3}{5}, 2\right)$

$f'(t) = \dfrac{3}{5t^2}$

$f'\left(\dfrac{3}{5}\right) = \dfrac{5}{3}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3, \left(0, -\frac{1}{2}\right)$

$f'(x) = \frac{21}{5}x^2$

$f'(0) = 0$

34. $y = 3x^3 - 10, (2, 14)$

$y' = 9x^2$

$y'(2) = 36$

35. $y = (4x + 1)^2, (0, 1)$

$= 16x^2 + 8x + 1$

$y' = 32x + 8$

$y'(0) = 8$

36. $f(x) = 3(5 - x)^2, (5, 0)$

$= 3x^2 - 30x + 75$

$f'(x) = 6x - 30$

$f'(5) = 0$

37. $f(\theta) = 4 \sin \theta - \theta, (0, 0)$

$f'(\theta) = 4 \cos \theta - 1$

$f'(0) = 4(1) - 1 = 3$

38. $g(t) = -2 \cos t + 5, (\pi, 7)$

$g'(t) = 2 \sin t$

$g'(\pi) = 0$

39. $f(x) = x^2 + 5 - 3x^{-2}$

$f'(x) = 2x + 6x^{-3} = 2x + \dfrac{6}{x^3}$

40. $f(x) = x^2 - 3x - 3x^{-2}$

$f'(x) = 2x - 3 + 6x^{-3}$

$= 2x - 3 + \dfrac{6}{x^3}$

41. $g(t) = t^2 - \dfrac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t + 12t^{-4} = 2t + \dfrac{12}{t^4}$

42. $f(x) = x + x^{-2}$

$f'(x) = 1 - 2x^{-3}$

$= 1 - \dfrac{2}{x^3}$

43. $f(x) = \dfrac{4x^3 + 3x^2}{x} = 4x^2 + 3x$

$f'(x) = 8x + 3$

44. $f(x) = \dfrac{x^3 - 6}{x^2} = x - 6x^{-2}$

$f'(x) = 1 + 12x^{-3} = 1 + \dfrac{12}{x^3}$

45. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \dfrac{8}{x^3} = \dfrac{x^3 - 8}{x^3}$

46. $h(x) = \dfrac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$h'(x) = 2 - \dfrac{1}{x^2} = \dfrac{2x^2 - 1}{x^2}$

47. $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

48. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = 36x - 45x^2$

49. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \dfrac{1}{2}x^{-1/2} - 2x^{-2/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$

50. $f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$

$f'(x) = \dfrac{1}{3}x^{-2/3} + \dfrac{1}{5}x^{-4/5} = \dfrac{1}{3x^{2/3}} + \dfrac{1}{5x^{4/5}}$

51. $h(s) = s^{4/5} - s^{2/3}$

$h'(s) = \dfrac{4}{5}s^{-1/5} - \dfrac{2}{3}s^{-1/3} = \dfrac{4}{5s^{1/5}} - \dfrac{2}{3s^{1/3}}$

52. $f(t) = t^{2/3} - t^{1/3} + 4$

$f'(t) = \dfrac{2}{3}t^{-1/3} - \dfrac{1}{3}t^{-2/3} = \dfrac{2}{3t^{1/3}} - \dfrac{1}{3t^{2/3}}$

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53. $f(x) = 6\sqrt{x} + 5\cos x = 6x^{1/2} + 5\cos x$

$f'(x) = 3x^{-1/2} - 5\sin x = \dfrac{3}{\sqrt{x}} - 5\sin x$

54. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = -\dfrac{2}{3}x^{-4/3} - 3\sin x = -\dfrac{2}{3x^{4/3}} - 3\sin x$

55. (a) $y = x^4 - 3x^2 + 2$

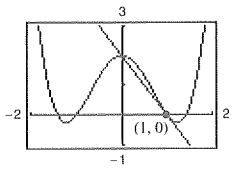
$y' = 4x^3 - 6x$

At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$

Tangent line: $y - 0 = -2(x - 1)$

$2x + y - 2 = 0$

(b)



56. (a) $y = x^3 + x$

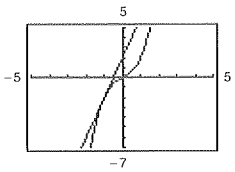
$y' = 3x^2 + 1$

At $(-1, -2)$: $y' = 3(-1)^2 + 1 = 4$

Tangent line: $y + 2 = 4(x + 1)$

$4x - y + 2 = 0$

(b)



57. (a) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\dfrac{3}{2}x^{-7/4} = -\dfrac{3}{2x^{7/4}}$

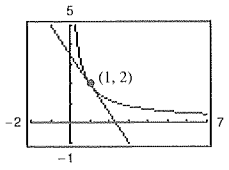
At $(1, 2)$: $f'(1) = -\dfrac{3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

$y = -\dfrac{3}{2}x + \dfrac{7}{2}$

$3x + 2y - 7 = 0$

(b)



58. (a) $y = (x^2 + 2x)(x + 1)$

$= x^3 + 3x^2 + 2x$

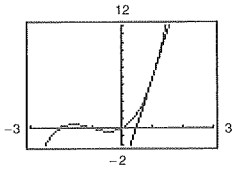
$y' = 3x^2 + 6x + 2$

At $(1, 6)$: $y' = 3(1)^2 + 6(1) + 2 = 11$

Tangent line: $y - 6 = 11(x - 1)$

$0 = 11x - y - 5$

(b)



59. $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x - 1)(x + 1)$

$y' = 0 \Rightarrow x = 0, \pm 1$

Horizontal tangents: $(0, 3), (1, 2), (-1, 2)$

60. $y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

61. $y = \dfrac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = -\dfrac{2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

62. $y = x^2 + 9$

$y' = 2x = 0 \Rightarrow x = 0$

At $x = 0$, $y = 1$.

Horizontal tangent: $(0, 9)$

63. $y = x + \sin x, 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi$: $y = \pi$

Horizontal tangent: (π, π)

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64. $y = \sqrt{3}x + 2\cos x,\ 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

At $x = \dfrac{\pi}{3}$: $y = \dfrac{\sqrt{3}\pi + 3}{3}$

At $x = \dfrac{2\pi}{3}$: $y = \dfrac{2\sqrt{3}\pi - 3}{3}$

Horizontal tangents: $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}\pi + 3}{3}\right), \left(\dfrac{2\pi}{3}, \dfrac{2\sqrt{3}\pi - 3}{3}\right)$

65. $x^2 - kx = 5x - 4$ Equate functions.

$2x - k = 5$ Equate derivatives.

So, $k = 2x - 5$ and

$x^2 - (2x - 5)x = 5x - 4 \Rightarrow -x^2 = -4 \Rightarrow x = \pm 2.$

For $x = 2$, $k = -1$ and for $x = -2$, $k = -9$.

66. $k - x^2 = -6x + 1$ Equate functions.

$-2x = -6$ Equate derivatives.

So, $x = 3$ and $k - 9 = -18 + 1 \Rightarrow k = -8$.

67. $\dfrac{k}{x} = -\dfrac{3}{4}x + 3$ Equate functions.

$-\dfrac{k}{x^2} = -\dfrac{3}{4}$ Equate derivatives.

So, $k = \dfrac{3}{4}x^2$ and

$$\frac{\frac{3}{4}x^2}{x} = -\frac{3}{4}x + 3 \Rightarrow \frac{3}{4}x = -\frac{3}{4}x + 3$$

$$\Rightarrow \frac{3}{2}x = 3 \Rightarrow x = 2 \Rightarrow k = 3.$$

68. $k\sqrt{x} = x + 4$ Equate functions.

$\dfrac{k}{2\sqrt{x}} = 1$ Equate derivatives.

So, $k = 2\sqrt{x}$ and

$\left(2\sqrt{x}\right)\sqrt{x} = x + 4 \Rightarrow 2x = x + 4 \Rightarrow x = 4 \Rightarrow k = 4.$

69. $kx^3 = x + 1$ Equate equations.

$3kx^2 = 1$ Equate derivatives.

So, $k = \dfrac{1}{3x^2}$ and

$$\left(\frac{1}{3x^2}\right)x^3 = x + 1$$

$$\frac{1}{3}x = x + 1$$

$$x = -\frac{3}{2},\ k = \frac{4}{27}.$$

70. $kx^4 = 4x - 1$ Equate equations.

$4kx^3 = 4$ Equate derivatives.

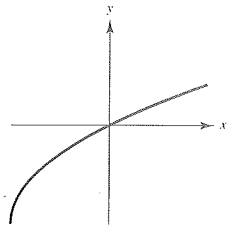
So, $k = \dfrac{1}{x^3}$ and

$$\left(\frac{1}{x^3}\right)x^4 = 4x - 1$$

$$x = 4x - 1$$

$$x = \frac{1}{3} \quad \text{and} \quad k = 27.$$

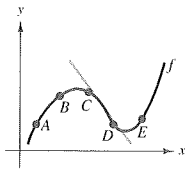
71. The graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing (i.e., $f'' < 0$) would, in general, look like the graph below.



72. (a) The slope appears to be steepest between A and B.

(b) The average rate of change between A and B is **greater** than the instantaneous rate of change at B.

(c)



73. $g(x) = f(x) + 6 \Rightarrow g'(x) = f'(x)$

74. $g(x) = -5f(x) \Rightarrow g'(x) = -5f'(x)$

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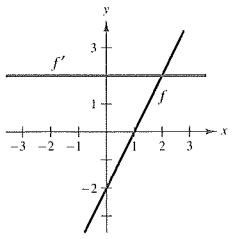
75.

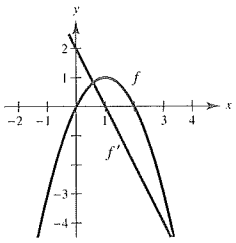


If f is linear then its derivative is a constant function.

$f(x) = ax + b$

$f'(x) = a$

76.



If f is quadratic, then its derivative is a linear function.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

77. Let (x_1, y_1) and (x_2, y_2) be the points of tangency on $y = x^2$ and $y = -x^2 + 6x - 5$, respectively.

The derivatives of these functions are:

$y' = 2x \Rightarrow m = 2x_1$ and $y' = -2x + 6 \Rightarrow m = -2x_2 + 6$

$m = 2x_1 = -2x_2 + 6$

$x_1 = -x_2 + 3$

Because $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(x_1^2\right)}{x_2 - x_1} = -2x_2 + 6$$

$$\frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(-x_2 + 3\right)^2}{x_2 - \left(-x_2 + 3\right)} = -2x_2 + 6$$

$$\left(-x_2^2 + 6x_2 - 5\right) - \left(x_2^2 - 6x_2 + 9\right) = \left(-2x_2 + 6\right)\left(2x_2 - 3\right)$$

$$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$$

$$2x_2^2 - 6x_2 + 4 = 0$$

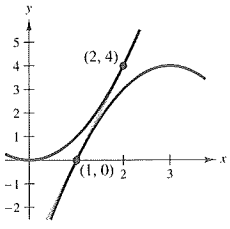
$$2\left(x_2 - 2\right)\left(x_2 - 1\right) = 0$$

$$x_2 = 1 \text{ or } 2$$

$x_2 = 1 \Rightarrow y_2 = 0, x_1 = 2$ and $y_1 = 4$

So, the tangent line through $(1, 0)$ and $(2, 4)$ is

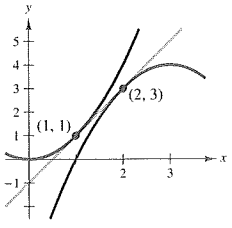
$$y - 0 = \left(\frac{4 - 0}{2 - 1}\right)(x - 1) \Rightarrow y = 4x - 4.$$



$x_2 = 2 \Rightarrow y_2 = 3, x_1 = 1$ and $y_1 = 1$

So, the tangent line through $(2, 3)$ and $(1, 1)$ is

$$y - 1 = \left(\frac{3 - 1}{2 - 1}\right)(x - 1) \Rightarrow y = 2x - 1.$$



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78. m_1 is the slope of the line tangent to $y = x$. m_2 is the slope of the line tangent to $y = 1/x$. Because

$$y = x \Rightarrow y' = 1 \Rightarrow m_1 = 1 \text{ and } y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} \Rightarrow m_2 = -\frac{1}{x^2}.$$

The points of intersection of $y = x$ and $y = 1/x$ are

$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$$

At $x = \pm 1$, $m_2 = -1$. Because $m_2 = -1/m_1$, these tangent lines are perpendicular at the points of intersection.

79. $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

Because $|\cos x| \leq 1$, $f'(x) \neq 0$ for all x and f does not have a horizontal tangent line.

80. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5$

Because $5x^4 + 9x^2 \geq 0$, $f'(x) \geq 5$. So, f does not have a tangent line with a slope of 3.

81. $f(x) = \sqrt{x}, (-4, 0)$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{0 - y}{-4 - x}$$

$$4 + x = 2\sqrt{x}y$$

$$4 + x = 2\sqrt{x}\sqrt{x}$$

$$4 + x = 2x$$

$$x = 4, y = 2$$

The point $(4, 2)$ is on the graph of f.

Tangent line: $y - 2 = \frac{0 - 2}{-4 - 4}(x - 4)$

$$4y - 8 = x - 4$$

$$0 = x - 4y + 4$$

82. $f(x) = \frac{2}{x}, (5, 0)$

$$f'(x) = -\frac{2}{x^2}$$

$$-\frac{2}{x^2} = \frac{0 - y}{5 - x}$$

$$-10 + 2x = -x^2y$$

$$-10 + 2x = -x^2\left(\frac{2}{x}\right)$$

$$-10 + 2x = -2x$$

$$4x = 10$$

$$x = \frac{5}{2}, y = \frac{4}{5}$$

The point $\left(\frac{5}{2}, \frac{4}{5}\right)$ is on the graph of f. The slope of the tangent line is $f'\left(\frac{5}{2}\right) = -\frac{8}{25}$.

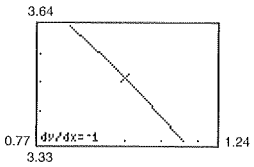
Tangent line: $y - \frac{4}{5} = -\frac{8}{25}\left(x - \frac{5}{2}\right)$

$$25y - 20 = -8x + 20$$

$$8x + 25y - 40 = 0$$

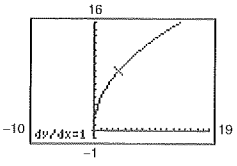
83. $f'(1)$ appears to be close to -1.

$f'(1) = -1$



84. $f'(4)$ appears to be close to 1.

$f'(4) = 1$



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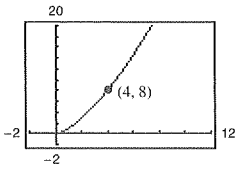
85. (a) One possible secant is between $(3.9, 7.7019)$ and $(4, 8)$:

$$y - 8 = \frac{8 - 7.7019}{4 - 3.9}(x - 4)$$
$$y - 8 = 2.981(x - 4)$$
$$y = S(x) = 2.981x - 3.924$$

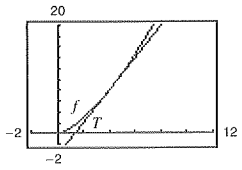


(b) $f'(x) = \frac{3}{2}x^{1/2} \Rightarrow f'(4) = \frac{3}{2}(2) = 3$

$T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c) As you move further away from $(4, 8)$, the accuracy of the approximation T gets worse.



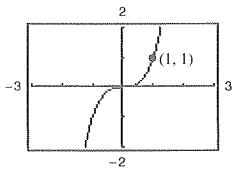
(d)

Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	−1	2	5	6.5	7.7	8	8.3	9.5	11	14	17

86. (a) Nearby point: $(1.0073138, 1.0221024)$

Secant line: $y - 1 = \frac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

$$y = 3.022(x - 1) + 1$$

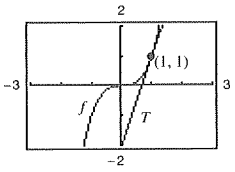


(Answers will vary.)

(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens as you move away from $(1, 1)$.



(d)

Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(x)$	−8	−1	0	0.125	0.729	1	1.331	3.375	8	27	64
$T(x)$	−8	−5	−2	−0.5	0.7	1	1.3	2.5	4	7	10

The accuracy decreases more rapidly than in Exercise 85 because $y = x^3$ is less "linear" than $y = x^{3/2}$.

87. False. Let $f(x) = x$ and $g(x) = x + 1$. Then $f'(x) = g'(x) = x$, but $f(x) \neq g(x)$.

88. True. If $f(x) = g(x) + c$, then $f'(x) = g'(x) + 0 = g'(x)$.

89. False. If $y = \pi^2$, then $dy/dx = 0$. (π^2 is a constant.)

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90. True. If $y = x/\pi = (1/\pi) \cdot x$, then
$dy/dx = (1/\pi)(1) = 1/\pi$.

91. True. If $g(x) = 3f(x)$, then $g'(x) = 3f'(x)$.

92. False. If $f(x) = \dfrac{1}{x_n} = x^{-n}$, then
$f'(x) = -nx^{-n-1} = \dfrac{-n}{x^{n+1}}$.

93. $f(t) = 4t + 5, \quad [1, 2]$

$f'(t) = 4$. So, $f'(1) = f'(2) = 4$.

Instantaneous rate of change is the constant 4. Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{13 - 9}{1} = 4$$

(These are the same because f is a line of slope 4)

94. $f(t) = t^2 - 7, \quad [3, 3.1]$

$f'(t) = 2t$

Instantaneous rate of change:

At $(3, 2)$: $f'(3) = 6$

At $(3.1, 2.61)$: $f'(3.1) = 6.2$

Average rate of change:

$$\frac{f(3.1) - f(3)}{3.1 - 3} = \frac{2.61 - 2}{0.1} = 6.1$$

95. $f(x) = -\dfrac{1}{x}, \quad [1, 2]$

$f'(x) = \dfrac{1}{x^2}$

Instantaneous rate of change:

$(1, -1) \Rightarrow f'(1) = 1$

$\left(2, -\dfrac{1}{2}\right) \Rightarrow f'(2) = \dfrac{1}{4}$

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{(-1/2) - (-1)}{2 - 1} = \frac{1}{2}$$

96. $f(x) = \sin x, \quad \left[0, \dfrac{\pi}{6}\right]$

$f'(x) = \cos x$

Instantaneous rate of change:

$(0, 0) \Rightarrow f'(0) = 1$

$\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right) \Rightarrow f'\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2} \approx 0.866$

Average rate of change:

$$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$$

97. (a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) $\dfrac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

(c) $v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec

When $t = 2$: $v(2) = -64$ ft/sec

(d) $-16t^2 + 1362 = 0$

$$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

(e) $v\left(\dfrac{\sqrt{1362}}{4}\right) = -32\left(\dfrac{\sqrt{1362}}{4}\right)$

$= -8\sqrt{1362} \approx -295.242$ ft/sec

98.

$$\begin{aligned} s(t) &= -16t^2 - 22t + 220 \\ v(t) &= -32t - 22 \\ v(3) &= -118 \text{ ft/sec} \\ s(t) &= -16t^2 - 22t + 220 \\ &= 112 \text{ (height after falling 108 ft)} \\ -16t^2 - 22t + 108 &= 0 \\ -2(t - 2)(8t + 27) &= 0 \\ t &= 2 \\ v(2) &= -32(2) - 22 \\ &= -86 \text{ ft/sec} \end{aligned}$$

99.

$$\begin{aligned} s(t) &= -4.9t^2 + v_0 t + s_0 \\ &= -4.9t^2 + 120t \\ v(t) &= -9.8t + 120 \\ v(5) &= -9.8(5) + 120 = 71 \text{ m/sec} \\ v(10) &= -9.8(10) + 120 = 22 \text{ m/sec} \end{aligned}$$

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100. $s(t) = -4.9t^2 + v_0t + s_0$

$= -4.9t^2 + s_0 = 0$ when $t = 5.6$.

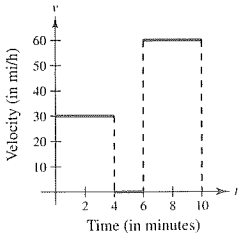
$s_0 = 4.9t^2 = 4.9(5.6)^2 \approx 153.7$ m

101. From $(0, 0)$ to $(4, 2)$, $s(t) = \frac{1}{2}t \Rightarrow v(t) = \frac{1}{2}$ mi/min.

$v(t) = \frac{1}{2}(60) = 30$ mi/h for $0 < t < 4$

Similarly, $v(t) = 0$ for $4 < t < 6$. Finally, from $(6, 2)$ to $(10, 6)$,

$s(t) = t - 4 \Rightarrow v(t) = 1$ mi/min. $= 60$ mi/h.



(The velocity has been converted to miles per hour.)

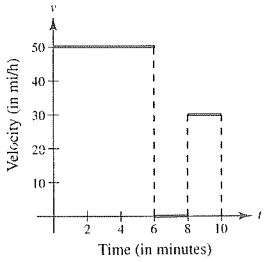
102. From $(0, 0)$ to $(6, 5)$, $s(t) = \frac{5}{6}t \Rightarrow v(t) = \frac{5}{6}$ mi/min.

$v(t) = \frac{5}{6}(60) = 50$ mi/h for $0 < t < 6$

Similarly, $v(t) = 0$ for $6 < t < 8$.

Finally, from $(8, 5)$ to $(10, 6)$,

$s(t) = \frac{1}{2}t + 1 \Rightarrow v(t) = \frac{1}{2}$ mi/min $= 30$ mi/h.



(The velocity has been converted to miles per hour.)

103. $v = 40$ mi/h $= \frac{2}{3}$ mi/min

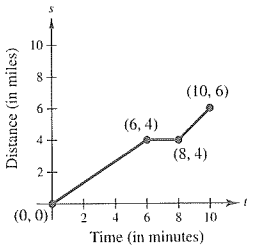
$(\frac{2}{3}$ mi/min$)(6$ min$) = 4$ mi

$v = 0$ mi/h $= 0$ mi/min

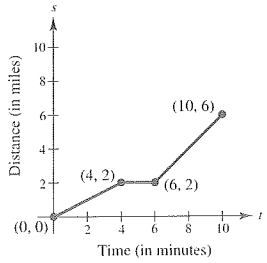
$(0$ mi/min$)(2$ min$) = 0$ mi

$v = 60$ mi/h $= 1$ mi/min

$(1$ mi/min$)(2$ min$) = 2$ mi



104. This graph corresponds with Exercise 101.



105. (a) Using a graphing utility,

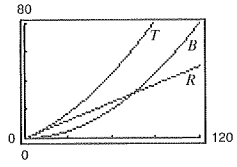
$R(v) = 0.417v - 0.02$.

(b) Using a graphing utility,

$B(v) = 0.0056v^2 + 0.001v + 0.04$.

(c) $T(v) = R(v) + B(v) = 0.0056v^2 + 0.418v + 0.02$

(d)



(e) $\frac{dT}{dv} = 0.0112v + 0.418$

For $v = 40$, $T'(40) \approx 0.866$

For $v = 80$, $T'(80) \approx 1.314$

For $v = 100$, $T'(100) \approx 1.538$

(f) For increasing speeds, the total stopping distance increases.

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106. $C = (\text{gallons of fuel used})(\text{cost per gallon})$

$$= \left(\frac{15{,}000}{x}\right)(2.76) = \frac{41{,}400}{x}$$

$$\frac{dC}{dx} = -\frac{41{,}400}{x^2}$$

x	10	15	20	25	30	35	40
C	4140	2760	2070	1656	1380	1183	1035
$\frac{dC}{dx}$	–414	–184	–103.5	–66.2	–46	–33.8	–25.9

The driver who gets 15 miles per gallon would benefit more. The rate of change at $x = 15$ is larger in absolute value than that at $x = 35$.

107. $V = s^3, \frac{dV}{ds} = 3s^2$

When $s = 6$ cm, $\frac{dV}{ds} = 108$ cm^3 per cm change in s.

108. $A = s^2, \frac{dA}{ds} = 2s$

When $s = 6$ m, $\frac{dA}{ds} = 12$ m^2 per m change in s.

109. $s(t) = -\frac{1}{2}at^2 + c$ and $s'(t) = -at$

$$\text{Average velocity: } \frac{s(t_0 + \Delta t) - s(t_0 - \Delta t)}{(t_0 + \Delta t) - (t_0 - \Delta t)} = \frac{\left[-(1/2)a(t_0 + \Delta t)^2 + c\right] - \left[-(1/2)a(t_0 - \Delta t)^2 + c\right]}{2\Delta t}$$

$$= \frac{-(1/2)a\left(t_0^2 + 2t_0\Delta t + (\Delta t)^2\right) + (1/2)a\left(t_0^2 - 2t_0\Delta t + (\Delta t)^2\right)}{2\,\Delta t}$$

$$= \frac{-2at_0\,\Delta t}{2\,\Delta t} = -at_0 = s'(t_0) \qquad \text{instantaneous velocity at } t = t_0$$

110. $C = \frac{1{,}008{,}000}{Q} + 6.3Q$

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q} + 6.3$$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350$, $\frac{dC}{dQ} \approx -\$1.93$.

111. $N = f(p)$

(a) $f'(2.979)$ is the rate of change of the number of gallons of gasoline sold when the price is \$2.979 per gallon.

(b) $f'(2.979)$ is usually negative. As prices go up, sales go down.

112. $\frac{dT}{dt} = K(T - T_a)$

113. $y = ax^2 + bx + c$

Because the parabola passes through $(0, 1)$ and $(1, 0)$, you have:

$(0, 1)$: $1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$(1, 0)$: $0 = a(1)^2 + b(1) + 1 \Rightarrow b = -a - 1$

So, $y = ax^2 + (-a - 1)x + 1$. From the tangent line $y = x - 1$, you know that the derivative is 1 at the point $(1, 0)$.

$y' = 2ax + (-a - 1)$

$1 = 2a(1) + (-a - 1)$

$1 = a - 1$

$a = 2$

$b = -a - 1 = -3$

Therefore, $y = 2x^2 - 3x + 1$.

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114. $y = \frac{1}{x}, x > 0$

$y' = -\frac{1}{x^2}$

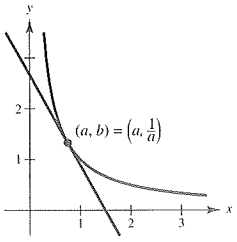
At (a, b), the equation of the tangent line is

$y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$ or $y = -\frac{x}{a^2} + \frac{2}{a}$.

The x-intercept is $(2a, 0)$.

The y-intercept is $\left(0, \frac{2}{a}\right)$.

The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.



115. $y = x^3 - 9x$

$y' = 3x^2 - 9$

Tangent lines through $(1, -9)$:

$y + 9 = (3x^2 - 9)(x - 1)$

$(x^3 - 9x) + 9 = 3x^3 - 3x^2 - 9x + 9$

$0 = 2x^3 - 3x^2 = x^2(2x - 3)$

$x = 0$ or $x = \frac{3}{2}$

The points of tangency are $(0, 0)$ and $\left(\frac{3}{2}, -\frac{81}{8}\right)$.

At $(0, 0)$, the slope is $y'(0) = -9$. At $\left(\frac{3}{2}, -\frac{81}{8}\right)$, the slope is $y'\left(\frac{3}{2}\right) = -\frac{9}{4}$.

Tangent Lines:

$y - 0 = -9(x - 0)$ and $y + \frac{81}{8} = -\frac{9}{4}\left(x - \frac{3}{2}\right)$

$y = -9x$ $y = -\frac{9}{4}x - \frac{27}{4}$

$9x + y = 0$ $9x + 4y + 27 = 0$

116. $y = x^2$

$y' = 2x$

(a) Tangent lines through $(0, a)$:

$y - a = 2x(x - 0)$

$x^2 - a = 2x^2$

$-a = x^2$

$\pm\sqrt{-a} = x$

The points of tangency are $\left(\pm\sqrt{-a}, -a\right)$. At $\left(\sqrt{-a}, -a\right)$, the slope is $y'\left(\sqrt{-a}\right) = 2\sqrt{-a}$.

At $\left(-\sqrt{-a}, -a\right)$, the slope is $y'\left(-\sqrt{-a}\right) = -2\sqrt{-a}$.

Tangent lines: $y + a = 2\sqrt{-a}\left(x - \sqrt{-a}\right)$ and $y + a = -2\sqrt{-a}\left(x + \sqrt{-a}\right)$

$y = 2\sqrt{-a}x + a$ $y = -2\sqrt{-a}x + a$

Restriction: a must be negative.

(b) Tangent lines through $(a, 0)$:

$y - 0 = 2x(x - a)$

$x^2 = 2x^2 - 2ax$

$0 = x^2 - 2ax = x(x - 2a)$

The points of tangency are $(0, 0)$ and $(2a, 4a^2)$. At $(0, 0)$, the slope is $y'(0) = 0$. At $(2a, 4a^2)$, the slope is $y'(2a) = 4a$.

Tangent lines: $y - 0 = 0(x - 0)$ and $y - 4a^2 = 4a(x - 2a)$

$y = 0$ $y = 4ax - 4a^2$

Restriction: None, a can be any real number.

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117. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax^3 = 8a \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} (x^2 + b) = 4 + b \end{aligned}\right\} \quad \begin{aligned} 8a &= 4 + b \\ 8a - 4 &= b \end{aligned}$$

$$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$$

For f to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$$\begin{aligned} 3a(2)^2 &= 2(2) \\ 12a &= 4 \\ a &= \tfrac{1}{3} \\ b &= 8a - 4 = -\tfrac{4}{3} \end{aligned}$$

118. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \ge 0 \end{cases}$

$f(0) = b = \cos(0) = 1 \Rightarrow b = 1$

$$f'(x) = \begin{cases} -\sin x, & x < 0 \\ a, & x > 0 \end{cases}$$

So, $a = 0$.

Answer: $a = 0, b = 1$

119. $f_1(x) = |\sin x|$ is differentiable for all $x \ne n\pi$, n an integer.

$f_2(x) = \sin|x|$ is differentiable for all $x \ne 0$.

You can verify this by graphing f_1 and f_2 and observing the locations of the sharp turns.

120. Let $f(x) = \cos x$.

$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x\to 0} \sin x\left(\frac{\sin \Delta x}{\Delta x}\right) \\ &= 0 - \sin x(1) = -\sin x \end{aligned}$$

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

1. $g(x) = (x^2 + 3)(x^2 - 4x)$

$$\begin{aligned} g'(x) &= (x^2 + 3)(2x - 4) + (x^2 - 4x)(2x) \\ &= 2x^3 - 4x^2 + 6x - 12 + 2x^3 - 8x^2 \\ &= 4x^3 - 12x^2 + 6x - 12 \\ &= 2(2x^3 - 6x^2 + 3x - 6) \end{aligned}$$

2. $f(x) = (6x + 5)(x^3 - 2)$

$$\begin{aligned} f'(x) &= (6x + 5)(3x^2) + (x^3 - 2)(6) \\ &= 18x^3 + 15x^2 + 6x^3 - 12 \\ &= 24x^3 + 15x^2 - 12 \end{aligned}$$

3. $h(t) = \sqrt{t}(1 - t^2) = t^{1/2}(1 - t^2)$

$$\begin{aligned} h'(t) &= t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2} \\ &= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2} \\ &= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}} \\ &= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}} \end{aligned}$$

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4. $g(s) = \sqrt{s}(s^2 + 8) = s^{1/2}(s^2 + 8)$

$$g'(s) = s^{1/2}(2s) + (s^2 + 8)\frac{1}{2}s^{-1/2}$$
$$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$$
$$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$$
$$= \frac{5s^2 + 8}{2\sqrt{s}}$$

5. $f(x) = x^3 \cos x$

$$f'(x) = x^3(-\sin x) + \cos x(3x^2)$$
$$= 3x^2 \cos x - x^3 \sin x$$
$$= x^2(3 \cos x - x \sin x)$$

6. $g(x) = \sqrt{x} \sin x$

$$g'(x) = \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$$
$$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$$

7. $f(x) = \dfrac{x}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

8. $g(t) = \dfrac{t^2 + 4}{5t - 3}$

$$g'(t) = \frac{(5t - 3)(2t) - (t^2 + 4)(5)}{(5t - 3)^2}$$
$$= \frac{10t^2 - 6t - 5t^2 - 20}{(5t - 3)^2} = \frac{5t^2 - 6t - 20}{(5t - 3)^2}$$

9. $h(x) = \dfrac{\sqrt{x}}{x^3 + 1} = \dfrac{x^{1/2}}{x^3 + 1}$

$$h'(x) = \frac{(x^3 + 1)\frac{1}{2}x^{-1/2} - x^{1/2}(3x^2)}{(x^3 + 1)^2}$$
$$= \frac{x^3 + 1 - 6x^3}{2x^{1/2}(x^3 + 1)^2}$$
$$= \frac{1 - 5x^3}{2\sqrt{x}(x^3 + 1)^2}$$

10. $h(s) = \dfrac{s}{\sqrt{s} - 1}$

$$h'(s) = \frac{(\sqrt{s} - 1)(1) - s\left(\frac{1}{2}s^{-1/2}\right)}{(\sqrt{s} - 1)^2}$$
$$= \frac{\sqrt{s} - 1 - \frac{1}{2}\sqrt{s}}{(\sqrt{s} - 1)^2} = \frac{\sqrt{s} - 2}{2(\sqrt{s} - 1)^2}$$

11. $g(x) = \dfrac{\sin x}{x^2}$

$$g'(x) = \frac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \frac{x \cos x - 2 \sin x}{x^3}$$

12. $f(t) = \dfrac{\cos t}{t^3}$

$$f'(t) = \frac{t^3(-\sin t) - \cos t(3t^2)}{(t^3)^2} = -\frac{t \sin t + 3 \cos t}{t^4}$$

13. $f(x) = (x^3 + 4x)(3x^2 + 2x - 5)$

$$f'(x) = (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4)$$
$$= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20$$
$$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$$
$$f'(0) = -20$$

14. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$

$$f'(x) = (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2)$$
$$= 3x^2(x - 1)^2 + 2(x - 1)^2(x^2 + x + 1)$$
$$= (x - 1)^2(5x^2 + 2x + 2)$$
$$f'(1) = 0$$

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15. $f(x) = \dfrac{x^2 - 4}{x - 3}$

$$f'(x) = \frac{(x-3)(2x) - (x^2-4)(1)}{(x-3)^2}$$

$$= \frac{2x^2 - 6x - x^2 + 4}{(x-3)^2}$$

$$= \frac{x^2 - 6x + 4}{(x-3)^2}$$

$$f'(1) = \frac{1 - 6 + 4}{(1-3)^2} = -\frac{1}{4}$$

16. $f(x) = \dfrac{x + 5}{x - 5}$

$$f'(x) = \frac{(x-5)(1) - (x+5)(1)}{(x-5)^2}$$

$$= \frac{x - 5 - x - 5}{(x-5)^2}$$

$$= \frac{-10}{(x-5)^2}$$

$$f'(4) = \frac{-10}{(4-5)^2} = -10$$

17. $f(x) = x \cos x$

$$f'(x) = (x)(-\sin x) + (\cos x)(1) = \cos x - x \sin x$$

$$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{8}(4 - \pi)$$

18. $f(x) = \dfrac{\sin x}{x}$

$$f'(x) = \frac{(x)(\cos x) - (\sin x)(1)}{x^2}$$

$$= \frac{x \cos x - \sin x}{x^2}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{(\pi/6)\left(\sqrt{3}/2\right) - (1/2)}{\pi^2/36}$$

$$= \frac{3\sqrt{3}\pi - 18}{\pi^2}$$

$$= \frac{3\left(\sqrt{3}\pi - 6\right)}{\pi^2}$$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 3x}{7}$	$y = \dfrac{1}{7}x^2 + \dfrac{3}{7}x$	$y' = \dfrac{2}{7}x + \dfrac{3}{7}$	$y' = \dfrac{2x + 3}{7}$
20.	$y = \dfrac{5x^2 - 3}{4}$	$y = \dfrac{5}{4}x^2 - \dfrac{3}{4}$	$y' = \dfrac{10}{4}x$	$y' = \dfrac{5x}{2}$
21.	$y = \dfrac{6}{7x^2}$	$y = \dfrac{6}{7}x^{-2}$	$y' = -\dfrac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
22.	$y = \dfrac{10}{3x^3}$	$y = \dfrac{10}{3}x^{-3}$	$y' = -\dfrac{30}{3}x^{-4}$	$y' = -\dfrac{10}{x^4}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}, x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}, x > 0$
24.	$y = \dfrac{5x^2 - 8}{11}$	$y = \dfrac{5}{11}x^2 - \dfrac{8}{11}$	$y' = \dfrac{5}{11}(2x)$	$y' = \dfrac{10x}{11}$

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25. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$

$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$

$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2}, \; x \neq 1$$

26. $f(x) = \dfrac{x^3 + 5x + 3}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(3x^2 + 5) - (x^3 + 5x + 3)(2x)}{(x^2 - 1)^2}$$

$$= \frac{3x^4 + 5x^2 - 3x^2 - 5 - 2x^4 - 10x^2 - 6x}{(x^2 - 1)^2}$$

$$= \frac{x^4 - 8x^2 - 6x - 5}{(x^2 - 1)^2}$$

27. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right) = x - \dfrac{4x}{x + 3}$

$$f'(x) = 1 - \frac{(x + 3)4 - 4x(1)}{(x + 3)^2}$$

$$= \frac{(x^2 + 6x + 9) - 12}{(x + 3)^2}$$

$$= \frac{x^2 + 6x - 3}{(x + 3)^2}$$

28. $f(x) = x^4\left[1 - \dfrac{2}{x + 1}\right] = x^4\left[\dfrac{x - 1}{x + 1}\right]$

$$f'(x) = x^4\left[\frac{(x + 1) - (x - 1)}{(x + 1)^2}\right] + \left[\frac{x - 1}{x + 1}\right](4x^3)$$

$$= x^4\left[\frac{2}{(x + 1)^2}\right] + \left[\frac{x^2 - 1}{(x + 1)^2}\right](4x^3)$$

$$= 2x^3\left[\frac{2x^2 + x - 2}{(x + 1)^2}\right]$$

29. $f(x) = \dfrac{3x - 1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

$$= \frac{3x + 1}{2x^{3/2}}$$

Alternate solution:

$$f(x) = \frac{3x - 1}{\sqrt{x}} = \frac{3x - 1}{x^{1/2}}$$

$$f'(x) = \frac{x^{1/2}(3) - (3x - 1)\left(\frac{1}{2}\right)\left(x^{-1/2}\right)}{x}$$

$$= \frac{\frac{1}{2}x^{-1/2}(3x + 1)}{x}$$

$$= \frac{3x + 1}{2x^{3/2}}$$

30. $f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}\left(x^{1/2} + 3\right)$

$$f'(x) = x^{1/3}\left(\frac{1}{2}x^{-1/2}\right) + \left(x^{1/2} + 3\right)\left(\frac{1}{3}x^{-2/3}\right)$$

$$= \frac{5}{6}x^{-1/6} + x^{-2/3}$$

$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

Alternate solution:

$$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right)$$

$$= x^{5/6} + 3x^{1/3}$$

$$f'(x) = \frac{5}{6}x^{-1/6} + x^{-2/3}$$

$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

31. $h(s) = \left(s^3 - 2\right)^2 = s^6 - 4s^3 + 4$

$$h'(s) = 6s^5 - 12s^2 = 6s^2\left(s^3 - 2\right)$$

32. $h(x) = \left(x^2 - 1\right)^2 = x^4 - 2x^2 + 1$

$$h'(x) = 4x^3 - 4x = 4x\left(x^2 - 1\right)$$

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33. $f(x) = \dfrac{2 - (1/x)}{x - 3} = \dfrac{2x - 1}{x(x - 3)} = \dfrac{2x - 1}{x^2 - 3x}$

$$f'(x) = \frac{(x^2 - 3x)2 - (2x - 1)(2x - 3)}{(x^2 - 3x)^2} = \frac{2x^2 - 6x - 4x^2 + 8x - 3}{(x^2 - 3x)^2}$$

$$= \frac{-2x^2 + 2x - 3}{(x^2 - 3x)^2} = -\frac{2x^2 - 2x + 3}{x^2(x - 3)^2}$$

34. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right) = 2x - \dfrac{x^2}{x + 1}$

$$g'(x) = 2 - \frac{(x + 1)2x - x^2(1)}{(x + 1)^2} = \frac{2(x^2 + 2x + 1) - x^2 - 2x}{(x + 1)^2} = \frac{x^2 + 2x + 2}{(x + 1)^2}$$

35. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

$$f'(x) = (6x^2 + 5)(x - 3)(x + 2) + (2x^3 + 5x)(1)(x + 2) + (2x^3 + 5x)(x - 3)(1)$$

$$= (6x^2 + 5)(x^2 - x - 6) + (2x^3 + 5x)(x + 2) + (2x^3 + 5x)(x - 3)$$

$$= (6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30) + (2x^4 + 4x^3 + 5x^2 + 10x) + (2x^4 + 5x^2 - 6x^3 - 15x)$$

$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

Note: You could simplify first:

$$f(x) = (2x^3 + 5x)(x^2 - x - 6)$$

36. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$$f'(x) = (3x^2 - 1)(x^2 + 2)(x^2 + x - 1) + (x^3 - x)(2x)(x^2 + x - 1) + (x^3 - x)(x^2 + 2)(2x + 1)$$

$$= (3x^4 + 5x^2 - 2)(x^2 + x - 1) + (2x^4 - 2x^2)(x^2 + x - 1) + (x^5 + x^3 - 2x)(2x + 1)$$

$$= (3x^6 + 5x^4 - 2x^2 + 3x^5 + 5x^3 - 2x - 3x^4 - 5x^2 + 2)$$
$$+ (2x^6 - 2x^4 + 2x^5 - 2x^3 - 2x^4 + 2x^2)$$
$$+ (2x^6 + 2x^4 - 4x^2 + x^5 + x^3 - 2x)$$

$$= 7x^6 + 6x^5 + 4x^3 - 9x^2 - 4x + 2$$

37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$

$$f'(x) = \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2}$$

$$= -\frac{4xc^2}{(x^2 - c^2)^2}$$

38. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$

$$f'(x) = \frac{(c^2 + x^2)(-2x) - (c^2 - x^2)(2x)}{(c^2 + x^2)^2}$$

$$= -\frac{4xc^2}{(c^2 + x^2)^2}$$

39. $f(t) = t^2 \sin t$

$$f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$$

40. $f(\theta) = (\theta + 1)\cos\theta$

$$f'(\theta) = (\theta + 1)(-\sin\theta) + (\cos\theta)(1)$$
$$= \cos\theta - (\theta + 1)\sin\theta$$

41. $f(t) = \dfrac{\cos t}{t}$

$$f'(t) = \frac{-t \sin t - \cos t}{t^2} = -\frac{t \sin t + \cos t}{t^2}$$

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42. $f(x) = \dfrac{\sin x}{x^3}$

$$f'(x) = \frac{x^3 \cos x - \sin x(3x^2)}{(x^3)^2} = \frac{x\cos x - 3\sin x}{x^4}$$

43. $f(x) = -x + \tan x$

$$f'(x) = -1 + \sec^2 x = \tan^2 x$$

44. $y = x + \cot x$

$$y' = 1 - \csc^2 x = -\cot^2 x$$

45. $g(t) = \sqrt[4]{t} + 6\csc t = t^{1/4} + 6\csc t$

$$g'(t) = \frac{1}{4}t^{-3/4} - 6\csc t \cot t$$

$$= \frac{1}{4t^{3/4}} - 6\csc t \cot t$$

46. $h(x) = \dfrac{1}{x} - 12\sec x = x^{-1} - 12\sec x$

$$h'(x) = -x^{-2} - 12\sec x \tan x$$

$$= \frac{-1}{x^2} - 12\sec x \tan x$$

47. $y = \dfrac{3(1-\sin x)}{2\cos x} = \dfrac{3 - 3\sin x}{2\cos x}$

$$y' = \frac{(-3\cos x)(2\cos x) - (3 - 3\sin x)(-2\sin x)}{(2\cos x)^2}$$

$$= \frac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x}$$

$$= \frac{3}{2}(-1 + \tan x \sec x - \tan^2 x)$$

$$= \frac{3}{2}\sec x(\tan x - \sec x)$$

48. $y = \dfrac{\sec x}{x}$

$$y' = \frac{x\sec x \tan x - \sec x}{x^2}$$

$$= \frac{\sec x(x\tan x - 1)}{x^2}$$

49. $y = -\csc x - \sin x$

$$y' = \csc x \cot x - \cos x$$

$$= \frac{\cos x}{\sin^2 x} - \cos x$$

$$= \cos x(\csc^2 x - 1)$$

$$= \cos x \cot^2 x$$

50. $y = x\sin x + \cos x$

$$y' = x\cos x + \sin x - \sin x = x\cos x$$

51. $f(x) = x^2 \tan x$

$$f'(x) = x^2 \sec^2 x + 2x\tan x$$

$$= x(x\sec^2 x + 2\tan x)$$

52. $f(x) = \sin x \cos x$

$$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$$

$$= \cos 2x$$

53. $y = 2x\sin x + x^2 \cos x$

$$y' = 2x\cos x + 2\sin x + x^2(-\sin x) + 2x\cos x$$

$$= 4x\cos x + (2 - x^2)\sin x$$

54. $h(\theta) = 5\theta\sec\theta + \theta\tan\theta$

$$h'(\theta) = 5\theta\sec\theta\tan\theta + 5\sec\theta + \theta\sec^2\theta + \tan\theta$$

55. $g(x) = \left(\dfrac{x+1}{x+2}\right)(2x-5)$

$$g'(x) = \left(\frac{x+1}{x+2}\right)(2) + (2x-5)\left[\frac{(x+2)(1) - (x+1)(1)}{(x+2)^2}\right]$$

$$= \frac{2x^2 + 8x - 1}{(x+2)^2}$$

(Form of answer may vary.)

56. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

$$f'(x) = 2\frac{x^5 + 2x^3 + 2x^2 - 2}{(x^2+1)^2}$$

(Form of answer may vary.)

57. $g(\theta) = \dfrac{\theta}{1 - \sin\theta}$

$$g'(\theta) = \frac{1 - \sin\theta + \theta\cos\theta}{(1-\sin\theta)^2}$$

(Form of answer may vary.)

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58. $f(\theta) = \dfrac{\sin\theta}{1-\cos\theta}$

$f'(\theta) = \dfrac{1}{\cos\theta - 1} = \dfrac{\cos\theta - 1}{(1-\cos\theta)^2}$ (Form of answer may vary.)

59. $y = \dfrac{1+\csc x}{1-\csc x}$

$y' = \dfrac{(1-\csc x)(-\csc x\cot x) - (1+\csc x)(\csc x\cot x)}{(1-\csc x)^2} = \dfrac{-2\csc x\cot x}{(1-\csc x)^2}$

$y'\left(\dfrac{\pi}{6}\right) = \dfrac{-2(2)(\sqrt{3})}{(1-2)^2} = -4\sqrt{3}$

60. $f(x) = \tan x\cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

61. $h(t) = \dfrac{\sec t}{t}$

$h'(t) = \dfrac{t(\sec t\tan t) - (\sec t)(1)}{t^2} = \dfrac{\sec t(t\tan t - 1)}{t^2}$

$h'(\pi) = \dfrac{\sec\pi(\pi\tan\pi - 1)}{\pi^2} = \dfrac{1}{\pi^2}$

62. $f(x) = \sin x(\sin x + \cos x)$

$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$

$= \sin x\cos x - \sin^2 x + \sin x\cos x + \cos^2 x$

$= \sin 2x + \cos 2x$

$f'\left(\dfrac{\pi}{4}\right) = \sin\dfrac{\pi}{2} + \cos\dfrac{\pi}{2} = 1$

63. (a) $f(x) = (x^3 + 4x - 1)(x-2)$, $(1, -4)$

$f'(x) = (x^3 + 4x - 1)(1) + (x-2)(3x^2 + 4)$

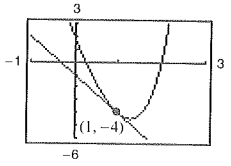
$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$

$= 4x^3 - 6x^2 + 8x - 9$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x-1) \Rightarrow y = -3x - 1$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

64. (a) $f(x) = (x+3)(x^2 - 2)$, $(-2, 2)$

$f'(x) = (x+3)(2x) + (x^2 - 2)(1)$

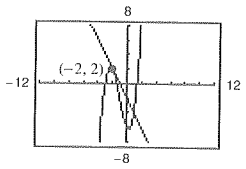
$= 3x^2 + 6x - 2$

$f'(-2) = -2$; Slope at $(-2, 2)$

Tangent line:

$y - 2 = -2(x+2) \Rightarrow y = -2x - 2$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -2$ at $(-2, 2)$.

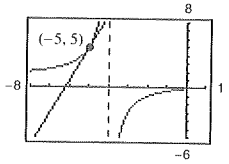
65. (a) $f(x) = \dfrac{x}{x+4}$, $(-5, 5)$

$f'(x) = \dfrac{(x+4)(1) - x(1)}{(x+4)^2} = \dfrac{4}{(x+4)^2}$

$f'(-5) = \dfrac{4}{(-5+4)^2} = 4$; Slope at $(-5, 5)$

Tangent line: $y - 5 = 4(x+5) \Rightarrow y = 4x + 25$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(-5, 5)$.

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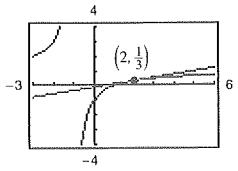
66. (a) $f(x) = \dfrac{x-1}{x+1}, \quad \left(2, \dfrac{1}{3}\right)$

$$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$f'(2) = \frac{2}{9}; \quad \text{Slope at } \left(2, \frac{1}{3}\right)$$

Tangent line: $y - \dfrac{1}{3} = \dfrac{2}{9}(x-2) \Rightarrow y = \dfrac{2}{9}x - \dfrac{1}{9}$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{2}{9}$ at $\left(2, \dfrac{1}{3}\right)$.

67. (a) $f(x) = \tan x, \quad \left(\dfrac{\pi}{4}, 1\right)$

$$f'(x) = \sec^2 x$$

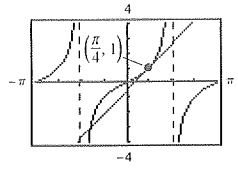
$$f'\left(\frac{\pi}{4}\right) = 2; \quad \text{Slope at } \left(\frac{\pi}{4}, 1\right)$$

Tangent line: $y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

$$y - 1 = 2x - \frac{\pi}{2}$$

$$4x - 2y - \pi + 2 = 0$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $\left(\dfrac{\pi}{4}, 1\right)$.

68. (a) $f(x) = \sec x, \quad \left(\dfrac{\pi}{3}, 2\right)$

$$f'(x) = \sec x \tan x$$

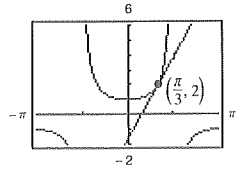
$$f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}; \quad \text{Slope at } \left(\frac{\pi}{3}, 2\right)$$

Tangent line:

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2\sqrt{3}$ at $\left(\dfrac{\pi}{3}, 2\right)$.

69. $f(x) = \dfrac{8}{x^2+4}; \quad (2, 1)$

$$f'(x) = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(4+4)^2} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x-2)$$

$$y = -\frac{1}{2}x + 2$$

$$2y + x - 4 = 0$$

70. $f(x) = \dfrac{27}{x^2+9}; \quad \left(-3, \dfrac{3}{2}\right)$

$$f'(x) = \frac{(x^2+9)(0) - 27(2x)}{(x^2+9)^2} = \frac{-54x}{(x^2+9)^2}$$

$$f'(-3) = \frac{-54(-3)}{(9+9)^2} = \frac{1}{2}$$

$$y - \frac{3}{2} = \frac{1}{2}(x+3)$$

$$y = \frac{1}{2}x + 3$$

$$2y - x - 6 = 0$$

71. $f(x) = \dfrac{16x}{x^2+16}; \quad \left(-2, -\dfrac{8}{5}\right)$

$$f'(x) = \frac{(x^2+16)(16) - 16x(2x)}{(x^2+16)^2} = \frac{256 - 16x^2}{(x^2+16)^2}$$

$$f'(-2) = \frac{256 - 16(4)}{20^2} = \frac{12}{25}$$

$$y + \frac{8}{5} = \frac{12}{25}(x+2)$$

$$y = \frac{12}{25}x - \frac{16}{25}$$

$$25y - 12x + 16 = 0$$

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72. $f(x) = \dfrac{4x}{x^2 + 6}; \left(2, \dfrac{4}{5}\right)$

$$f'(x) = \frac{(x^2 + 6)(4) - 4x(2x)}{(x^2 + 6)^2} = \frac{24 - 4x^2}{(x^2 + 6)^2}$$

$$f'(2) = \frac{24 - 16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x - 2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$$25y - 2x - 16 = 0$$

73. $f(x) = \dfrac{2x - 1}{x^2} = 2x^{-1} - x^{-2}$

$$f'(x) = -2x^{-2} + 2x^{-3} = \frac{2(-x + 1)}{x^3}$$

$f'(x) = 0$ when $x = 1$, and $f(1) = 1$.

Horizontal tangent at $(1, 1)$.

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(2x) - (x^2)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x}{(x^2 + 1)^2}$$

$f'(x) = 0$ when $x = 0$.

Horizontal tangent is at $(0, 0)$.

75. $f(x) = \dfrac{x^2}{x - 1}$

$$f'(x) = \frac{(x - 1)(2x) - x^2(1)}{(x - 1)^2}$$

$$= \frac{x^2 - 2x}{(x - 1)^2} = \frac{x(x - 2)}{(x - 1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

$$f'(x) = \frac{(x^2 - 7)(1) - (x - 4)(2x)}{(x^2 - 7)^2}$$

$$= \frac{x^2 - 7 - 2x^2 + 8x}{(x^2 - 7)^2}$$

$$= \frac{-x^2 - 8x + 7}{(x^2 - 7)^2} = -\frac{(x - 7)(x - 1)}{(x^2 - 7)^2}$$

$f'(x) = 0$ for $x = 1, 7$; $f(1) = \dfrac{1}{2}$, $f(7) = \dfrac{1}{14}$

f has horizontal tangents at $\left(1, \dfrac{1}{2}\right)$ and $\left(7, \dfrac{1}{14}\right)$.

77. $f(x) = \dfrac{x + 1}{x - 1}$

$$f'(x) = \frac{(x - 1) - (x + 1)}{(x - 1)^2} = \frac{-2}{(x - 1)^2}$$

$2y + x = 6 \Rightarrow y = -\dfrac{1}{2}x + 3$; Slope: $-\dfrac{1}{2}$

$$\frac{-2}{(x - 1)^2} = -\frac{1}{2}$$

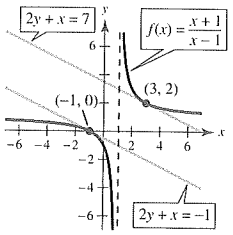
$$(x - 1)^2 = 4$$

$$x - 1 = \pm 2$$

$x = -1, 3$; $f(-1) = 0$, $f(3) = 2$

$$y - 0 = -\frac{1}{2}(x + 1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

$$y - 2 = -\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$



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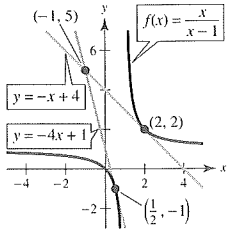
78. $f(x) = \dfrac{x}{x-1}$

$$f'(x) = \frac{(x-1)-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

Let $(x, y) = \left(x, x/(x-1)\right)$ be a point of tangency on the graph of f.



$$\frac{5 - \left(x/(x-1)\right)}{-1-x} = \frac{-1}{(x-1)^2}$$

$$\frac{4x-5}{(x-1)(x+1)} = \frac{1}{(x-1)^2}$$

$$(4x-5)(x-1) = x+1$$

$$4x^2 - 10x + 4 = 0$$

$$(x-2)(2x-1) = 0 \Rightarrow x = \frac{1}{2}, 2$$

$$f\left(\frac{1}{2}\right) = -1,\ f(2) = 2;\ f'\left(\frac{1}{2}\right) = -4,\ f'(2) = -1$$

Two tangent lines:

$$y + 1 = -4\left(x - \frac{1}{2}\right) \Rightarrow y = -4x + 1$$

$$y - 2 = -1(x-2) \Rightarrow y = -x + 4$$

79. $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$$g'(x) = \frac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \frac{6}{(x+2)^2}$$

$$g(x) = \frac{5x+4}{(x+2)} = \frac{3x}{(x+2)} + \frac{2x+4}{(x+2)} = f(x) + 2$$

f and g differ by a constant.

80. $f'(x) = \dfrac{x(\cos x - 3) - (\sin x - 3x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$$g'(x) = \frac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \frac{x\cos x - \sin x}{x^2}$$

$$g(x) = \frac{\sin x + 2x}{x} = \frac{\sin x - 3x + 5x}{x} = f(x) + 5$$

f and g differ by a constant.

81. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\frac{1}{2}\right) = 1$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(4) = \frac{3(-1) - 7(0)}{3^2} = -\frac{1}{3}$$

82. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(4) = \frac{1}{2}(8) + 1(0) = 4$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(7) = \frac{4(2) - 4(-1)}{4^2} = \frac{12}{16} = \frac{3}{4}$$

83. Area $= A(t) = (6t+5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$

$$A'(t) = 9t^{1/2} + \frac{5}{2}t^{-1/2} = \frac{18t+5}{2\sqrt{t}} \text{ cm}^2/\text{sec}$$

84. $V = \pi r^2 h = \pi(t+2)\left(\dfrac{1}{2}\sqrt{t}\right)$

$$= \frac{1}{2}\left(t^{3/2} + 2t^{1/2}\right)\pi$$

$$V'(t) = \frac{1}{2}\left(\frac{3}{2}t^{1/2} + t^{-1/2}\right)\pi = \frac{3t+2}{4t^{1/2}}\pi \text{ in.}^3/\text{sec}$$

85. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right)$$

(a) When $x = 10$: $\dfrac{dC}{dx} = -\$38.13$ thousand/100 components

(b) When $x = 15$: $\dfrac{dC}{dx} = -\$10.37$ thousand/100 components

(c) When $x = 20$: $\dfrac{dC}{dx} = -\$3.80$ thousand/100 components

As the order size increases, the cost per item decreases.

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86. $P = \dfrac{k}{V}$

$$\frac{dP}{dV} = -\frac{k}{V^2}$$

87. $P(t) = 500\left[1 + \dfrac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ bacteria/h

88. $F = \dfrac{Gm_1m_2}{d^2} = Gm_1m_2d^{-2}$

$$\frac{dF}{dd} = F'(d) = \frac{-2\,Gm_1m_2}{d^3}$$

89. (a) $\sec x = \dfrac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

(b) $\csc x = \dfrac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

(c) $\cot x = \dfrac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

90. $f(x) = \sec x$

$g(x) = \csc x,\ [0, 2\pi)$

$f'(x) = g'(x)$

$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x} \cdot \dfrac{\cos x}{\sin x}} = -1 \Rightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

$x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

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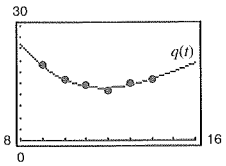
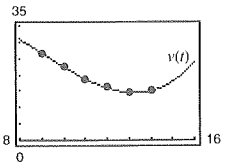
91. (a) Using a graphing utility,

$$q(t) = -0.0546t^3 + 2.529t^2 - 36.89t + 186.6$$

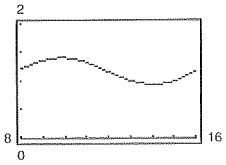
$$v(t) = 0.0796t^3 - 2.162t^2 + 15.32t + 5.9.$$

(b)





(c) $A = \dfrac{v(t)}{q(t)} = \dfrac{0.0796t^3 - 2.162t^2 + 15.32t + 5.9}{-0.0546t^3 + 2.529t^2 - 36.89t + 186.6}$



A represents the average value (in billions of dollars) per one million personal computers.

(d) $A'(t)$ represents the rate of change of the average value per one million personal computers for the given year t.

92. (a) $\sin\theta = \dfrac{r}{r+h}$

$$r + h = r\csc\theta$$

$$h = r\csc\theta - r = r(\csc\theta - 1)$$

(b) $h'(\theta) = r(-\csc\theta \cdot \cot\theta)$

$$h'(30°) = h'\left(\frac{\pi}{6}\right)$$

$$= -3960\left(2 \cdot \sqrt{3}\right) = -7920\sqrt{3} \text{ mi/rad}$$

93. $f(x) = x^4 + 2x^3 - 3x^2 - x$

$f'(x) = 4x^3 + 6x^2 - 6x - 1$

$f''(x) = 12x^2 + 12x - 6$

94. $f(x) = 8x^6 - 10x^5 + 5x^3$

$f'(x) = 48x^5 - 50x^4 + 15x^2$

$f''(x) = 240x^4 - 200x^3 + 30x$

95. $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

96. $f(x) = x + \dfrac{32}{x^2}$

$f'(x) = 1 - \dfrac{64}{x^3}$

$f''(x) = \dfrac{192}{x^4}$

97. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

$f''(x) = \dfrac{2}{(x-1)^3}$

98. $f(x) = \dfrac{x^2 + 2x - 1}{x} = x + 2 - \dfrac{1}{x}$

$f'(x) = 1 + \dfrac{1}{x^2}$

$f''(x) = -\dfrac{2}{x^3}$

99. $f(x) = x\sin x$

$f'(x) = x\cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= -x\sin x + 2\cos x$

100. $f(x) = \sec x$

$f'(x) = \sec x\tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x\tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

101. $f'(x) = x^2$

$f''(x) = 2x$

102. $f''(x) = 2 - 2x^{-1}$

$f'''(x) = 2x^{-2} = \dfrac{2}{x^2}$

103. $f'''(x) = 2\sqrt{x}$

$f^{(4)}(x) = \dfrac{1}{2}(2)x^{-1/2} = \dfrac{1}{\sqrt{x}}$

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104. $f^{(4)}(x) = 2x + 1$

$f^{(5)}(x) = 2$

$f^{(6)}(x) = 0$

105. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4$

$= 0$

106. $f(x) = 4 - h(x)$

$f'(x) = -h'(x)$

$f'(2) = -h'(2) = -4$

107. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$

$f'(2) = \dfrac{h(2)g'(2) - g(2)h'(2)}{\left[h(2)\right]^2}$

$= \dfrac{(-1)(-2) - (3)(4)}{(-1)^2}$

$= -10$

108. $f(x) = g(x)h(x)$

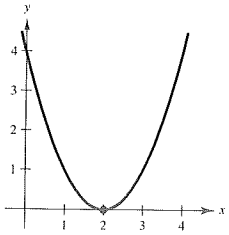
$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = g(2)h'(2) + h(2)g'(2)$

$= (3)(4) + (-1)(-2)$

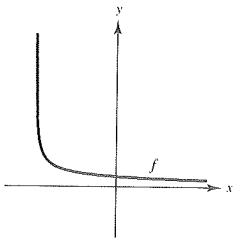
$= 14$

109. The graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$ would, in general, look like the graph below.

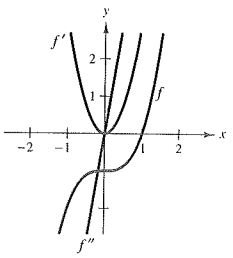


One such function is $f(x) = (x - 2)^2$.

110. The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would, in general, look like the graph below.

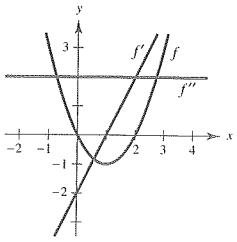


111.



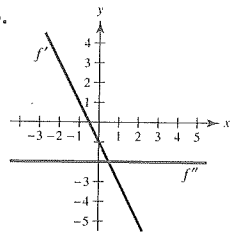
It appears that f is cubic, so f' would be quadratic and f'' would be linear.

112.

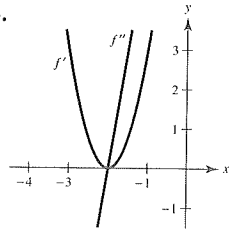


It appears that f is quadratic so f' would be linear and f'' would be constant.

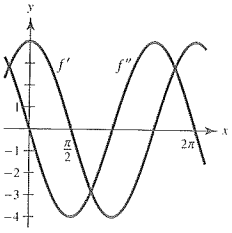
113.



114.



115.



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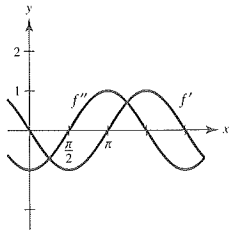
116.



117. $v(t) = 36 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec^2

The speed of the object is decreasing.

118. $v(t) = \dfrac{100t}{2t + 15}$

$$a(t) = v'(t) = \frac{(2t + 15)(100) - (100t)(2)}{(2t + 15)^2} = \frac{1500}{(2t + 15)^2}$$

(a) $a(5) = \dfrac{1500}{[2(5) + 15]^2} = 2.4$ ft/sec^2

(b) $a(10) = \dfrac{1500}{[2(10) + 15]^2} \approx 1.2$ ft/sec^2

(c) $a(20) = \dfrac{1500}{[2(20) + 15]^2} \approx 0.5$ ft/sec^2

119. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = 16.50t + 66$

$a(t) = v'(t) = -16.50$

t(sec)	0	1	2	3	4
$s(t)$ (ft)	0	57.75	99	123.75	132
$v(t) = s'(t)$ (ft/sec)	66	49.5	33	16.5	0
$a(t) = v'(t)$ (ft/sec^2)	−16.5	−16.5	−16.5	−16.5	−16.5

Average velocity on:

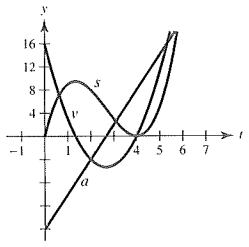
$[0, 1]$ is $\dfrac{57.75 - 0}{1 - 0} = 57.75$

$[1, 2]$ is $\dfrac{99 - 57.75}{2 - 1} = 41.25$

$[2, 3]$ is $\dfrac{123.75 - 99}{3 - 2} = 24.75$

$[3, 4]$ is $\dfrac{132 - 123.75}{4 - 3} = 8.25$

120. (a)

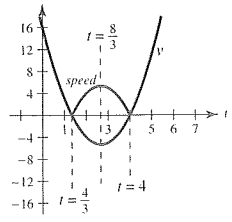


s position function

v velocity function

a acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$ and $(8/3, 4)$ and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.



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121. $f(x) = x^n$

$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

Note: $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "n factorial")

122. $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

123. $f(x) = g(x)h(x)$

(a) $f'(x) = g(x)h'(x) + h(x)g'(x)$

$$\begin{aligned} f''(x) &= g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x) \\ &= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x) \\ f'''(x) &= g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x) \\ &= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x) \\ f^{(4)}(x) &= g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x) \\ &\quad + g'''(x)h'(x) + g^{(4)}(x)h(x) \\ &= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x) \end{aligned}$$

(b) $$\begin{aligned} f^{(n)}(x) &= g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1\left[(n-1)(n-2)\cdots(2)(1)\right]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)\left[(n-2)(n-3)\cdots(2)(1)\right]}g''(x)h^{(n-2)}(x) \\ &\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)\left[(n-3)(n-4)\cdots(2)(1)\right]}g'''(x)h^{(n-3)}(x) + \cdots \\ &\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{\left[(n-1)(n-2)\cdots(2)(1)\right](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x) \\ &= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots \\ &\quad + \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x) \end{aligned}$$

Note: $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "n factorial")

124. $\left[xf(x)\right]' = xf'(x) + f(x)$

$\left[xf(x)\right]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$

$\left[xf(x)\right]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$

In general, $\left[xf(x)\right]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

125. $f(x) = x^n \sin x$

$f'(x) = x^n \cos x + nx^{n-1} \sin x$

When $n = 1$: $f'(x) = x \cos x + \sin x$

When $n = 2$: $f'(x) = x^2 \cos x + 2 \sin x$

When $n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$

When $n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$

For general n, $f'(x) = x^n \cos x + nx^{n-1} \sin x$.

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126. $f(x) = \dfrac{\cos x}{x^n} = x^{-n}\cos x$

$f'(x) = -x^{-n}\sin x - nx^{-n-1}\cos x$

$= -x^{-n-1}(x\sin x + n\cos x)$

$= -\dfrac{x\sin x + n\cos x}{x^{n+1}}$

When $n = 1$: $f'(x) = -\dfrac{x\sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x\sin x + 2\cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x\sin x + 3\cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x\sin x + 4\cos x}{x^5}$

For general n, $f'(x) = -\dfrac{x\sin x + n\cos x}{x^{n+1}}$.

127. $y = \dfrac{1}{x}, y' = -\dfrac{1}{x^2}, y'' = \dfrac{2}{x^3}$

$x^3y'' + 2x^2y' = x^3\left[\dfrac{2}{x^3}\right] + 2x^2\left[-\dfrac{1}{x^2}\right] = 2 - 2 = 0$

128.

$y = 2x^3 - 6x + 10$

$y' = 6x^2 - 6$

$y'' = 12x$

$y''' = 12$

$-y''' - xy'' - 2y' = -12 - x(12x) - 2(6x^2 - 6) = -24x^2$

129.

$y = 2\sin x + 3$

$y' = 2\cos x$

$y'' = -2\sin x$

$y'' + y = -2\sin x + (2\sin x + 3) = 3$

130.

$y = 3\cos x + \sin x$

$y' = -3\sin x + \cos x$

$y'' = -3\cos x - \sin x$

$y'' + y = (-3\cos x - \sin x) + (3\cos x + \sin x) = 0$

131. False. If $y = f(x)g(x)$, then

$\dfrac{dy}{dx} = f(x)g'(x) + g(x)f'(x)$.

132. True. y is a fourth-degree polynomial.

$\dfrac{d^ny}{dx^n} = 0$ when $n > 4$.

133. True

$h'(c) = f(c)g'(c) + g(c)f'(c)$

$= f(c)(0) + g(c)(0)$

$= 0$

134. True

135. True

136. True. If $v(t) = c$ then $a(t) = v'(t) = 0$.

137. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

x-intercept at $(1, 0)$: $0 = a + b + c$

$(2, 7)$ on graph: $7 = 4a + 2b + c$

Slope 10 at $(2, 7)$: $10 = 4a + b$

Subtracting the third equation from the second, $-3 = b + c$. Subtracting this equation from the first, $3 = a$. Then, $10 = 4(3) + b \Rightarrow b = -2$. Finally, $-3 = (-2) + c \Rightarrow c = -1$.

$f(x) = 3x^2 - 2x - 1$

138. $f(x) = ax^3 + bx^2 + cx + d, a \neq 0$

$f'(x) = 3ax^2 + 2bx + c$

$f'(x) = 0 \Rightarrow x = \dfrac{-2b \pm \sqrt{4b^2 - 12ac}}{6a}$

(a) No horizontal tangents: $f'(x) \neq 0$

$4b^2 - 12ac < 0$

Example: $a = c = 1, b = 0$:

$f(x) = x^3 + x$

(b) Exactly one horizontal tangent $4b^2 - 12ac = 0$

Example: $a = 1, b = 3, c = 3$:

$f(x) = x^3 + 3x^2 + 3x$

(c) Exactly two horizontal tangents

$4b^2 - 12ac > 0$

Example: $b = 1, a = 1, c = 0$:

$f(x) = x^3 + x^2$

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139. $f(x) = x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

$$f'(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases} = 2|x|$$

$$f''(x) = \begin{cases} 2, & \text{if } x > 0 \\ -2, & \text{if } x < 0 \end{cases}$$

$f''(0)$ does not exist since the left and right derivatives are not equal.

140. (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$

$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$

$= fg'' + f'g' + f'g' + f''g$

$= fg'' + 2f'g' + f''g$

$\neq fg'' + f''g$ False

141. $\dfrac{d}{dx}\big[f(x)g(x)h(x)\big] = \dfrac{d}{dx}\big[(f(x)g(x))h(x)\big]$

$= \dfrac{d}{dx}\big[f(x)g(x)\big]h(x) + f(x)g(x)h'(x)$

$= \big[f(x)g'(x) + g(x)f'(x)\big]h(x) + f(x)g(x)h'(x)$

$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

Section 2.4 The Chain Rule

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (5x - 8)^4$	$u = 5x - 8$	$y = u^4$
2. $y = \dfrac{1}{\sqrt{x+1}}$	$u = x + 1$	$y = u^{-1/2}$
3. $y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
4. $y = 3\tan(\pi x^2)$	$u = \pi x^2$	$y = 3\tan u$
5. $y = \csc^3 x$	$u = \csc x$	$y = u^3$
6. $y = \sin\dfrac{5x}{2}$	$u = \dfrac{5x}{2}$	$y = \sin u$

7. $y = (4x - 1)^3$

$y' = 3(4x - 1)^2(4) = 12(4x - 1)^2$

8. $y = 2(6 - x^2)^5$

$y' = 2(5)(6 - x^2)^4(-2x) = -20x(6 - x^2)^4$

9. $g(x) = 3(4 - 9x)^4$

$g'(x) = 12(4 - 9x)^3(-9) = -108(4 - 9x)^3$

10. $f(t) = (9t + 2)^{2/3}$

$f'(t) = \dfrac{2}{3}(9t + 2)^{-1/3}(9) = \dfrac{6}{\sqrt[3]{9t + 2}}$

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11. $f(t) = \sqrt{5-t} = (5-t)^{1/2}$

$$f'(t) = \frac{1}{2}(5-t)^{-1/2}(-1) = \frac{-1}{2\sqrt{5-t}}$$

12. $g(x) = \sqrt{9-4x} = (9-4x)^{1/2}$

$$g'(x) = \frac{1}{2}(9-4x)^{-1/2}(-4) = \frac{-2}{\sqrt{9-4x}}$$

13. $y = \sqrt[3]{6x^2+1} = (6x^2+1)^{1/3}$

$$y' = \frac{1}{3}(6x^2+1)^{-2/3}(12x) = \frac{4x}{(6x^2+1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2+1)^2}}$$

14. $g(x) = \sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1|$

$$g'(x) = \begin{cases} 1, & x > 1 \\ -1, & x < 1 \end{cases}$$

15. $y = 2\sqrt[4]{9-x^2} = 2(9-x^2)^{1/4}$

$$y' = 2\left(\frac{1}{4}\right)(9-x^2)^{-3/4}(-2x)$$

$$= \frac{-x}{(9-x^2)^{3/4}} = \frac{-x}{\sqrt[4]{(9-x^2)^3}}$$

16. $f(x) = -3\sqrt[4]{2-9x}$

$$f(x) = -3(2-9x)^{1/4}$$

$$f'(x) = -\frac{3}{4}(2-9x)^{-3/4}(-9)$$

$$= \frac{27}{4(2-9x)^{3/4}} = \frac{27}{4\sqrt[4]{(2-9x)^3}}$$

17. $y = (x-2)^{-1}$

$$y' = -1(x-2)^{-2}(1) = \frac{-1}{(x-2)^2}$$

18. $s(t) = \dfrac{1}{t^2+3t-1}$

$$s(t) = (t^2+3t-1)^{-1}$$

$$s'(t) = -1(t^2+3t-1)^{-2}(2t+3) = \frac{-(2t+3)}{(t^2+3t-1)^2}$$

19. $f(t) = (t-3)^{-2}$

$$f'(t) = -2(t-3)^{-3}(1) = \frac{-2}{(t-3)^3}$$

20. $y = -\dfrac{5}{(t+3)^3}$

$$y = -5(t+3)^{-3}$$

$$y' = 15(t+3)^{-4} = \frac{15}{(t+3)^4}$$

21. $y = (x+2)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2}(x+2)^{-3/2}$$

$$= -\frac{1}{2(x+2)^{3/2}}$$

$$= -\frac{1}{2\sqrt{(x+2)^3}}$$

22. $g(t) = \sqrt{\dfrac{1}{t^2-2}}$

$$g(t) = (t^2-2)^{-1/2}$$

$$g'(t) = -\frac{1}{2}(t^2-2)^{-3/2}(2t)$$

$$= -\frac{t}{(t^2-2)^{3/2}} = -\frac{t}{\sqrt{(t^2-2)^3}}$$

23. $f(x) = x^2(x-2)^4$

$$f'(x) = x^2\left[4(x-2)^3(1)\right] + (x-2)^4(2x)$$

$$= 2x(x-2)^3\left[2x + (x-2)\right]$$

$$= 2x(x-2)^3(3x-2)$$

24. $f(x) = x(3x-9)^3$

$$f'(x) = x\left[3(3x-9)^2(3)\right] + (3x-9)^3(1)$$

$$= (3x-9)^2\left[9x + 3x - 9\right]$$

$$= 27(x-3)^2(4x-3)$$

25. $y = x\sqrt{1-x^2} = x(1-x^2)^{1/2}$

$$y' = x\left[\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right] + (1-x^2)^{1/2}(1)$$

$$= -x^2(1-x^2)^{-1/2} + (1-x^2)^{1/2}$$

$$= (1-x^2)^{-1/2}\left[-x^2 + (1-x^2)\right]$$

$$= \frac{1-2x^2}{\sqrt{1-x^2}}$$

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26. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$

$$y' = \frac{1}{2}x^2\left(\frac{1}{2}(16 - x^2)^{-1/2}(-2x)\right) + x(16 - x^2)^{1/2}$$

$$= \frac{-x^3}{2\sqrt{16 - x^2}} + x\sqrt{16 - x^2} = \frac{-x(3x^2 - 32)}{2\sqrt{16 - x^2}}$$

27. $y = \frac{x}{\sqrt{x^2 + 1}} = \frac{x}{(x^2 + 1)^{1/2}}$

$$y' = \frac{(x^2 + 1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2 + 1)^{-1/2}(2x)}{\left[(x^2 + 1)^{1/2}\right]^2}$$

$$= \frac{(x^2 + 1)^{1/2} - x^2(x^2 + 1)^{-1/2}}{x^2 + 1}$$

$$= \frac{(x^2 + 1)^{-1/2}\left[x^2 + 1 - x^2\right]}{x^2 + 1}$$

$$= \frac{1}{(x^2 + 1)^{3/2}} = \frac{1}{\sqrt{(x^2 + 1)^3}}$$

28. $y = \frac{x}{\sqrt{x^4 + 4}}$

$$y' = \frac{(x^4 + 4)^{1/2}(1) - x\frac{1}{2}(x^4 + 4)^{-1/2}(4x^3)}{x^4 + 4}$$

$$= \frac{x^4 + 4 - 2x^4}{(x^4 + 4)^{3/2}} = \frac{4 - x^4}{(x^4 + 4)^{3/2}} = \frac{4 - x^4}{\sqrt{(x^4 + 4)^3}}$$

29. $g(x) = \left(\frac{x + 5}{x^2 + 2}\right)^2$

$$g'(x) = 2\left(\frac{x + 5}{x^2 + 2}\right)\left(\frac{(x^2 + 2) - (x + 5)(2x)}{(x^2 + 2)^2}\right)$$

$$= \frac{2(x + 5)(2 - 10x - x^2)}{(x^2 + 2)^3}$$

$$= \frac{-2(x + 5)(x^2 + 10x - 2)}{(x^2 + 2)^3}$$

30. $h(t) = \left(\frac{t^2}{t^3 + 2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3 + 2}\right)\left(\frac{(t^3 + 2)(2t) - t^2(3t^2)}{(t^3 + 2)^2}\right)$$

$$= \frac{2t^2(4t - t^4)}{(t^3 + 2)^3} = \frac{2t^3(4 - t^3)}{(t^3 + 2)^3}$$

31. $f(v) = \left(\frac{1 - 2v}{1 + v}\right)^3$

$$f'(v) = 3\left(\frac{1 - 2v}{1 + v}\right)^2\left(\frac{(1 + v)(-2) - (1 - 2v)}{(1 + v)^2}\right)$$

$$= \frac{-9(1 - 2v)^2}{(1 + v)^4}$$

32. $g(x) = \left(\frac{3x^2 - 2}{2x + 3}\right)^3$

$$g'(x) = 3\left(\frac{3x^2 - 2}{2x + 3}\right)^2\left(\frac{(2x + 3)(6x) - (3x^2 - 2)(2)}{(2x + 3)^2}\right)$$

$$= \frac{3(3x^2 - 2)^2(6x^2 + 18x + 4)}{(2x + 3)^4}$$

$$= \frac{6(3x^2 - 2)^2(3x^2 + 9x + 2)}{(2x + 3)^4}$$

33. $f(x) = \left((x^2 + 3)^5 + x\right)^2$

$$f'(x) = 2\left((x^2 + 3)^5 + x\right)\left(5(x^2 + 3)^4(2x) + 1\right)$$

$$= 2\left[10x(x^2 + 3)^9 + (x^2 + 3)^5 + 10x^2(x^2 + 3)^4 + x\right] = 20x(x^2 + 3)^9 + 2(x^2 + 3)^5 + 20x^2(x^2 + 3)^4 + 2x$$

34. $g(x) = \left(2 + (x^2 + 1)^4\right)^3$

$$g'(x) = 3\left(2 + (x^2 + 1)^4\right)^2\left(4(x^2 + 1)^3(2x)\right) = 24x(x^2 + 1)^3\left(2 + (x^2 + 1)^4\right)^2$$

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35. $f(x) = \sqrt{2 + \sqrt{2 + \sqrt{x}}}$

$$= \left[2 + \left(2 + x^{1/2}\right)^{1/2}\right]^{1/2}$$

$$f'(x) = \frac{1}{2}\left[2 + \left(2 + x^{1/2}\right)^{1/2}\right]^{-1/2}\left[\frac{1}{2}\left(2 + x^{1/2}\right)^{-1/2}\left(\frac{1}{2}x^{-1/2}\right)\right]$$

$$= \frac{1}{8\sqrt{x}\sqrt{2 + \sqrt{x}}\sqrt{2 + \sqrt{2 + \sqrt{x}}}}$$

36. $g(t) = \sqrt{\sqrt{t + 1} + 1} = \left[(t + 1)^{1/2} + 1\right]^{1/2}$

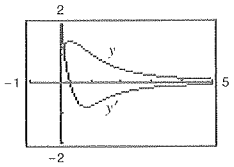
$$g'(t) = \frac{1}{2}\left[(t + 1)^{1/2} + 1\right]^{-1/2}\left[\frac{1}{2}(t + 1)^{-1/2}\right]$$

$$= \frac{1}{4\sqrt{t + 1}\sqrt{\sqrt{t + 1} + 1}}$$

37. $y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

$$y' = \frac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}\left(x^2 + 1\right)^2}$$

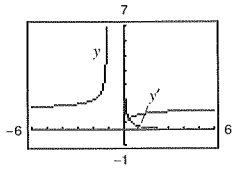
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



38. $y = \sqrt{\dfrac{2x}{x + 1}}$

$$y' = \frac{1}{\sqrt{2x}(x + 1)^{3/2}}$$

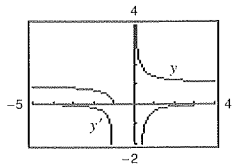
y' has no zeros.



39. $y = \sqrt{\dfrac{x + 1}{x}}$

$$y' = -\frac{\sqrt{(x + 1)/x}}{2x(x + 1)}$$

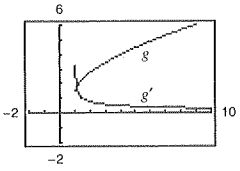
y' has no zeros.



40. $g(x) = \sqrt{x - 1} + \sqrt{x + 1}$

$$g'(x) = \frac{1}{2\sqrt{x - 1}} + \frac{1}{2\sqrt{x + 1}}$$

g' has no zeros.

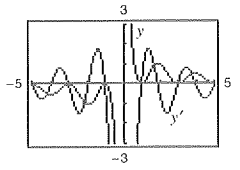


41. $y = \dfrac{\cos \pi x + 1}{x}$

$$\frac{dy}{dx} = \frac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$$

$$= -\frac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$$

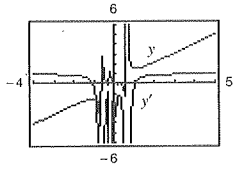
The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



42. $y = x^2 \tan \dfrac{1}{x}$

$$\frac{dy}{dx} = 2x \tan \frac{1}{x} - \sec^2 \frac{1}{x}$$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



43. (a) $y = \sin x$

$y' = \cos x$

$y'(0) = 1$

1 cycle in $[0, 2\pi]$

(b) $y = \sin 2x$

$y' = 2 \cos 2x$

$y'(0) = 2$

2 cycles in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

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44. (a) $y = \sin 3x$

$y' = 3\cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

(b) $y = \sin\left(\frac{x}{2}\right)$

$y' = \left(\frac{1}{2}\right)\cos\left(\frac{x}{2}\right)$

$y'(0) = \frac{1}{2}$

Half cycle in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

45. $y = \cos 4x$

$\frac{dy}{dx} = -4\sin 4x$

46. $y = \sin \pi x$

$\frac{dy}{dx} = \pi \cos \pi x$

47. $g(x) = 5\tan 3x$

$g'(x) = 15\sec^2 3x$

48. $h(x) = \sec(x^2)$

$h'(x) = 2x\sec(x^2)\tan(x^2)$

49. $y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$

$y' = \cos(\pi^2 x^2)[2\pi^2 x] = 2\pi^2 x\cos(\pi^2 x^2)$

$= 2\pi^2 x\cos(\pi x)^2$

50. $y = \cos(1 - 2x)^2 = \cos\left((1 - 2x)^2\right)$

$y' = -\sin(1 - 2x)^2(2(1 - 2x)(-2))$

$= 4(1 - 2x)\sin(1 - 2x)^2$

51. $h(x) = \sin 2x\cos 2x$

$h'(x) = \sin 2x(-2\sin 2x) + \cos 2x(2\cos 2x)$

$= 2\cos^2 2x - 2\sin^2 2x$

$= 2\cos 4x$

Alternate solution: $h(x) = \frac{1}{2}\sin 4x$

$h'(x) = \frac{1}{2}\cos 4x(4) = 2\cos 4x$

52. $g(\theta) = \sec\frac{1}{2}\theta\tan\frac{1}{2}\theta$

$g'(\theta) = \sec\left(\frac{1}{2}\theta\right)\sec^2\left(\frac{1}{2}\theta\right)\frac{1}{2} + \tan\left(\frac{1}{2}\theta\right)\sec\left(\frac{1}{2}\theta\right)\tan\left(\frac{1}{2}\theta\right)\frac{1}{2}$

$= \frac{1}{2}\sec\left(\frac{1}{2}\theta\right)\left[\sec^2\left(\frac{1}{2}\theta\right) + \tan^2\left(\frac{1}{2}\theta\right)\right]$

53. $f(x) = \frac{\cot x}{\sin x} = \frac{\cos x}{\sin^2 x}$

$f'(x) = \frac{\sin^2 x(-\sin x) - \cos x(2\sin x\cos x)}{\sin^4 x}$

$= \frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} = \frac{-1 - \cos^2 x}{\sin^3 x}$

54. $g(v) = \frac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$= \cos^2 v - \sin^2 v = \cos 2v$

55. $y = 4\sec^2 x$

$y' = 8\sec x \cdot \sec x\tan x = 8\sec^2 x\tan x$

56. $g(t) = 5\cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10\cos \pi t(-\sin \pi t)(\pi)$

$= -10\pi(\sin \pi t)(\cos \pi t)$

$= -5\pi \sin 2\pi t$

57. $f(\theta) = \tan^2 5\theta = (\tan 5\theta)^2$

$f'(\theta) = 2(\tan 5\theta)(\sec^2 5\theta)5 = 10\tan 5\theta\sec^2 5\theta$

58. $g(\theta) = \cos^2 8\theta = (\cos 8\theta)^2$

$g'(\theta) = 2(\cos 8\theta)(-\sin 8\theta)8 = -16\cos 8\theta\sin 8\theta$

59. $f(\theta) = \frac{1}{4}\sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta\cos 2\theta = \frac{1}{2}\sin 4\theta$

60. $h(t) = 2\cot^2(\pi t + 2)$

$h'(t) = 4\cot(\pi t + 2)(-\csc^2(\pi t + 2)(\pi))$

$= -4\pi\cot(\pi t + 2)\csc^2(\pi t + 2)$

61. $f(t) = 3\sec^2(\pi t - 1)$

$f'(t) = 6\sec(\pi t - 1)\sec(\pi t - 1)\tan(\pi t - 1)(\pi)$

$= 6\pi\sec^2(\pi t - 1)\tan(\pi t - 1) = \frac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$

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62. $y = 3x - 5\cos(\pi x)^2 = 3x - 5\cos(\pi^2 x^2)$

$$\frac{dy}{dx} = 3 + 5\sin(\pi^2 x^2)(2\pi^2 x) = 3 + 10\pi^2 x\sin(\pi x)^2$$

63. $y = \sqrt{x} + \frac{1}{4}\sin(2x)^2 = \sqrt{x} + \frac{1}{4}\sin(4x^2)$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{1}{4}\cos(4x^2)(8x) = \frac{1}{2\sqrt{x}} + 2x\cos(2x)^2$$

64. $y = \sin x^{1/3} + (\sin x)^{1/3}$

$$y' = \cos x^{1/3}\left(\frac{1}{3}x^{-2/3}\right) + \frac{1}{3}(\sin x)^{-2/3}\cos x$$

$$= \frac{1}{3}\left[\frac{\cos x^{1/3}}{x^{2/3}} + \frac{\cos x}{(\sin x)^{2/3}}\right]$$

65. $y = \sin(\tan 2x)$

$$y' = \cos(\tan 2x)(\sec^2 2x)(2) = 2\cos(\tan 2x)\sec^2 2x$$

66. $y = \cos\sqrt{\sin(\tan \pi x)}$

$$y' = -\sin\sqrt{\sin(\tan \pi x)} \cdot \frac{1}{2}(\sin(\tan \pi x))^{-1/2}\cos(\tan \pi x)\sec^2 \pi x(\pi) = \frac{-\pi \sin\sqrt{\sin(\tan \pi x)}\cos(\tan \pi x)\sec^2 \pi x}{2\sqrt{\sin(\tan \pi x)}}$$

67. $s(t) = (t^2 + 6t - 2)^{1/2}$, $(3, 5)$

$$s'(t) = \frac{1}{2}(t^2 + 6t - 2)^{-1/2}(2t + 6)$$

$$= \frac{t + 3}{\sqrt{t^2 + 6t - 2}}$$

$$s'(3) = \frac{6}{5}$$

68. $y = (3x^3 + 4x)^{1/5}$, $(2, 2)$

$$y' = \frac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$$

$$= \frac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$$

$$y'(2) = \frac{1}{2}$$

69. $f(x) = \frac{5}{x^3 - 2} = 5(x^3 - 2)^{-1}$, $\left(-2, -\frac{1}{2}\right)$

$$f'(x) = -5(x^3 - 2)^{-2}(3x^2) = \frac{-15x^2}{(x^3 - 2)^2}$$

$$f'(-2) = -\frac{60}{100} = -\frac{3}{5}$$

70. $f(x) = \frac{1}{(x^2 - 3x)^2} = (x^2 - 3x)^{-2}$, $\left(4, \frac{1}{16}\right)$

$$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3) = \frac{-2(2x - 3)}{(x^2 - 3x)^3}$$

$$f'(4) = -\frac{5}{32}$$

71. $f(t) = \frac{3t + 2}{t - 1}$, $(0, -2)$

$$f'(t) = \frac{(t - 1)(3) - (3t + 2)(1)}{(t - 1)^2} = \frac{-5}{(t - 1)^2}$$

$$f'(0) = -5$$

72. $f(x) = \frac{x + 1}{2x - 3}$, $(2, 3)$

$$f'(x) = -\frac{(2x - 3)(1) - (x + 1)(2)}{(2x - 3)^2} = \frac{-5}{(2x - 3)^2}$$

$$f'(2) = -5$$

73. $y = 26 - \sec^3 4x$, $(0, 25)$

$$y' = -3\sec^2 4x \sec 4x \tan 4x\, 4$$

$$= -12\sec^3 4x \tan 4x$$

$$y'(0) = 0$$

74. $y = \frac{1}{x} + \sqrt{\cos x}$, $\left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

$$y' = -\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$$

$y'(\pi/2)$ is undefined.

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75. (a) $f(x) = (2x^2 - 7)^{1/2}$, $(4, 5)$

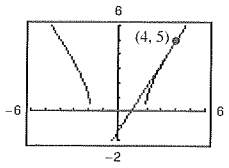
$$f'(x) = \frac{1}{2}(2x^2 - 7)^{-1/2}(4x) = \frac{2x}{\sqrt{2x^2 - 7}}$$

$$f'(4) = \frac{8}{5}$$

Tangent line:

$$y - 5 = \frac{8}{5}(x - 4) \Rightarrow 8x - 5y - 7 = 0$$

(b)



76. (a) $f(x) = \frac{1}{3}x\sqrt{x^2 + 5}$, $(2, 2)$

$$f'(x) = \frac{1}{3}x\left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x)\right] + \frac{1}{3}(x^2 + 5)^{1/2}$$

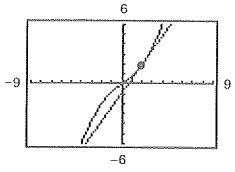
$$= \frac{x^2}{2\sqrt{x^2 + 5}} + \frac{1}{3}\sqrt{x^2 + 5}$$

$$f'(2) = \frac{4}{3(3)} + \frac{1}{3}(3) = \frac{13}{9}$$

Tangent line:

$$y - 2 = \frac{13}{9}(x - 2) \Rightarrow 13x - 9y - 8 = 0$$

(b)



77. (a) $y = (4x^3 + 3)^2$, $(-1, 1)$

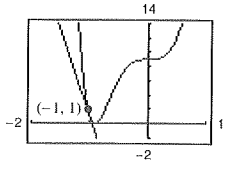
$$y' = 2(4x^3 + 3)(12x^2) = 24x^2(4x^3 + 3)$$

$$y'(-1) = -24$$

Tangent line:

$$y - 1 = -24(x + 1) \Rightarrow 24x + y + 23 = 0$$

(b)



78. (a) $f(x) = (9 - x^2)^{2/3}$, $(1, 4)$

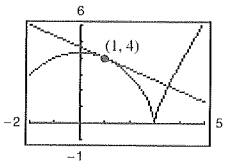
$$f'(x) = \frac{2}{3}(9 - x^2)^{-1/3}(-2x) = \frac{-4x}{3(9 - x^2)^{1/3}}$$

$$f'(1) = \frac{-4}{3(8)^{1/3}} = -\frac{2}{3}$$

Tangent line:

$$y - 4 = -\frac{2}{3}(x - 1) \Rightarrow 2x + 3y - 14 = 0$$

(b)



79. (a) $f(x) = \sin 2x$, $(\pi, 0)$

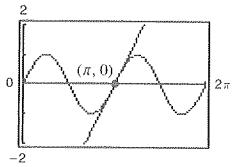
$$f'(x) = 2\cos 2x$$

$$f'(\pi) = 2$$

Tangent line:

$$y = 2(x - \pi) \Rightarrow 2x - y - 2\pi = 0$$

(b)



80. (a) $y = \cos 3x$, $\left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

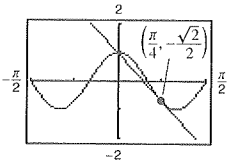
$$y' = -3\sin 3x$$

$$y'\left(\frac{\pi}{4}\right) = -3\sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$$

Tangent line: $y + \frac{\sqrt{2}}{2} = \frac{-3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$$

(b)



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81. (a) $f(x) = \tan^2 x, \quad \left(\frac{\pi}{4}, 1\right)$

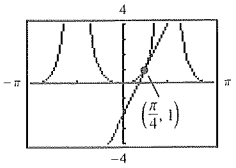
$f'(x) = 2 \tan x \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

Tangent line:

$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$

(b)



82. (a) $y = 2 \tan^3 x, \quad \left(\frac{\pi}{4}, 2\right)$

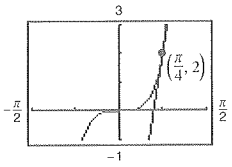
$y' = 6 \tan^2 x \cdot \sec^2 x$

$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$

Tangent line:

$y - 2 = 12\left(x - \frac{\pi}{4}\right) \Rightarrow 12x - y + (2 - 3\pi) = 0$

(b)



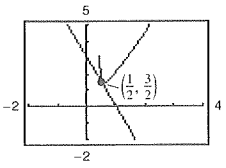
83. (a) $g(t) = \frac{3t^2}{\sqrt{t^2 + 2t - 1}}, \quad \left(\frac{1}{2}, \frac{3}{2}\right)$

$g'(t) = \frac{3t(t^2 + 3t - 2)}{(t^2 + 2t - 1)^{3/2}}$

$g'\left(\frac{1}{2}\right) = -3$

(b) $y - \frac{3}{2} = -3\left(x - \frac{1}{2}\right) \Rightarrow 3x + y - 3 = 0$

(c)



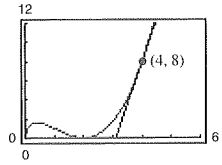
84. (a) $f(x) = \sqrt{x}(2 - x)^2, \quad (4, 8)$

$f'(x) = \frac{(x - 2)(5x - 2)}{2\sqrt{x}}$

$f'(4) = 9$

(b) $y - 8 = 9(x - 4) \Rightarrow 9x - y - 28 = 0$

(c)



85. (a) $s(t) = \frac{(4 - 2t)\sqrt{1 + t}}{3}, \quad \left(0, \frac{4}{3}\right)$

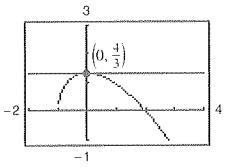
$s'(t) = \frac{-2\sqrt{1 + t}}{3} + \frac{2 - t}{3\sqrt{1 + t}}$

$s'(0) = 0$

(b) $y - \frac{4}{3} = 0(x - 0)$

$y = \frac{4}{3}$

(c)



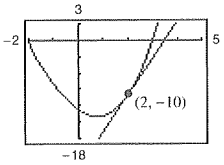
86. (a) $y = (t^2 - 9)\sqrt{t + 2}, \quad (2, -10)$

$y' = \frac{5t^2 + 8t - 9}{2\sqrt{t + 2}}$

$y'(2) = \frac{27}{4}$

(b) $y + 10 = \frac{27}{4}(t - 2) \Rightarrow 27t - 4y - 94$

(c)



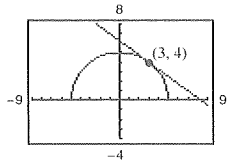
87. $f(x) = \sqrt{25 - x^2}, \quad (3, 4)$

$f'(x) = \frac{-x}{\sqrt{25 - x^2}}$

$f'(3) = -\frac{3}{4}$

Tangent line:

$y - 4 = -\frac{3}{4}(x - 3) \Rightarrow 3x + 4y - 25 = 0$



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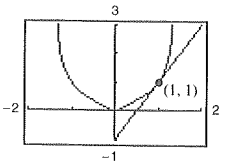
88. $f(x) = \dfrac{|x|}{\sqrt{2 - x^2}}$, $(1, 1)$

$f'(x) = \dfrac{2}{(2 - x^2)^{3/2}}$ for $x > 0$

$f'(1) = 2$



Tangent line: $y - 1 = 2(x - 1) \Rightarrow 2x - y - 1 = 0$

89.
$$f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$$
$$f'(x) = -2\sin x + 2\cos 2x$$
$$= -2\sin x + 2 - 4\sin^2 x = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(\sin x + 1)(2\sin x - 1) = 0$$
$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$$
$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Horizontal tangents at $x = \dfrac{\pi}{6}, \dfrac{3\pi}{2}, \dfrac{5\pi}{6}$

Horizontal tangent at the points $\left(\dfrac{\pi}{6}, \dfrac{3\sqrt{3}}{2}\right)$, $\left(\dfrac{3\pi}{2}, 0\right)$, and $\left(\dfrac{5\pi}{6}, -\dfrac{3\sqrt{3}}{2}\right)$

90.
$$f(x) = \frac{x}{\sqrt{2x - 1}}$$
$$f'(x) = \frac{(2x - 1)^{1/2} - x(2x - 1)^{-1/2}}{2x - 1}$$
$$= \frac{2x - 1 - x}{(2x - 1)^{3/2}}$$
$$= \frac{x - 1}{(2x - 1)^{3/2}}$$
$$\frac{x - 1}{(2x - 1)^{3/2}} = 0 \Rightarrow x = 1$$

Horizontal tangent at $(1, 1)$

91.
$$f(x) = 5(2 - 7x)^4$$
$$f'(x) = 20(2 - 7x)^3(-7) = -140(2 - 7x)^3$$
$$f''(x) = -420(2 - 7x)^2(-7) = 2940(2 - 7x)^2$$

92.
$$f(x) = 4(x^2 - 2)^3$$
$$f'(x) = 12(x^2 - 2)^2(2x) = 24x(x^2 - 2)^2$$
$$f''(x) = 24x(2)(x^2 - 2)(2x) + 24(x^2 - 2)^2$$
$$= (x^2 - 2)(96x^2 + 24(x^2 - 2))$$
$$= (x^2 - 2)(120x^2 - 48)$$
$$= 24(x^2 - 2)(5x^2 - 2)$$

93.
$$f(x) = \frac{1}{x - 6} = (x - 6)^{-1}$$
$$f'(x) = -(x - 6)^{-2}$$
$$f''(x) = 2(x - 6)^{-3} = \frac{2}{(x - 6)^3}$$

94.
$$f(x) = \frac{4}{(x + 2)^3} = 4(x + 2)^{-3}$$
$$f'(x) = -12(x + 2)^{-4}$$
$$f''(x) = 48(x + 2)^{-5} = \frac{48}{(x + 2)^5}$$

95.
$$f(x) = \sin x^2$$
$$f'(x) = 2x\cos x^2$$
$$f''(x) = 2x\left[2x(-\sin x^2)\right] + 2\cos x^2$$
$$= 2(\cos x^2 - 2x^2 \sin x^2)$$

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96. $f(x) = \sec^2 \pi x$

$$f'(x) = 2 \sec \pi x(\pi \sec \pi x \tan \pi x)$$
$$= 2\pi \sec^2 \pi x \tan \pi x$$
$$f''(x) = 2\pi \sec^2 \pi x(\sec^2 \pi x)(\pi) + 2\pi \tan \pi x(2\pi \sec^2 \pi x \tan \pi x)$$
$$= 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$$
$$= 2\pi^2 \sec^2 \pi x(\sec^2 \pi x + 2 \tan^2 \pi x)$$
$$= 2\pi^2 \sec^2 \pi x(3 \sec^2 \pi x - 2)$$

97. $h(x) = \frac{1}{9}(3x + 1)^3, \quad \left(1, \frac{64}{9}\right)$

$$h'(x) = \tfrac{1}{9}3(3x + 1)^2(3) = (3x + 1)^2$$
$$h''(x) = 2(3x + 1)(3) = 18x + 6$$
$$h''(1) = 24$$

98. $f(x) = \dfrac{1}{\sqrt{x + 4}} = (x + 4)^{-1/2}, \quad \left(0, \dfrac{1}{2}\right)$

$$f'(x) = -\frac{1}{2}(x + 4)^{-3/2}$$
$$f''(x) = \frac{3}{4}(x + 4)^{-5/2} = \frac{3}{4(x + 4)^{5/2}}$$
$$f''(0) = \frac{3}{128}$$

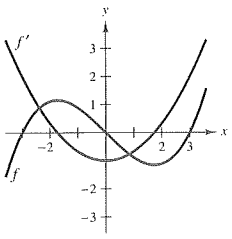
99. $f(x) = \cos x^2, \quad (0, 1)$

$$f'(x) = -\sin(x^2)(2x) = -2x \sin(x^2)$$
$$f''(x) = -2x \cos(x^2)(2x) - 2 \sin(x^2)$$
$$= -4x^2 \cos(x^2) - 2 \sin(x^2)$$
$$f''(0) = 0$$

100. $g(t) = \tan 2t, \quad \left(\dfrac{\pi}{6}, \sqrt{3}\right)$

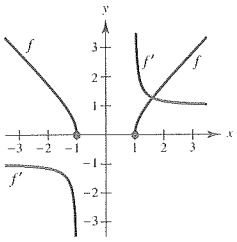
$$g'(t) = 2 \sec^2(2t)$$
$$g''(t) = 4 \sec(2t) \cdot \sec(2t) \tan(2t)2$$
$$= 8 \sec^2(2t) \tan(2t)$$
$$g''\left(\frac{\pi}{6}\right) = 32\sqrt{3}$$

101.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

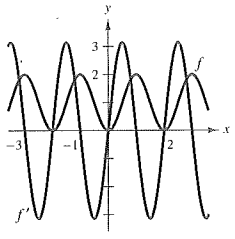
102.



f is decreasing on $(-\infty, -1)$ so f' must be negative there.

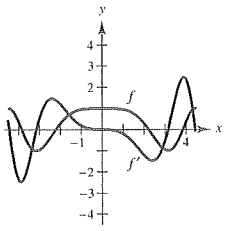
f is increasing on $(1, \infty)$ so f' must be positive there.

103.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

104.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

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105. $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \Rightarrow g'(x) = 3f'(3x)$

106. $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \Rightarrow g'(x) = 2xf'(x^2)$

107. (a) $g(x) = f(x) - 2 \Rightarrow g'(x) = f'(x)$

(b) $h(x) = 2f(x) \Rightarrow h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \Rightarrow r'(x) = f'(-3x)(-3) = -3f'(-3x)$

So, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = (-3)(-4) = 12$

(d) $s(x) = f(x + 2) \Rightarrow s'(x) = f'(x + 2)$

So, you need to know $f'(x + 2)$.

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

108. (a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible, you need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(5) = \dfrac{(3)(6) - (-3)(-2)}{(3)^2} = \dfrac{12}{9} = \dfrac{4}{3}$

(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

109. (a) $h(x) = f(g(x)), g(1) = 4, g'(1) = -\frac{1}{2}, f'(4) = -1$

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(4)g'(1) = (-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$

(b) $s(x) = g(f(x)), f(5) = 6, f'(5) = -1, g'(6)$ does not exist.

$s'(x) = g'(f(x))\, f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

$s'(5)$ does not exist because g is not differentiable at 6.

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110. (a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3) = f'(5)(1) = \frac{1}{2}$

(b) $s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9) = g'(8)(2) = (-1)(2) = -2$

111. (a) $F = 132{,}400(331 - v)^{-1}$

$$F' = (-1)(132{,}400)(331 - v)^{-2}(-1) = \frac{132{,}400}{(331 - v)^2}$$

When $v = 30$, $F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$$F' = (-1)(132{,}400)(331 + v)^{-2}(-1) = \frac{-132{,}400}{(331 + v)^2}$$

When $v = 30$, $F' \approx -1.016$.

112. $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$

$v = y' = \frac{1}{3}[-12 \sin 12t] - \frac{1}{4}[12 \cos 12t]$

$= -4 \sin 12t - 3 \cos 12t$

When $t = \pi/8$, $y = 0.25$ ft and $v = 4$ ft/sec.

113. $\theta = 0.2 \cos 8t$

The maximum angular displacement is $\theta = 0.2$ (because $-1 \leq \cos 8t \leq 1$).

$$\frac{d\theta}{dt} = 0.2[-8 \sin 8t] = -1.6 \sin 8t$$

When $t = 3$, $d\theta/dt = -1.6 \sin 24 \approx 1.4489$ rad/sec.

114. $y = A \cos \omega t$

(a) Amplitude: $A = \dfrac{3.5}{2} = 1.75$

$$y = 1.75 \cos \omega t$$

Period: $10 \Rightarrow \omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

$$y = 1.75 \cos \frac{\pi t}{5}$$

(b) $$v = y' = 1.75\left[-\frac{\pi}{5} \sin \frac{\pi t}{5}\right] = -0.35\pi \sin \frac{\pi t}{5}$$

115. $S = C(R^2 - r^2)$

$$\frac{dS}{dt} = C\left(2R\frac{dR}{dt} - 2r\frac{dr}{dt}\right)$$

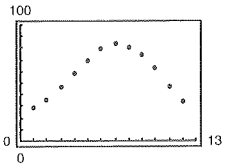
Because r is constant, you have $dr/dt = 0$ and

$$\frac{dS}{dt} = (1.76 \times 10^5)(2)(1.2 \times 10^{-2})(10^{-5})$$

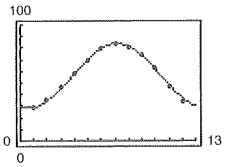
$= 4.224 \times 10^{-2} = 0.04224$ cm/sec^2.

116. (a) Using a graphing utility, you obtain a model similar to $T(t) = 56.1 + 27.6 \sin(0.48t - 1.86)$.

(a)

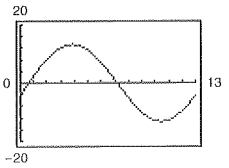


(b)



The model is a good fit.

(c)



$T'(t) \approx 13.25 \cos(0.48t - 1.86)$

(d) The temperature changes most rapidly around spring (March–May), and fall (Oct–Nov).

117. (a) $x = -1.637t^3 + 19.31t^2 - 0.5t - 1$

(b) $C = 60x + 1350$

$$= 60(-1.637t^3 + 19.31t^2 - 0.5t - 1) + 1350$$

$$\frac{dC}{dt} = 60(-4.911t^2 + 38.62t - 0.5)$$

$$= -294.66t^2 + 2317.2t - 30$$

(c) The function dC/dt is quadratic, not linear. The cost function levels off at the end of the day, perhaps due to fatigue.

118. $f(x) = \sin \beta x$

(a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)} = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

119. (a) Yes, if $f(x + p) = f(x)$ for all x, then $f'(x + p) = f'(x)$, which shows that f' is periodic as well.

(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

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120. (a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \dfrac{5-0}{6-2} = \dfrac{5}{4}$.

Also, $g'(1) = 0$. So, $r'(1) = 0$.

(b) $s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \dfrac{5}{2}$, $g'\left(\dfrac{5}{2}\right) = \dfrac{6-4}{6-2} = \dfrac{1}{2}$ and

$f'(4) = \dfrac{5}{4}$. So, $s'(4) = \dfrac{1}{2}\left(\dfrac{5}{4}\right) = \dfrac{5}{8}$.

121. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Rightarrow g'(x) = 0$

$g'(x) = 2\sin x \cos x + 2\cos x(-\sin x) = 0$

(b) $\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$.

Equivalently,

$f'(x) = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$ and

$g'(x) = 2\tan x \cdot \sec^2 x = 2\sec^2 x \tan x$, which are the same.

122. (a) If $f(-x) = -f(x)$, then

$$\frac{d}{dx}\left[f(-x)\right] = \frac{d}{dx}\left[-f(x)\right]$$

$$f'(-x)(-1) = -f'(x)$$

$$f'(-x) = f'(x).$$

So, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$$\frac{d}{dx}\left[f(-x)\right] = \frac{d}{dx}\left[f(x)\right]$$

$$f'(-x)(-1) = f'(x)$$

$$f'(-x) = -f'(x).$$

So, f' is odd.

123. $|u| = \sqrt{u^2}$

$$\frac{d}{dx}\left[|u|\right] = \frac{d}{dx}\left[\sqrt{u^2}\right] = \frac{1}{2}\left(u^2\right)^{-1/2}(2uu')$$

$$= \frac{uu'}{\sqrt{u^2}} = u'\frac{u}{|u|}, \quad u \neq 0$$

124. $g(x) = |3x - 5|$

$$g'(x) = 3\left(\frac{3x-5}{|3x-5|}\right), \quad x \neq \frac{5}{3}$$

125. $f(x) = |x^2 - 9|$

$$f'(x) = 2x\left(\frac{x^2-9}{|x^2-9|}\right), \quad x \neq \pm 3$$

126. $h(x) = |x|\cos x$

$$h'(x) = -|x|\sin x + \frac{x}{|x|}\cos x, \quad x \neq 0$$

127. $f(x) = |\sin x|$

$$f'(x) = \cos x\left(\frac{\sin x}{|\sin x|}\right), \; x \neq k\pi$$

128. (a) $f(x) = \tan x$ $\qquad f(\pi/4) = 1$

$f'(x) = \sec^2 x$ $\qquad f'(\pi/4) = 2$

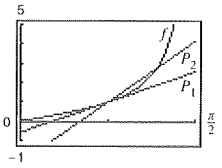
$f''(x) = 2\sec^2 x \tan x$ $\qquad f''(\pi/4) = 4$

$P_1(x) = 2(x - \pi/4) + 1$

$$P_2(x) = \frac{1}{2}(4)(x - \pi/4)^2 + 2(x - \pi/4) + 1$$

$$= 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/4$.

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129. (a) $f(x) = \sec x$ $\qquad f(\pi/6) = \dfrac{2}{\sqrt{3}}$

$f'(x) = \sec x \tan x$ $\qquad f'(\pi/6) = \dfrac{2}{3}$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$ $\qquad f''(\pi/6) = \dfrac{10\sqrt{3}}{9}$

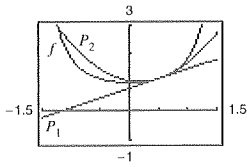
$= \sec^3 x + \sec x \tan^2 x$

$P_1(x) = \dfrac{2}{3}(x - \pi/6) + \dfrac{2}{\sqrt{3}}$

$P_2(x) = \dfrac{1}{2} \cdot \left(\dfrac{10}{3\sqrt{3}}\right)\left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right) + \dfrac{2}{\sqrt{3}}$

$= \left(\dfrac{5}{3\sqrt{3}}\right)\left(x - \dfrac{\pi}{6}\right)^2 + \dfrac{2}{3}\left(x - \dfrac{\pi}{6}\right) + \dfrac{2}{\sqrt{3}}$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/6$.

130. False. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}(-1)$.

131. False. If $f(x) = \sin^2 2x$, then $f'(x) = 2(\sin 2x)(2 \cos 2x)$.

132. True

133.
$$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$$
$$f'(x) = a_1 \cos x + 2a_2 \cos 2x + \cdots + na_n \cos nx$$
$$f'(0) = a_1 + 2a_2 + \cdots + na_n$$
$$|a_1 + 2a_2 + \cdots + na_n| = |f'(0)| = \lim_{x\to 0}\left|\frac{f(x) - f(0)}{x - 0}\right| = \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \cdot \left|\frac{\sin x}{x}\right| = \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \le 1$$

134. $\dfrac{d}{dx}\left[\dfrac{P_n(x)}{(x^k - 1)^{n+1}}\right] = \dfrac{(x^k - 1)^{n+1}P_n'(x) - P_n(x)(n + 1)(x^k - 1)^n kx^{k-1}}{(x^k - 1)^{2n+2}} = \dfrac{(x^k - 1)P_n'(x) - (n + 1)kx^{k-1}P_n(x)}{(x^k - 1)^{n+2}}$

$$P_n(x) = (x^k - 1)^{n+1}\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right] \Rightarrow$$

$$P_{n+1}(x) = (x^k - 1)^{n+2}\frac{d}{dx}\left[\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right]\right] = (x^k - 1)P_n'(x) - (n + 1)kx^{k-1}P_n(x)$$

$$P_{n+1}(1) = -(n + 1)kP_n(1)$$

For $n = 1$, $\dfrac{d}{dx}\left[\dfrac{1}{x^k - 1}\right] = \dfrac{-kx^{k-1}}{(x^k - 1)^2} = \dfrac{P_1(x)}{(x^k - 1)^2} \Rightarrow P_1(1) = -k$. Also, $P_0(1) = 1$.

You now use mathematical induction to verify that $P_n(1) = (-k)^n n!$ for $n \ge 0$. Assume true for n. Then

$$P_{n+1}(1) = -(n + 1)k\,P_n(1) = -(n + 1)k(-k)^n n! = (-k)^{n+1}(n + 1)!.$$

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Section 2.5 Implicit Differentiation

1. $x^2 + y^2 = 9$

$2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$

2. $x^2 - y^2 = 25$

$2x - 2yy' = 0$

$$y' = \frac{x}{y}$$

3. $x^{1/2} + y^{1/2} = 16$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = -\frac{x^{-1/2}}{y^{-1/2}} = -\sqrt{\frac{y}{x}}$$

4. $x^3 + y^3 = 64$

$3x^2 + 3y^2y' = 0$

$$y' = -\frac{x^2}{y^2}$$

5. $x^3 - xy + y^2 = 7$

$3x^2 - xy' - y + 2yy' = 0$

$$(2y - x)y' = y - 3x^2$$

$$y' = \frac{y - 3x^2}{2y - x}$$

6. $x^2y + y^2x = -2$

$x^2y' + 2xy + y^2 + 2yxy' = 0$

$$(x^2 + 2xy)y' = -(y^2 + 2xy)$$

$$y' = \frac{-y(y + 2x)}{x(x + 2y)}$$

7. $x^3y^3 - y - x = 0$

$3x^3y^2y' + 3x^2y^3 - y' - 1 = 0$

$$(3x^3y^2 - 1)y' = 1 - 3x^2y^3$$

$$y' = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

8. $\sqrt{xy} = x^2y + 1$

$$\frac{1}{2}(xy)^{-1/2}(xy' + y) = 2xy + x^2y'$$

$$\frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} = 2xy + x^2y'$$

$$\left(\frac{x}{2\sqrt{xy}} - x^2\right)y' = 2xy - \frac{y}{2\sqrt{xy}}$$

$$y' = \frac{2xy - \dfrac{y}{2\sqrt{xy}}}{\dfrac{x}{2\sqrt{xy}} - x^2}$$

$$y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$$

9. $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$

$$(4xy - 3x^2)y' = 6xy - 3x^2 - 2y^2$$

$$y' = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$$

10. $4 \cos x \sin y = 1$

$4[-\sin x \sin y + \cos x \cos y\, y'] = 0$

$$\cos x \cos y\, y' = \sin x \sin y$$

$$y' = \frac{\sin x \sin y}{\cos x \cos y} = \tan x \tan y$$

11. $\sin x + 2 \cos 2y = 1$

$\cos x - 4(\sin 2y)y' = 0$

$$y' = \frac{\cos x}{4 \sin 2y}$$

12. $(\sin \pi x + \cos \pi y)^2 = 2$

$2(\sin \pi x + \cos \pi y)[\pi \cos \pi x - \pi(\sin \pi y)y'] = 0$

$$\pi \cos \pi x - \pi(\sin \pi y)y' = 0$$

$$y' = \frac{\cos \pi x}{\sin \pi y}$$

13. $\sin x = x(1 + \tan y)$

$\cos x = x(\sec^2 y)y' + (1 + \tan y)(1)$

$$y' = \frac{\cos x - \tan y - 1}{x \sec^2 y}$$

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