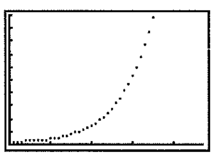
24. To find the approximate values, set $y_1 = x + y$ and use IMPEULT with initial values $x = 0$ and $y = 1$ and step size -0.1 for 10 points. The exact values are given by $y = 2e^x - x - 1$.

x	$y\left(\begin{matrix}\text{improved}\\\text{Euler}\end{matrix}\right)$	y (exact)	Error
0	1	1.0	0
−0.1	0.9100	0.9097	0.0003
−0.2	0.8381	0.8375	0.0006
−0.3	0.7824	0.7816	0.0008
−0.4	0.7416	0.7406	0.0010
−0.5	0.7142	0.7131	0.0011
−0.6	0.6988	0.6976	0.0012
−0.7	0.6944	0.6932	0.0012
−0.8	0.7000	0.6987	0.0013
−0.9	0.7145	0.7131	0.0013
−1.0	0.7371	0.7358	0.0013

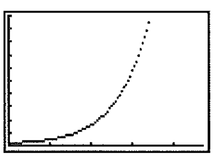
25. Set $y_1 = y + e^x - 2$ and EULERG, with initial values $x = 0$ and $y = 2$ and step sizes 0.1 and 0.05.

(a)



[0, 4.7] by [0, 100]

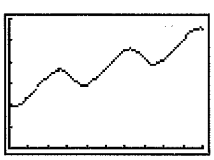
(b)



[0, 4.7] by [0, 100]

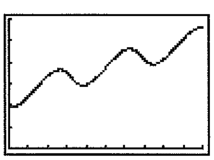
26. Set $y_1 = \cos(2x - y)$ and use EULERG with initial values $x = 0$ and $y = 2$ and step sizes 0.1 and 0.05.

(a)



[0, 10] by [0, 6]

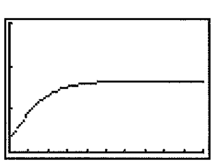
(b)



[0, 10] by [0, 6]

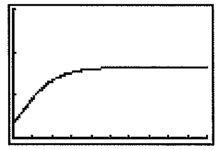
27. Set $y_1 = y\left(\frac{1}{2} - \ln|y|\right)$ and use IMPEULG with initial values $x = 0$ and $y = \frac{1}{3}$ and step size 0.1 and 0.05.

(a)



[0, 10] by [0, 3]

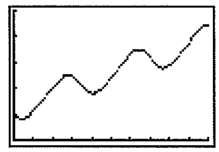
(b)



[0, 10] by [0, 3]

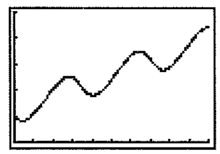
28. Set $y = \sin(2x - y)$ and use IMPEULG with initial values $x = 0$ and $y = 1$ and step sizes 0.1 and 0.05.

(a)



[0, 10] by [0, 5]

(b)



[0, 10] by [0, 5]

29. To find the approximate values, let $y_1 = y$ and use EULERT with initial values $x = 0$ and $y = 1$ and step size 0.05 for 20 points. This gives $y(1) \approx 2.6533$.

Since the exact solution to the initial value problem is $y = e^x$, the exact value of $y(1)$ is e.

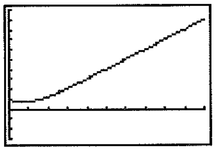
30. To find the approximate values, let $y_1 = 3y$ and use IMPEULT with initial values $x = 0$ and $y = 1$ and step size 0.05 for 20 points. This gives $y(1) \approx 19.8845$.

Since the exact solution to the initial value problem is $y = e^{3x}$, the exact value of $y(1)$ is e^3.

31. To find the approximate values, let $y_1 = 1 + y$ and use RUNKUTT with initial values $x = 0$ and $y = 1$ and step size 0.1 for 10 points. The exact values are given by $y = 2e^x - 1$.

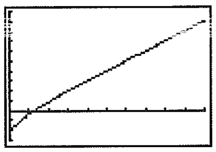
x	y (Runge-Kutta)	y (exact)	Error
0	1	1	0
0.1	1.2103	1.2103	0.0000002
0.2	1.4428	1.4428	0.0000004
0.3	1.6997	1.6997	0.0000006
0.4	1.9836	1.9836	0.0000009
0.5	2.2974	2.2974	0.0000013
0.6	2.6442	2.6442	0.0000017
0.7	3.0275	3.0275	0.0000022
0.8	3.4511	3.4511	0.0000027
0.9	3.9192	3.9192	0.0000034
1.0	4.4366	4.4366	0.0000042

32. (a) Set $y_1 = x - y$ and use RUNKUTT with initial values $x = 0$ and $y = 1$ and step size 0.1.



[0, 10] by [−3, 10]

(b) Use RUNKUTT with initial values $x = 0$ and $y = -2$ and step size 0.1.



[0, 10] by [−3, 10]

Chapter 6 Review Exercises
(pp. 358 – 361)

1. $\int_0^{\pi/3} \sec^2 \theta \, d\theta = \tan \theta \Big]_0^{\pi/3} = \tan \frac{\pi}{3} - \tan 0 = \sqrt{3}$

2. $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \left[\frac{1}{2}x^2 - x^{-1}\right]_1^2$

$= \left(\frac{1}{2}(4) - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right)$

$= \frac{3}{2} + \frac{1}{2}$

$= \frac{4}{2} = 2$

3. Let $u = 2x + 1$

$du = 2\, dx$

$\frac{1}{2} du = dx$

$\int_0^1 \frac{36}{(2x+1)^3} dx = 18 \int_1^3 \frac{1}{u^3} du$

$= 18\left(-\frac{1}{2}\right) u^{-2} \Big]_1^3$

$= -9\left(\frac{1}{9} - 1\right)$

$= -9\left(-\frac{8}{9}\right)$

$= 8$

4. Let $u = 1 - x^2$

$du = -2x\, dx$

$-du = 2x\, dx$

$\int_{-1}^1 2x \sin(1 - x^2)\, dx = -\int_0^0 \sin u\, du = 0$

5. Let $u = \sin x$

$du = \cos x\, dx$

$\int_0^{\pi/2} 5 \sin^{3/2} x \cos x\, dx = \int_0^1 5u^{3/2}\, du$

$= 5 \cdot \frac{2}{5} u^{5/2} \Big]_0^1$

$= 2(1 - 0)$

$= 2$

6. $\int_{1/2}^4 \frac{x^2 + 3x}{x} dx = \int_{1/2}^4 (x + 3)\, dx \ (x \neq 0)$

$= \left(\frac{1}{2}x^2 + 3x\right)\Big]_{1/2}^4$

$= \left(\frac{1}{2}(16) + 3(4)\right) - \left(\frac{1}{2}\left(\frac{1}{4}\right) + \frac{3}{2}\right)$

$= 20 - \left(\frac{1}{8} + \frac{12}{8}\right)$

$= 20 - \frac{13}{8}$

$= \frac{147}{8}$

7. Let $u = \tan x$

$du = \sec^2 x\, dx$

$\int_0^{\pi/4} e^{\tan x} \sec^2 x\, dx = \int_0^1 e^u\, du$

$= e^u \Big]_0^1$

$= e^1 - e^0$

$= e - 1$

8. Let $u = \ln r$

$du = \frac{1}{r} dr$

$\int_1^e \frac{\sqrt{\ln r}}{r} dr = \int_0^1 u^{1/2}\, du$

$= \frac{2}{3} u^{3/2} \Big]_0^1$

$= \frac{2}{3}(1 - 0)$

$= \frac{2}{3}$

9. Let $u = 2 - \sin x$

$du = -\cos x\, dx$

$-du = \cos x\, dx$

$\int \frac{\cos x}{2 - \sin x} dx = -\int \frac{1}{u} du$

$= -\ln |u| + C$

$= -\ln |2 - \sin x| + C$

10. Let $u = 3x + 4$

$$du = 3\,dx$$

$$\frac{1}{3}du = dx$$

$$\int \frac{dx}{\sqrt[3]{3x+4}} = \frac{1}{3}\int u^{-1/3}\,du$$
$$= \frac{1}{3}\cdot\frac{3}{2}u^{2/3} + C$$
$$= \frac{1}{2}(3x+4)^{2/3} + C$$

11. Let $u = t^2 + 5$

$$du = 2t\,dt$$

$$\frac{1}{2}du = t\,dt$$

$$\int \frac{t\,dt}{t^2+5} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$
$$= \frac{1}{2}\ln|t^2+5| + C$$
$$= \frac{1}{2}\ln(t^2+5) + C$$

12. Let $u = \frac{1}{\theta}$

$$du = -\frac{1}{\theta^2}\,d\theta$$

$$\int \frac{1}{\theta^2}\sec\frac{1}{\theta}\tan\frac{1}{\theta}\,d\theta = -\int \sec u\tan u\,du$$
$$= -\sec u + C$$
$$= -\sec\frac{1}{\theta} + C$$

13. Let $u = \ln y$

$$du = \frac{1}{y}\,dy$$

$$\int \frac{\tan(\ln y)}{y}\,dy = \int \tan u\,du$$
$$= \int \frac{\sin u}{\cos u}\,du$$

Let $w = \cos u$

$$dw = -\sin u\,du$$

$$= -\int \frac{1}{w}\,dw$$
$$= -\ln|w| + C$$
$$= -\ln|\cos u| + C$$
$$= -\ln|\cos(\ln y)| + C$$

14. Let $u = e^x$

$$du = e^x\,dx$$

$$\int e^x\sec(e^x)\,dx = \int \sec u\,du$$
$$= \ln|\sec u + \tan u| + C$$
$$= \ln|\sec(e^x) + \tan(e^x)| + C$$

15. Let $u = \ln x$

$$du = \frac{1}{x}\,dx$$

$$\int \frac{dx}{x\ln x} = \int \frac{1}{u}\,du$$
$$= \ln|u| + C$$
$$= \ln|\ln x| + C$$

16. $\int \frac{dt}{t\sqrt{t}} = \int \frac{dt}{t^{3/2}}$

$$= \int t^{-3/2}\,dt$$

$$= -2t^{-1/2} + C$$

$$= -\frac{2}{\sqrt{t}} + C$$

17. Use tabular integration with $f(x) = x^3$ and $g(x) = \cos x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	$\cos x$
$3x^2$	(−)	$\sin x$
$6x$	(+)	$-\cos x$
6	(−)	$-\sin x$
0		$\cos x$

$$\int x^3\cos x\,dx$$
$$= x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$$

18. Let $u = \ln x$ $\qquad dv = x^4\,dx$

$$du = \frac{1}{x}\,dx \qquad v = \frac{1}{5}x^5$$

$$\int x^4\ln x\,dx = \frac{1}{5}x^5\ln x - \int \frac{1}{5}x^5\left(\frac{1}{x}\right)dx$$
$$= \frac{1}{5}x^5\ln x - \frac{1}{5}\int x^4\,dx$$
$$= \frac{1}{5}x^5\ln x - \frac{1}{25}x^5 + C$$

19. Let $u = e^{3x}$ $\qquad dv = \sin x\, dx$

$du = 3e^{3x}\, dx \qquad v = -\cos x$

$$\int e^{3x} \sin x\, dx = -e^{3x}\cos x + \int 3\cos x\, e^{3x}\, dx$$

Integrate by parts again.

Let $u = 3e^{3x}$ $\qquad dv = \cos x\, dx$

$du = 9e^{3x}\, dx \qquad v = \sin x$

$$\int e^{3x}\sin x\, dx = -e^{3x}\cos x + 3e^{3x}\sin x - \int 9e^{3x}\sin x\, dx$$

$$10\int e^{3x}\sin x\, dx = -e^{3x}\cos x + 3e^{3x}\sin x + C$$

$$\int e^{3x}\sin x\, dx = \frac{1}{10}[-e^{3x}\cos x + 3e^{3x}\sin x] + C$$

$$= \left(\frac{3\sin x}{10} - \frac{\cos x}{10}\right)e^{3x} + C$$

20. Let $u = x^2$ $\qquad dv = e^{-3x}\, dx$

$du = 2x\, dx \qquad v = -\frac{1}{3}e^{-3x}$

$$\int x^2 e^{-3x}\, dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int e^{-3x}x\, dx$$

Let $u = x$ $\qquad dv = e^{-3x}\, dx$

$du = dx \qquad v = -\frac{1}{3}e^{-3x}$

$$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\, dx\right]$$

$$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}\, dx$$

$$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$$

$$= \left(-\frac{x^2}{3} - \frac{2x}{9} - \frac{2}{27}\right)e^{-3x} + C$$

21. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}$

$dy = \left(1 + x + \frac{x^2}{2}\right)dx$

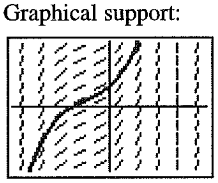
$\int dy = \int\left(1 + x + \frac{x^2}{2}\right)dx$

$y = x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + C$

$y(0) = C = 1$

$y = \frac{x^3}{6} + \frac{x^2}{2} + x + 1$

Graphical support:



[−4, 4] by [−3, 3]

22. $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$

$dy = \left(x + \frac{1}{x}\right)^2 dx$

$\int dy = \int\left(x + \frac{1}{x}\right)^2 dx$

$y = \int\left(x^2 + 2 + \frac{1}{x^2}\right)dx$

$y = \frac{1}{3}x^3 + 2x - x^{-1} + C$

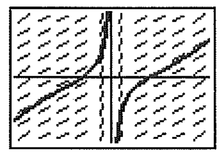
$y(1) = \frac{1}{3} + 2 - 1 + C = 1$

$\frac{4}{3} + C = 1$

$C = -\frac{1}{3}$

$y = \frac{x^3}{3} + 2x - \frac{1}{x} - \frac{1}{3}$

Graphical support:



[−2, 2] by [−10, 10]

23. $\frac{dy}{dt} = \frac{1}{t+4}$

$dy = \frac{1}{t+4}\, dt$

$\int dy = \int \frac{1}{t+4}\, dt$

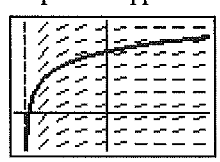
$y = \ln|t + 4| + C$

$y(-3) = \ln(1) + C = 2$

$C = 2$

$y = \ln(t + 4) + 2$

Graphical Support:



[−4.5, 5] by [−2, 5]

24. $\frac{dy}{d\theta} = \csc 2\theta \cot 2\theta$

$dy = \csc 2\theta \cot 2\theta\, d\theta$

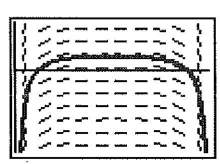
$\int dy = \int \csc 2\theta \cot 2\theta\, d\theta$

$y = -\frac{1}{2}\csc 2\theta + C$

$y\left(\frac{\pi}{4}\right) = -\frac{1}{2} + C = 1$

$C = \frac{3}{2}$

$y = -\frac{1}{2}\csc 2\theta + \frac{3}{2}$



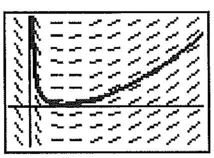
[0, 1.57] by [−5, 3]

25.
$$\frac{d(y')}{dx} = 2x - \frac{1}{x^2}$$
$$d(y') = \left(2x - \frac{1}{x^2}\right) dx$$
$$\int d(y') = \int\left(2x - \frac{1}{x^2}\right) dx$$
$$y' = x^2 + x^{-1} + C$$
$$y'(1) = 2 + C = 1$$
$$C = -1$$
$$y' = x^2 + x^{-1} - 1$$
$$\int dy = \int(x^2 + x^{-1} - 1)\,dx$$
$$y = \frac{1}{3}x^3 + \ln x - x + C$$
$$y(1) = \frac{1}{3} + 0 - 1 + C = 0$$
$$-\frac{2}{3} + C = 0$$
$$C = \frac{2}{3}$$
$$y = \frac{x^3}{3} + \ln x - x + \frac{2}{3}$$

Graphical support:

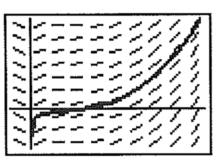
Let $f(x) = \frac{x^3}{3} + \ln x - x + \frac{2}{3}$.

We first show the graph of $y = f'(x) = x^2 + x^{-1} - 1$, $x > 0$, along with the slope field for $y' = f''(x) = 2x - \frac{1}{x^2}$.



[−0.5, 4.21] by [−9, 21]

We now show the graph of $y = f(x)$ along with the slope field for $y' = f'(x) = x^2 + x^{-1} - 1$.

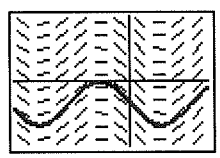


[−0.5, 4.21] by [−9, 21]

26.
$$\frac{d(r'')}{dt} = -\cos t$$
$$d(r'') = -\cos t\,dt$$
$$\int d(r'') = \int -\cos t\,dt$$
$$r'' = -\sin t + C$$
$$r''(0) = C = -1$$
$$r'' = -\sin t - 1$$
$$\int d(r') = \int(-\sin t - 1)\,dt$$
$$r' = \cos t - t + C$$
$$r'(0) = 1 + C = -1$$
$$C = -2$$
$$r' = \cos t - t - 2$$
$$\int dr = \int(\cos t - t - 2)\,dt$$
$$r = \sin t - \frac{t^2}{2} - 2t + C$$
$$r(0) = C = -1$$
$$r = \sin t - \frac{t^2}{2} - 2t - 1$$

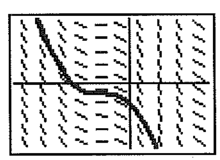
Graphical support:

We first show the graph of $y = r'' = -\sin t - 1$ along with the slope field for $y' = r''' = -\cos t$.



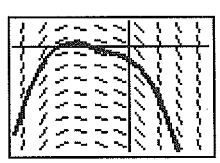
[−6, 4] by [−3, 3]

Next, we show the graph of $y = r' = \cos t - t - 2$ along with the slope field for $y' = r'' = -\sin t - 1$.



[−6, 4] by [−3, 3]

Finally we show the graph of $y = r = \sin t - \frac{t^2}{2} - 2t - 1$ along with the slope field for $y' = r' = \cos t - t - 2$.



[−6, 4] by [−8, 2]

27. $\frac{dy}{dx} = y + 2$

$\frac{dy}{y+2} = dx$

$\int \frac{dy}{y+2} = \int dx$

$\ln|y + 2| = x + C$

$y + 2 = Ce^x$

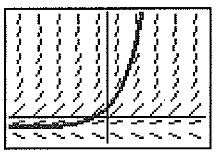
$y = Ce^x - 2$

$y(0) = C - 2 = 2$

$C = 4$

$y = 4e^x - 2$

Graphical support:



[−5, 5] by [−5, 20]

28. $\frac{dy}{dx} = (2x + 1)(y + 1)$

$\frac{dy}{y+1} = (2x + 1)\,dx$

$\int \frac{dy}{y+1} = \int (2x + 1)\,dx$

$\ln|y + 1| = x^2 + x + C$

$y + 1 = Ce^{x^2+x}$

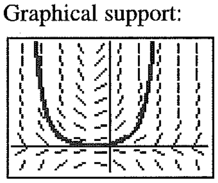
$y = Ce^{x^2+x} - 1$

$y(-1) = C - 1 = 1$

$C = 2$

$y = 2e^{x^2+x} - 1$

Graphical support:



[−3, 3] by [−10, 40]

29. $\int -f(x)\,dx = -\int f(x)\,dx$

$= -(1 - \sqrt{x}) + C$

$= -1 + \sqrt{x} + C$

Since $-1 + C$ is an arbitrary constant, we may write the indefinite integral as $\sqrt{x} + C$.

30. $\int [x + f(x)]\,dx = \int x\,dx + \int f(x)\,dx$

$= \frac{x^2}{2} + (1 - \sqrt{x}) + C$

$= \frac{x^2}{2} + 1 - \sqrt{x} + C$

Since $1 + C$ is an arbitrary constant, we may write the indefinite integral as $\frac{x^2}{2} - \sqrt{x} + C$.

31. $\int [2f(x) - g(x)]\,dx = 2\int f(x)\,dx - \int g(x)\,dx$

$= 2(1 - \sqrt{x}) - (x + 2) + C$

$= -2\sqrt{x} - x + C$

32. $\int [g(x) - 4]\,dx = \int g(x)\,dx - \int 4\,dx$

$= (x + 2) - 4x + C$

$= 2 - 3x + C$

Since $2 + C$ is an arbitrary constant, we may write the indefinite integral as $-3x + C$.

33. We seek the graph of a function whose derivative is $\frac{\sin x}{x}$. Graph (b) is increasing on $[-\pi, \pi]$, where $\frac{\sin x}{x}$ is positive, and oscillates slightly outside of this interval. This is the correct choice, and this can be verified by graphing $\text{NINT}\left(\frac{\sin x}{x}, x, 0, x\right)$.

34. We seek the graph of a function whose derivative is e^{-x^2}. Since $e^{-x^2} > 0$ for all x, the desired graph is increasing for all x. Thus, the only possibility is graph (d), and we may verify that this is correct by graphing $\text{NINT}(e^{-x^2}, x, 0, x)$.

35. (iv) The given graph looks like the graph of $y = x^2$, which satisfies $\frac{dy}{dx} = 2x$ and $y(1) = 1$.

36. Yes, $y = x$ is a solution.

37. (a) $\frac{dv}{dt} = 2 + 6t$

$\int dv = \int (2 + 6t)\,dt$

$v = 2t + 3t^2 + C$

Initial condition: $v = 4$ when $t = 0$

$4 = 0 + C$

$4 = C$

$v = 2t + 3t^2 + 4$

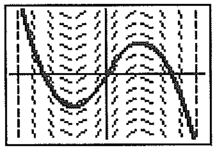
(b) $\int_0^1 v(t)\,dt = \int_0^1 (2t + 3t^2 + 4)\,dt$

$= \left[t^2 + t^3 + 4t\right]_0^1$

$= 6 - 0$

$= 6$

The particle moves 6 m.

38. 

[−10, 10] by [−10, 10]

39. Set $y_1 = y + \cos x$ and use EULERT with initial values $x = 0$ and $y = 0$ and step size 0.1 for 20 points.

x	y
0	0
0.1	0.1000
0.2	0.2095
0.3	0.3285
0.4	0.4568
0.5	0.5946
0.6	0.7418
0.7	0.8986
0.8	1.0649
0.9	1.2411
1.0	1.4273
1.1	1.6241
1.2	1.8319
1.3	2.0513
1.4	2.2832
1.5	2.5285
1.6	2.7884
1.7	3.0643
1.8	3.3579
1.9	3.6709
2.0	4.0057

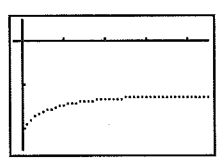
40. Set $y_1 = (2 - y)(2x + 3)$ and use IMPEULT with intial values $x = -3$ and $y = 1$ and step size 0.1 for 20 points.

x	y
−3	1
−2.9	0.6680
−2.8	0.2599
−2.7	−0.2294
−2.6	−0.8011
−2.5	−1.4509
−2.4	−2.1687
−2.3	−2.9374
−2.2	−3.7333
−2.1	−4.5268
−2.0	−5.2840
−1.9	−5.9686
−1.8	−6.5456
−1.7	−6.9831
−1.6	−7.2562
−1.5	−7.3488
−1.4	−7.2553
−1.3	−6.9813
−1.2	−6.5430
−1.1	−5.9655
−1.0	−5.2805

41. To estimate $y(3)$, set $y_1 = \frac{x - 2y}{x + 1}$ and use IMPEULT with initial values $x = 0$ and $y = 1$ and step size 0.05 for 60 points. This gives $y(3) \approx 0.9063$.

42. To estimate $y(4)$, set $y_1 = \frac{x^2 - 2y + 1}{x}$ and use EULERT with initial values $x = 1$ and $y = 1$ and step size 0.05 for 60 points. This gives $y(4) \approx 4.4974$.

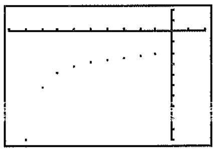
43. Set $y_1 = e^{-(x+y+2)}$ and use EULERG with initial values $x = 0$ and $y = -2$ and step sizes 0.1 and −0.1.

(a) 

[−0.2, 4.5] by [−2.5, 0.5]

43. continued

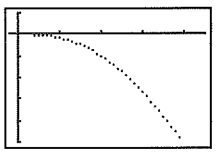
(b) Note that we choose a small interval of x-values because the y-values decrease very rapidly and our calculator cannot handle the calculations for $x \le -1$. (This occurs because the analytic solution is $y = -2 + \ln(2 - e^{-x})$, which has an asymptote at $x = -\ln 2 \approx -0.69$. Obviously, the Euler approximations are misleading for $x \le -0.7$.)



$[-1, 0.2]$ by $[-10, 2]$

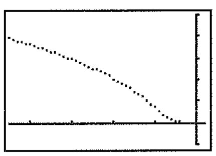
44. Set $y_1 = -\dfrac{x^2 + y}{e^y + x}$ and use IMPEULG with initial values $x = 0$ and $y = 0$ and step sizes 0.1 and -0.1.

(a)



$[-0.2, 4.5]$ by $[-5, 1]$

(b)



$[-4.5, 0.2]$ by $[-1, 5]$

45. (a) Half-life $= \dfrac{\ln 2}{k}$

$$2.645 = \frac{\ln 2}{k}$$

$$k = \frac{\ln 2}{2.645} \approx 0.262059$$

(b) Mean life $= \dfrac{1}{k} \approx 3.81593$ years

46. $T - T_s = (T_0 - T_s)e^{-kt}$

$T - 40 = (220 - 40)e^{-kt}$

Use the fact that $T = 180$ and $t = 15$ to find k.

$$180 - 40 = (220 - 40)e^{-(k)(15)}$$

$$e^{15k} = \frac{180}{140} = \frac{9}{7}$$

$$k = \frac{1}{15}\ln\frac{9}{7}$$

$$T - 40 = (220 - 40)e^{-((1/15)\ln(9/7))t}$$

$$70 - 40 = (220 - 40)e^{-((1/15)\ln(9/7))t}$$

$$e^{((1/15)\ln(9/7))t} = \frac{180}{30} = 6$$

$$\left(\frac{1}{15}\ln\frac{9}{7}\right)t = \ln 6$$

$$t = \frac{15\ln 6}{\ln(9/7)} \approx 107 \text{ min}$$

It took a total of about 107 minutes to cool from 220°F to 70°F. Therefore, the time to cool from 180°F to 70°F was about 92 minutes.

47. $T - T_s = (T_0 - T_s)e^{-kt}$

We have the system:

$$\begin{cases} 39 - T_s = (46 - T_s)e^{-10k} \\ 33 - T_s = (46 - T_s)e^{-20k} \end{cases}$$

Thus, $\dfrac{39 - T_s}{46 - T_s} = e^{-10k}$ and $\dfrac{33 - T_s}{46 - T_s} = e^{-20k}$.

Since $(e^{-10k})^2 = e^{-20k}$, this means:

$$\left(\frac{39 - T_s}{46 - T_s}\right)^2 = \frac{33 - T_s}{46 - T_s}$$

$$(39 - T_s)^2 = (33 - T_s)(46 - T_s)$$

$$1521 - 78T_s + T_s^2 = 1518 - 79T_s + T_s^2$$

$$T_s = -3$$

The refrigerator temperature was -3°C.

48. Use the method of Example 3 in Section 6.4.

$$e^{-kt} = 0.995$$

$$-kt = \ln 0.995$$

$$t = -\frac{1}{k}\ln 0.995 = -\frac{5700}{\ln 2}\ln 0.995 \approx 41.2$$

The painting is about 41.2 years old.

49. Use the method of Example 3 in Section 6.4.

Since 90% of the carbon-14 has decayed, 10% remains.

$$e^{-kt} = 0.1$$

$$-kt = \ln 0.1$$

$$t = -\frac{1}{k}\ln 0.1 = -\frac{5700}{\ln 2}\ln 0.1 \approx 18{,}935$$

The charcoal sample is about 18,935 years old.

50. Use $t = 1988 - 1924 = 64$ years.

$$250\,e^{rt} = 7500$$

$$e^{rt} = 30$$

$$rt = \ln 30$$

$$r = \frac{\ln 30}{t} = \frac{\ln 30}{64} \approx 0.053$$

The rate of appreciation is about 0.053, or 5.3%.

51. Using the Law of Exponential Change in Section 6.4 with appropriate changes of variables, the solution to the differential equation is $L(x) = L_0 e^{-kx}$, where $L_0 = L(0)$ is the surface intensity. We know $0.5 = e^{-18k}$, so $k = \frac{\ln 0.5}{-18}$ and our equation becomes $L(x) = L_0 e^{(\ln 0.5)(x/18)} = L_0\left(\frac{1}{2}\right)^{x/18}$. We now find the depth where the intensity is one-tenth of the surface value.

$$0.1 = \left(\frac{1}{2}\right)^{x/18}$$
$$\ln 0.1 = \frac{x}{18}\ln\left(\frac{1}{2}\right)$$
$$x = \frac{18 \ln 0.1}{\ln 0.5} \approx 59.8 \text{ ft}$$

You can work without artificial light to a depth of about 59.8 feet.

52. (a)
$$\frac{dy}{dt} = \frac{kA}{V}(c - y)$$
$$\int \frac{dy}{c-y} = \int \frac{kA}{V}\,dt$$
$$-\ln|c - y| = \frac{kA}{V}t + C$$
$$\ln|c - y| = -\frac{kA}{V}t - C$$
$$|c - y| = e^{-(kA/V)t-C}$$
$$c - y = \pm e^{-(kA/V)t-C}$$
$$y = c \pm e^{-(kA/V)t-C}$$
$$y = c + De^{-(kA/V)t}$$

Initial condition $y = y_0$ when $t = 0$

$$y_0 = c + D$$
$$y_0 - c = D$$

Solution: $y = c + (y_0 - c)e^{-(kA/V)t}$

(b) $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} [c + (y_0 - c)e^{-(kA/V)t}] = c$

53. (a) $P(t) = \frac{150}{1 + e^{4.3-t}} = \frac{150}{1 + e^{4.3}e^{-t}}$

This is $P = \frac{M}{1 + Ae^{-kt}}$ where $M = 150$, $A = e^{4.3}$, and $k = 1$. Therefore, it is a solution of the logistic differential equation.

$\frac{dP}{dt} = \frac{k}{M}P(M - P)$, or $\frac{dP}{dt} = \frac{1}{150}P(150 - P)$. The carrying capacity is 150.

(b) $P(0) = \frac{150}{1 + e^{4.3}} \approx 2$

Initially there were 2 infected students.

(c)
$$\frac{150}{1 + e^{4.3-t}} = 125$$
$$\frac{6}{5} = 1 + e^{4.3-t}$$
$$\frac{1}{5} = e^{4.3-t}$$
$$-\ln 5 = 4.3 - t$$
$$t = 4.3 + \ln 5 \approx 5.9 \text{ days}$$

It took about 6 days.

54. Use the Fundamental Theorem of Calculus.

$$y' = \frac{d}{dx}\left(\int_0^x \sin t^2\,dt\right) + \frac{d}{dx}(x^3 + x + 2)$$
$$= (\sin x^2) + (3x^2 + 1)$$
$$y'' = \frac{d}{dx}(\sin x^2 + 3x^2 + 1)$$
$$= (\cos x^2)(2x) + 6x$$
$$= 2x\cos(x^2) + 6x$$

Thus, the differential equation is satisfied.

Verify the initial conditions:

$$y'(0) = (\sin 0^2) + 3(0)^2 + 1 = 1$$
$$y(0) = \int_0^0 \sin(t^2)\,dt + 0^3 + 0 + 2 = 2$$

55.
$$\frac{dP}{dt} = 0.002P\left(1 - \frac{P}{800}\right)$$
$$\frac{dP}{dt} = 0.002P\left(\frac{800 - P}{800}\right)$$
$$\frac{800}{P(800 - P)}\,dP = 0.002\,dt$$
$$\frac{(800 - P) + P}{P(800 - P)}\,dP = 0.002\,dt$$
$$\int\left(\frac{1}{P} + \frac{1}{800 - P}\right)dP = \int 0.002\,dt$$
$$\ln|P| - \ln|800 - P| = 0.002t + C$$
$$\ln\left|\frac{P}{800 - P}\right| = 0.002t + C$$
$$\ln\left|\frac{800 - P}{P}\right| = -0.002t - C$$
$$\left|\frac{800 - P}{P}\right| = e^{-0.002t-C}$$
$$\frac{800 - P}{P} = \pm e^{-C}e^{-0.002t}$$
$$\frac{800}{P} - 1 = Ae^{-0.002t}$$
$$P = \frac{800}{1 + Ae^{-0.002t}}$$

Initial condition: $P(0) = 50$

$$50 = \frac{800}{1 + Ae^0}$$
$$1 + A = 16$$
$$A = 15$$

Solution: $P = \frac{800}{1 + 15e^{-0.002t}}$

56. Method 1–Compare graph of $y_1 = x^2 \ln x$ with $y_2 = \text{NDER}\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right)$. The graphs should be the same.

Method 2–Compare graph of $y_1 = \text{NINT}(x^2 \ln x)$ with $y_2 = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$. The graphs should be the same or differ only by a vertical translation.

57. (a)

$$20{,}000 = 10{,}000(1.063)^t$$
$$2 = 1.063^t$$
$$\ln 2 = t \ln 1.063$$
$$t = \frac{\ln 2}{\ln 1.063} \approx 11.345$$

It will take about 11.3 years.

(b)

$$20{,}000 = 10{,}000e^{0.063t}$$
$$2 = e^{0.063t}$$
$$\ln 2 = 0.063t$$
$$t = \frac{\ln 2}{0.063} \approx 11.002$$

It will take about 11.0 years.

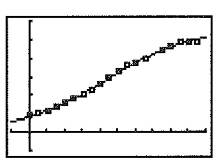
58. (a) $f'(x) = \frac{d}{dx}\int_0^x u(t)\,dt = u(x)$

$g'(x) = \frac{d}{dx}\int_3^x u(t)\,dt = u(x)$

(b) $C = f(x) - g(x)$

$$= \int_0^x u(t)\,dt - \int_3^x u(t)\,dt$$
$$= \int_0^x u(t)\,dt + \int_x^3 u(t)\,dt$$
$$= \int_0^3 u(t)\,dt$$

59. (a) $y = \frac{56.0716}{1 + 5.894e^{-0.0205x}}$

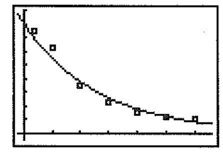


[−20, 200] by [−10, 60]

(b) The carrying capacity is about 56.0716 million people.

(c) Use NDER twice to solve $y'' = 0$. The solution is $x \approx 86.52$, representing (approximately) the year 1887. The population at this time was approximately $P(86.52) \approx 28.0$ million people.

60. (a) $T = 79.961(0.9273)^t$



[−1, 33] by [−5, 90]

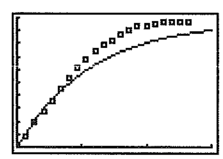
(b) Solving $T(t) = 40$ graphically, we obtain $t \approx 9.2$ sec. The temperature will reach 40° after about 9.2 seconds.

(c) When the probe was removed, the temperature was about $T(0) \approx 79.96$°C.

61.

$$\frac{v_0 m}{k} = \text{coasting distance}$$
$$\frac{(0.86)(30.84)}{k} = 0.97$$
$$k \approx 27.343$$
$$s(t) = \frac{v_0 m}{k}(1 - e^{-(k/m)t})$$
$$s(t) = 0.97(1 - e^{-(27.343/30.84)t})$$
$$s(t) = 0.97(1 - e^{-0.8866t})$$

A graph of the model is shown superimposed on a graph of the data.



[0, 3] by [0,1]

Chapter 7
Applications of Definite Integrals

Section 7.1 Integral as Net Change (pp. 363–374)

Exploration 1 Revisiting Example 2

1. $s(t) = \int\left(t^2 - \frac{8}{(t+1)^2}\right)dt = \frac{t^3}{3} + \frac{8}{t+1} + C$

$s(0) = \frac{0^3}{3} + \frac{8}{0+1} + C = 9 \Rightarrow C = 1$

Thus, $s(t) = \frac{t^3}{3} + \frac{8}{t+1} + 1$.

2. $s(1) = \frac{1^3}{3} + \frac{8}{1+1} + 1 = \frac{16}{3}$. This is the same as the answer we found in Example 2a.

3. $s(5) = \frac{5^3}{3} + \frac{8}{5+1} + 1 = 44$. This is the same answer we found in Example 2b.