Rocket City Math League 2017-2018

Discovery Solutions Round 1

1. Answer: $x = -2, x = 3$

Factor the denominator into $(x - 3)(x + 2) = 0$.

2. Answer: 10 Glaks

The price of 4 Gloobs and 1 Glog is equal to 90 Glaks. Her payment of 3 Glorks equals 100 Glaks. Therefore, her change will be 10 Glaks.

3. **Answer: 75 minutes**

Imagine the moon as a clock and the satellite is a hand in the clock. Its orbital period of 20 minutes means the satellite travels one quarter of a cycle in 5 minutes. Let's say the satellite starts in the 12 o'clock position and takes a photo to start the timer. This covers everything from 10 to 2 o'clock. 25 minutes later, the satellite is at the 3 o'clock position and takes another picture. This covers everything from 1 to 5 o'clock. 25 minutes later, the third photograph is taken from the 6 o'clock position, which covers everything from 4 to 8. 25 minutes later, the fourth photograph is taken from the 9 o'clock position, covering 7 to 11 o'clock and therefore the whole equator. This occurs 75 minutes from the time of the first photograph.

4. **Answer:** $\begin{bmatrix} \mathbf{-2} & \mathbf{-1} \\ \mathbf{7/4} & \mathbf{3/4} \end{bmatrix}$

Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. In this case, the $ad - bc = 4$, so $A^{-1} = \frac{1}{4}\begin{bmatrix} 4 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1/4 & 1/2 \end{bmatrix}$. Multiply this by matrix B to get $\begin{bmatrix} -2 & -1 \\ 7/4 & 3/4 \end{bmatrix}$.

5. **Answer:** $\dfrac{3\sqrt{6}+2}{50}$

Using the identity $sin^2A + cos^2A = 1$, and plugging in the given value of cosine, $\sin A = \sqrt{1-\frac{25}{49}} = \sqrt{\frac{24}{49}} = \frac{2\sqrt{6}}{7}$. Plug in sin A to get $\dfrac{2}{21*\frac{2\sqrt{6}}{7} - 4}$ which simplifies to $\dfrac{2}{6\sqrt{6}-4} = \dfrac{1}{3\sqrt{6}-2}$ which needs to be rationalized by multiplying the numerator and denominator by the conjugate of the denominator: $3\sqrt{6}+2$, simplifying the expression to $\dfrac{3\sqrt{6}+2}{50}$.

6. **Answer: 7th position**

The astronaut should stand in the 7th position. The first player teleports the 4th player, the second teleports the 6th, the 3rd teleports the 8th, the fifth teleports the tenth, the seventh teleports the second, the 9th teleports the 5th, the first teleports the 9th, the 3rd teleports himself, and the seventh teleports the first, leaving him as the last remaining prisoner to be set free.

7. **Answer: 6,521,488** or 13,042,976

The formula for the area of a square is s^2. Let's set the area of Commander Cool's ship equal to a^2, with a= 6,521,493. The area of Commander Starlight's ship will be equal to b^2, with b = 6,521,483. We are asked for the difference between the two areas, so the resulting equation will be $a^2 - b^2$, which factors into $(a + b)(a - b)$. This will be $13{,}042{,}976 \cdot 10 = 130{,}429{,}760$ m^2. Divide the product by 20 m^2 to get 6,521,488 Bloops. If you consider the ship "two sided" like a flat paper ship, its area will be twice that, or 13,042,976.

8. Answer: $a = -3, 1/2$

Both trig functions = ½. Let b = ½ and the equation becomes: $a^2 - ab + 3a - 3b = 0$, which factors out into $a(a - b) + 3(a - b) = 0$. This becomes $(a + 3)(a - b) = 0$. Therefore, $a = -3$ and $a = 1/2$.

9. **Answer: 33/8**

The solutions are 2, 17/8, and two imaginary roots which won't be x-intercepts. So the sum of the x values of the x-intercepts is 33/8.

10. **Answer:** $\frac{-44}{125}$

Method I: $\cos 3A = 4(\cos A)^3 - 3\cos A$. Using $\sin A = \frac{3}{5}$, solve for cosine using the Pythagorean Theorem. $c^2 - a^2 = b^2$ or $\cos A = \frac{4}{5}$. Plugging this into the formula gives $\cos 3A = (4)\left(\frac{4}{5}\right)^3 - (3)\left(\frac{4}{5}\right)$. Therefore, $\cos 3A = \frac{-44}{125}$.

Method 2:

Use double angle formulas to get $\sin 2A = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$ and

$\cos 2A = 2\left(\frac{4}{5}\right)^2 - 1 = \frac{7}{25}$. So cos(3A)=cos A cos 2A − sin A sin 2A.

Plugging in values gives cos (3A) = $\left(\frac{4}{5}\right)\left(\frac{7}{25}\right) - \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = \frac{-44}{125}$.