Rocket City Math League
Discovery Solutions

2018-2019
Round 1

1. **Answer:** $\begin{bmatrix} \mathbf{43} & \mathbf{41} \\ \mathbf{33} & \mathbf{57} \end{bmatrix}$

$$\begin{bmatrix} 5 \cdot 7 + 4 \cdot 2 & 5 \cdot 1 + 4 \cdot 9 \\ 3 \cdot 7 + 6 \cdot 2 & 3 \cdot 1 + 6 \cdot 9 \end{bmatrix}$$

2. **Answer: 5140**

Julia will have $55 \cdot 90 + 190 = 5140$ Grepples after working for 90 days.

3. **Answer: 264**

Since the probability of randomly selecting a Gazorpazorp is 20%, the probability of randomly selecting a Plutonian is 80%. In addition, the probability of randomly selecting a regular member given that the member is a Plutonian is 45%, so the probability of randomly selecting an administrator from the Plutonians is 55%. Thus, the number of Plutonian administrators is $.8 \cdot .55 \cdot 600 = 264$.

4. **Answer:** $\frac{3}{2}$ **or 1.5**

The number of gallons of water that Mark adds represents an infinite geometric series, $a_n = a_1(r)^{n-1}$, where $a_1 = 1$ and $r = \frac{1}{3}$. Thus, the sum is:

$$S_\infty = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

5. **Answer**: **200**

The rover's path represents a 3-4-5 right triangle, where the legs of the triangle have lengths of $40 \cdot 3$ and $40 \cdot 4$. Thus, the length of the hypotenuse, which equals the distance from its starting point, is simply $40 \cdot 5$.

6. **Answer: 96**

Use the binomial expansion formula: $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$

$$(3x^3)^4 + \frac{4}{1!}(3x^3)^3(2y^2) + \frac{4(3)}{2!}(3x^3)^2(2y^2)^2 + \frac{4(3)(2)}{3!}(3x^3)(2y^2)^3 + \frac{4(3)(2)(1)}{4!}(2y^2)^4$$

7. **Answer: 1024**

Solving $\log_2 N = 10$ gives $N = 2^{10}$.

8. **Answer: 0.35**

For independent events A and B, $P(A|B) = P(A)$. Thus, the probability Zuro will be attacked,given Teno is not attacked is equal to the probability that Zuro will be attacked, or 0.35.

9. **Answer: 65**

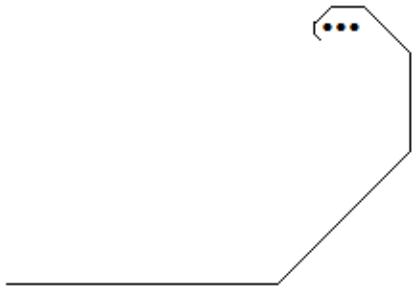
The amplitude is $\sqrt{33^2 + 56^2}$.

10. **Answer:** $(\mathbf{4}, \mathbf{4})$

If written out, the distances travelled by Maggy can be grouped into different directions of movement.

Distances Grouped by Movement				
_	4	1	$\frac{1}{4}$	•••
/	$\frac{4\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{8}$	•••
|	2	$\frac{1}{2}$	$\frac{1}{8}$	•••
\	$\frac{2\sqrt{2}}{2}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{16}$	•••

The path would look similar to the figure below (not drawn to scale):



The path distances can be further divided into x and y components which determine the position approached in the as Maggy's movement iteration count approaches infinity. Each segment of the path can be written as an infinite geometric sum, $S = \frac{a_1}{1-r}$, and the total resultant coordinate is:

$$(total\ sum\ of\ x\ series, total\ sum\ of\ y\ series)$$

X Components				
_	4	-1	$\frac{1}{4}$	•••
/	2	$-\frac{1}{2}$	$\frac{1}{8}$	•••
|	0	0	0	•••
\	-1	$\frac{1}{4}$	$-\frac{1}{16}$	•••

Strictly horizontal component: $S = \frac{4}{1-(-\frac{1}{4})} = \frac{16}{5}$

Diagonally right component: $S = \frac{2}{1-(-\frac{1}{4})} = \frac{8}{5}$

Diagonally left component: $S = \frac{-1}{1-(-\frac{1}{4})} = -\frac{4}{5}$

Y Components				
_	0	0	0	•••
/	2	$-\frac{1}{2}$	$\frac{1}{8}$	•••
|	2	$-\frac{1}{2}$	$\frac{1}{8}$	•••
\	1	$-\frac{1}{4}$	$\frac{1}{16}$	•••

Diagonally right component: $S = \frac{2}{1-(-\frac{1}{4})} = \frac{8}{5}$

Strictly vertical component: $S = \frac{2}{1-(-\frac{1}{4})} = \frac{8}{5}$

Diagonally left component: $S = \frac{1}{1-(-\frac{1}{4})} = \frac{4}{5}$

Thus the solution is: $(S_x, S_y) = \left(\frac{16}{5} + \frac{8}{5} - \frac{4}{5}, \frac{8}{5} + \frac{8}{5} + \frac{4}{5}\right) = (4,4)$.