



Rocket City Math League Gemini Solutions

2018-2019
Round 1

Note: Units are not necessary in answers.

1. **Answer: 42π feet**

If the surface area of the faces of the tire is 441π , then $A = \pi r^2 = 441\pi$, where r is the radius of the tire. Solving for r gives $r = 21$. Circumference equals $2\pi r$, so the circumference is 42π .

2. **Answer: $\frac{324}{\pi}$ inches**

9 feet is the length of $\frac{60^\circ}{360^\circ} = \frac{1}{6}$ of the circumference of the circle. Thus, the circumference of the circle is 54 feet. Also, circumference equals $2\pi r$, so $2\pi r = 54$ gives $r = \frac{27}{\pi}$ feet. In inches, $r = \frac{27}{\pi} \cdot 12 = \frac{324}{\pi}$.

3. **Answer: 288π mi³**

Since Planet Xerov's circumference at its widest point is 48π miles and circumference equals $2\pi r$, where r is its radius, Planet Xerov's radius is 24 miles. Thus, Planet Zorb's radius is $\frac{1}{4} \cdot 24 = 6$ miles. The volume of Planet Zorb is $\frac{4}{3}\pi r^3 = 288\pi$.

4. **Answer: $50\sqrt{3}$ m²**

The surface area of one face of the octahedron houses is equal to $\frac{5^2\sqrt{3}}{4} = \frac{25\sqrt{3}}{4}$ square meters. Since each of the houses has 8 faces, the total surface area of one house is $\frac{25\sqrt{3}}{4} \cdot 8 = 50\sqrt{3}$ square meters.

5. **Answer: $\sqrt{31}$ feet**

The diameter of the circle with length $2r = 16$ is the diagonal of the inscribed rectangle and forms a right triangle with hypotenuse 16 and side length 15. The width of the rectangle is equal to the length of the other leg of the triangle, which can be found using Pythagorean Theorem: $\sqrt{16^2 - 15^2} = \sqrt{31}$.

6. **Answer: $\frac{400\pi}{3}$ ft³**

The volume of the right circular conical homes is equal to $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height. The total volume of Zorbog and Fritz's home is thus $\frac{1}{3}\pi(10^2)(8) = \frac{800\pi}{3}$. Zorbog's part of the home is equal to $\frac{1}{2} \cdot \frac{800\pi}{3} = \frac{400\pi}{3}$.

7. **Answer: $4\sqrt{2}$ feet**

The area of the rectangular-shaped garden is $16 \cdot 9 = 144$ square feet. Since the new, square-shaped garden has twice the area, its area must be 288 square feet. Thus, $s^2 = 288$, where s is the side length in feet. Solving for s gives $s = 12\sqrt{2}$ feet. In yards, $s = \frac{12\sqrt{2}}{3} = 4\sqrt{2}$.

8. **Answer: 210**

The number of diagonals in a polygon is equal to $\frac{n(n-3)}{2}$, where n is the number of sides. Hence, a 23-gon has $\frac{23 \cdot 20}{2} = 230$ diagonals. A regular dodecahedron has 20 vertices. The positive difference between these two values is 210.

9. **Answer: $10\sqrt{10}$ miles**

On the xy -plane, if Planet Zorb is located at $(0, 0)$, then Planet Xerov is located at $(20, 0)$ and Planet Yolum is located at $(0, 60)$. Planet Alonzo is located at the midpoint of the line segment connecting $(20, 0)$ and $(0, 60)$, so Alonzo is located at $(10, 30)$. The distance between Planet Zorb, where Amy is located, and Planet Alonzo can be found using the distance formula: $\sqrt{(10 - 0)^2 + (30 - 0)^2} = \sqrt{100 + 900} = \sqrt{1000} = 10\sqrt{10}$ miles.

10. **Answer: 231 inches**

The ratio of Amy's height to the length of Amy's shadow must be equal to the ratio of the flagpole's height to the length of the flagpole's shadow. Let x be the height of the flagpole. Also, the length of the flagpole's shadow, in inches, is $22 \cdot 12 = 264$. Then, $\frac{63}{72} = \frac{x}{264}$. Solving for x gives $x = 231$ inches.