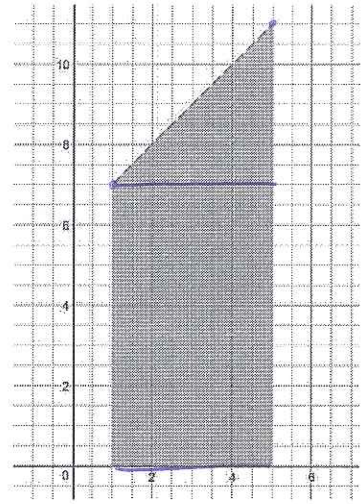
Review 5.1 – 5.2 Antiderivatives and Area Name: Answer key Pd: __

A **calculator may be used**, but **Show all work**.

1. Find the area of the region bounded by $y = x + 6$, the x – axis, $x = 1$ and $x = 5$. Write a definite integral to represent the total area, then find the area geometrically.

$$\int_1^5 (x+6)\,dx = \frac{7+11}{2}(4) = 36$$



2. Evaluate $\sum_{i=1}^{5}(i^2 + 2i)$ $= (1+2)+(4+4)+(9+6)+(16+8)+(25+10)$

$= 3+8+15+24+35$

$= 85$

3. Let $f(x) = x^3 + 2x$.

a. Find a midpoint Riemann Sum using 2 equal subintervals to approximate $\int_0^4 f(x)dx$ (Hint: Don't use a summation. Make a table and find the area of each rectangle.

x	f(x)
0	
1	3
2	
3	33
4	

$\Delta x = \frac{4-0}{2} = 2$

$2 \cdot 3 + 2 \cdot 33 = 72$

b. Does the Right Riemann Sum over approximate $\int_0^4 f(x)dx$? Why or why not?

over-estimate, since f is inc on [0,4]

$f'(x) = 3x^2 + 2 \geq 0$

$f'(x) > 0 \ \forall x.$

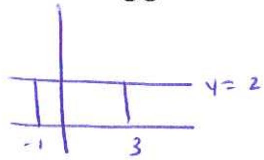
4. Evaluate $\int x^2 - 3x + \sqrt{x}\,dx$

$$\frac{x^3}{3} - \frac{3x^2}{2} + \frac{2}{3}x^{3/2} + C$$

5. Evaluate each of the following geometrically.

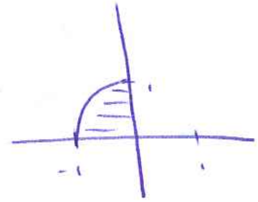
a. $\int_{-1}^{3} 2dx$

$2(3+1) = \boxed{8}$

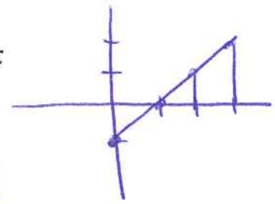


c. $\int_{-1}^{0} \sqrt{1-x^2}dx$

$\frac{1}{4}\pi(1)^2 = \boxed{\frac{\pi}{4}}$

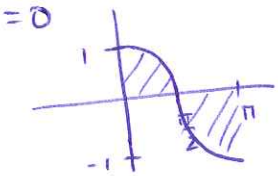


b. $\int_{0}^{3} (x-1)dx$



$-\frac{1}{2} + 2 = \frac{3}{2}$

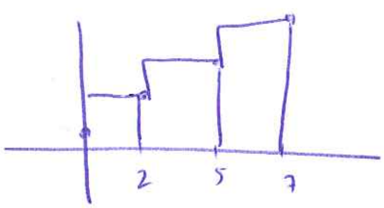
d. $\int_{0}^{\pi} \cos x\, dx = 0$



6. Let $y = f(x)$ be a twice differentiable strictly increasing function with selected values provided in the table below.

x	0	2	5	7
$f(x)$	1	4	6	10

Use a Right Riemann Sum to approximate $\int_{0}^{7} f(x)dx$. $\approx 4(2) + 6(3) + 10(2) = 46$



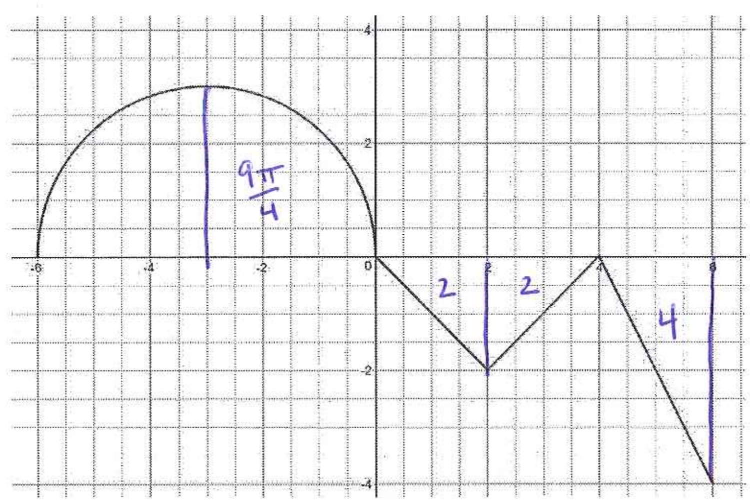
7. Use the graph of $y = f'(x)$ defined on the interval [-6,6], which consists of line segments and a semicircle as shown below, to answer the following questions. $f(0) = 2$.

a. $\int_{-3}^{2} f'(x)dx = \frac{9\pi}{4} - 2$

b. $\int_{1}^{-1} f'(x)dx = 0$

c. $\int_{6}^{0} f'(x)dx = -\int_{0}^{6} f'(x)dx = \boxed{+8}$

d. What is the absolute maximum value of f? Justify. 2 @ $x=0$



Since $f' \geq 0$ on $(-6,0)$ and $f' \leq 0$ on $(0,6)$ thus abs. max @ $x = 0$.