Rocket City Math League

Senior Division

2013-2014

Inter-School Test

Answers must be written inside the corresponding box on the answer sheet. All answers must be written in exact, reduced, simplified, and rationalized form. **No calculators, books, or other aides may be used.**

1. Marcella bought some new pink astronaut boots. She paid $153 after three discounts. The boots were marked down to 50% of the original price, and were found on the clearance rack which promised 20% off the marked price. She used a 15% off coupon at the cash register which deducted 15% of the discounted price. What was the original price, to the nearest cent? *(1 point)*

2. If $\tan\theta = \frac{-2}{7}$ and $0 \le \theta \le \pi$, then what is $\csc\theta$? *(1 point)*

3. A rectangular prism has dimensions 5, 8 and q. Find an expression that describes the surface area of the prism. *(1 point)*

4. Solve this equation for x: $\log_6 x = 2(\log_3 9)$ *(1 point)*

5. What is the volume of a sphere with a center at the origin and whose surface passes through the point (7, 4, 3)? *(1 point)*

6. Write the following in order from least to greatest: 2^{100}, 3^{60}, 10^{30}, or 31^{20}. *(2 points)*

7. Evaluate $\sum_{n=1}^{18} \sin^2(5n)$, where angles are measured in degrees. *(2 points)*

8. Find the volume of a parallelepiped with the following points: A (1, 3, 5), B (5, 2, 4), C (4,-3,4), and D (2, 3, 1), where $\overline{AB}$, $\overline{BC}$, and $\overline{BD}$ are edges of the parallelepiped. *(2 points)*

9. Let a and b be the legs of a right triangle with hypotenuse 4. Find $a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8$. *(2 points)*

10. A scientist is experimenting with space slime, which can change volume. When the volume is measured in quontonians, the scientist finds that the volume of the slime can be determined by $\frac{5x^2+3x+10}{(x-5)(x^2+4x+5)}$ where x represents the time in days since the slime was created. Decompose this expression using partial fractions. *(3 points)*

11 Solve for matrix A: $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} A + \begin{bmatrix} -2 & 3 \\ 2 & 4 \end{bmatrix} = 2\begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ *(3 points)*

12. Find M+AR+S for $y = 5\frac{(6x^2+18x-168)(x^2-8x-65)}{18(x+17)(x+7)(x-4)^2}$ if M = the sum of the x-coordinates of the x-intercepts, A = the total number of asymptotes, R = the y-coordinate of the y-intercept of any horizontal asymptotes, and S = the y-coordinate of the point on the graph for which x = 10. *(3 points)*

13. Find the sum of the slopes of the lines tangent to the curve $x^2 + y^2 - 4x + 4y = 92$ that pass through the point (7, 8). *(4 points)*

14. If the probability of event A occurring is 1/5, what is the probability that during 5 identical trials, event A occurred at least three times? *(4 points)*

15. Juan and Han are both going to Giacomo's Italy to eat lunch. They both arrive sometime between 11 a.m. and 2 p.m., and both eat for half an hour. What is the probability that at some point they are both eating at the same time, given that they begin eating as soon as they arrive? *(5 points)*