

14-3 Study Guide and Intervention *(continued)***Trigonometric Identities**

Simplify Expressions The simplified form of a trigonometric expression is written as a numerical value or in terms of a single trigonometric function, if possible. Any of the trigonometric identities on page 849 can be used to simplify expressions containing trigonometric functions.

Example 1 Simplify $(1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta$.

$$\begin{aligned} (1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta &= \sin^2 \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \cdot \cos^2 \theta \\ &= \sin \theta + \sin \theta \\ &= 2 \sin \theta \end{aligned}$$

Example 2 Simplify $\frac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \frac{\csc \theta}{1 + \sin \theta}$.

$$\begin{aligned} \frac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \frac{\csc \theta}{1 + \sin \theta} &= \frac{\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - \frac{\frac{1}{\sin \theta}}{1 + \sin \theta} \\ &= \frac{\frac{1}{\sin \theta}(1 + \sin \theta) - \frac{1}{\sin \theta}(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{\frac{1}{\sin \theta} + 1 - \frac{1}{\sin \theta} + 1}{1 - \sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} \end{aligned}$$

Exercises

Simplify each expression.

1. $\frac{\tan \theta \cdot \csc \theta}{\sec \theta}$

2. $\frac{\sin \theta \cdot \cot \theta}{\sec^2 \theta - \tan^2 \theta}$

3. $\frac{\sin^2 \theta - \cot \theta \cdot \tan \theta}{\cot \theta \cdot \sin \theta}$

4. $\frac{\cos \theta}{\sec \theta - \tan \theta}$

5. $\frac{\tan \theta \cdot \cos \theta}{\sin \theta} + \cot \theta \cdot \sin \theta \cdot \tan \theta \cdot \csc \theta$

6. $\frac{\csc^2 \theta - \cot^2 \theta}{\tan \theta \cdot \cos \theta}$

7. $3 \tan \theta \cdot \cot \theta + 4 \sin \theta \cdot \csc \theta + 2 \cos \theta \cdot \sec \theta$

8. $\frac{1 - \cos^2 \theta}{\tan \theta \cdot \sin \theta}$

14-3 Skills Practice**Trigonometric Identities**

Find the value of each expression.

1. $\sin \theta$, if $\cos \theta = -\frac{4}{5}$ and $90^\circ < \theta < 180^\circ$

2. $\cos \theta$, if $\tan \theta = 1$ and $180^\circ < \theta < 270^\circ$

3. $\sec \theta$, if $\tan \theta = 1$ and $0^\circ \leq \theta < 90^\circ$

4. $\cos \theta$, if $\tan \theta = \frac{1}{2}$ and $0^\circ \leq \theta < 90^\circ$

5. $\tan \theta$, if $\sin \theta = -\frac{\sqrt{2}}{2}$ and $180^\circ < \theta < 270^\circ$

6. $\cos \theta$, if $\sec \theta = 2$ and $270^\circ < \theta < 360^\circ$

7. $\cos \theta$, if $\csc \theta = -2$ and $180^\circ < \theta < 270^\circ$

8. $\tan \theta$, if $\cos \theta = -\frac{2\sqrt{5}}{5}$ and $180^\circ < \theta < 270^\circ$

9. $\cos \theta$, if $\cot \theta = -\frac{3}{2}$ and $90^\circ < \theta < 180^\circ$

10. $\csc \theta$, if $\cos \theta = \frac{8}{17}$ and $0^\circ < \theta < 90^\circ$

11. $\cot \theta$, if $\csc \theta = -2$ and $180^\circ < \theta < 270^\circ$

12. $\tan \theta$, if $\sin \theta = -\frac{5}{13}$ and $180^\circ < \theta < 270^\circ$

Simplify each expression.

13. $\sin \theta \sec \theta$

14. $\csc \theta \sin \theta$

15. $\cot \theta \sec \theta$

16. $\frac{\cos \theta}{\sec \theta}$

17. $\tan \theta + \cot \theta$

18. $\csc \theta \tan \theta - \tan \theta \sin \theta$

19. $\frac{1 - \sin^2 \theta}{\sin \theta + 1}$

20. $\csc \theta + \cot \theta$

21. $\frac{\sin^2 \theta + \cos^2 \theta}{1 - \cos^2 \theta}$

22. $1 + \frac{\tan^2 \theta}{1 + \sec \theta}$