

129. (a)  $f(x) = \sec x$

$$f(\pi/6) = \frac{2}{\sqrt{3}}$$

$$f'(x) = \sec x \tan x$$

$$f'(\pi/6) = \frac{2}{3}$$

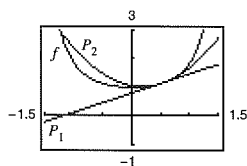
$$\begin{aligned} f''(x) &= \sec x(\sec^2 x) + \tan x(\sec x \tan x) \\ &= \sec^3 x + \sec x \tan^2 x \end{aligned}$$

$$f''(\pi/6) = \frac{10\sqrt{3}}{9}$$

$$P_1(x) = \frac{2}{3}(x - \pi/6) + \frac{2}{\sqrt{3}}$$

$$\begin{aligned} P_2(x) &= \frac{1}{2} \cdot \left(\frac{10}{3\sqrt{3}}\right) \left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3} \left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}} \\ &= \left(\frac{5}{3\sqrt{3}}\right) \left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3} \left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}} \end{aligned}$$

(b)


 (c)  $P_2$  is a better approximation than  $P_1$ .

 (d) The accuracy worsens as you move away from  $x = \pi/6$ .

130. False. If  $y = (1 - x)^{1/2}$ , then  $y' = \frac{1}{2}(1 - x)^{-1/2}(-1)$ .

131. False. If  $f(x) = \sin^2 2x$ , then  $f'(x) = 2(\sin 2x)(2 \cos 2x)$ .

132. True

133.  $f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$

$$f'(x) = a_1 \cos x + 2a_2 \cos 2x + \cdots + na_n \cos nx$$

$$f'(0) = a_1 + 2a_2 + \cdots + na_n$$

$$|a_1 + 2a_2 + \cdots + na_n| = |f'(0)| = \lim_{x \rightarrow 0} \left| \frac{f(x) - f(0)}{x - 0} \right| = \lim_{x \rightarrow 0} \left| \frac{f(x)}{\sin x} \right| \cdot \left| \frac{\sin x}{x} \right| = \lim_{x \rightarrow 0} \left| \frac{f(x)}{\sin x} \right| \leq 1$$

134. 
$$\frac{d}{dx} \left[ \frac{P_n(x)}{(x^k - 1)^{n+1}} \right] = \frac{(x^k - 1)^{n+1} P_n'(x) - P_n(x)(n+1)(x^k - 1)^n kx^{k-1}}{(x^k - 1)^{2n+2}} = \frac{(x^k - 1)P_n'(x) - (n+1)kx^{k-1}P_n(x)}{(x^k - 1)^{n+2}}$$

$$P_n(x) = (x^k - 1)^{n+1} \frac{d^n}{dx^n} \left[ \frac{1}{x^k - 1} \right] \Rightarrow$$

$$P_{n+1}(x) = (x^k - 1)^{n+2} \frac{d}{dx} \left[ \frac{d^n}{dx^n} \left[ \frac{1}{x^k - 1} \right] \right] = (x^k - 1)P_n'(x) - (n+1)kx^{k-1}P_n(x)$$

$$P_{n+1}(1) = -(n+1)kP_n(1)$$

$$\text{For } n = 1, \frac{d}{dx} \left[ \frac{1}{x^k - 1} \right] = \frac{-kx^{k-1}}{(x^k - 1)^2} = \frac{P_1(x)}{(x^k - 1)^2} \Rightarrow P_1(1) = -k. \text{ Also, } P_0(1) = 1.$$

 You now use mathematical induction to verify that  $P_n(1) = (-k)^n n!$  for  $n \geq 0$ . Assume true for  $n$ . Then

$$P_{n+1}(1) = -(n+1)k P_n(1) = -(n+1)k(-k)^n n! = (-k)^{n+1}(n+1)!$$

## Section 2.5 Implicit Differentiation

1.  $x^2 + y^2 = 9$

$2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$

2.  $x^2 - y^2 = 25$

$2x - 2yy' = 0$

$$y' = \frac{x}{y}$$

3.  $x^{1/2} + y^{1/2} = 16$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$

$$y' = -\frac{x^{-1/2}}{y^{-1/2}} \\ = -\sqrt{\frac{y}{x}}$$

4.  $x^3 + y^3 = 64$

$3x^2 + 3y^2y' = 0$

$$y' = -\frac{x^2}{y^2}$$

5.  $x^3 - xy + y^2 = 7$

$3x^2 - xy' - y + 2yy' = 0$

$(2y - x)y' = y - 3x^2$

$$y' = \frac{y - 3x^2}{2y - x}$$

6.  $x^2y + y^2x = -2$

$x^2y' + 2xy + y^2 + 2yxy' = 0$

$(x^2 + 2xy)y' = -(y^2 + 2xy)$

$$y' = \frac{-y(y + 2x)}{x(x + 2y)}$$

7.  $x^3y^3 - y - x = 0$

$3x^3y^2y' + 3x^2y^3 - y' - 1 = 0$

$(3x^3y^2 - 1)y' = 1 - 3x^2y^3$

$$y' = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

8.  $\sqrt{xy} = x^2y + 1$

$\frac{1}{2}(xy)^{-1/2}(xy' + y) = 2xy + x^2y'$

$\frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} = 2xy + x^2y'$

$$\left(\frac{x}{2\sqrt{xy}} - x^2\right)y' = 2xy - \frac{y}{2\sqrt{xy}}$$

$$y' = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2}$$

$$y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$$

9.  $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$

$(4xy - 3x^2)y' = 6xy - 3x^2 - 2y^2$

$$y' = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$$

10.  $4 \cos x \sin y = 1$

$4[-\sin x \sin y + \cos x \cos y y'] = 0$

$\cos x \cos y y' = \sin x \sin y$

$$y' = \frac{\sin x \sin y}{\cos x \cos y} \\ = \tan x \tan y$$

11.  $\sin x + 2 \cos 2y = 1$

$\cos x - 4(\sin 2y)y' = 0$

$$y' = \frac{\cos x}{4 \sin 2y}$$

12.  $(\sin \pi x + \cos \pi y)^2 = 2$

$2(\sin \pi x + \cos \pi y)[\pi \cos \pi x - \pi(\sin \pi y)y'] = 0$

$\pi \cos \pi x - \pi(\sin \pi y)y' = 0$

$$y' = \frac{\cos \pi x}{\sin \pi y}$$

13.  $\sin x = x(1 + \tan y)$

$\cos x = x(\sec^2 y)y' + (1 + \tan y)(1)$

$$y' = \frac{\cos x - \tan y - 1}{x \sec^2 y}$$

14.  $\cot y = x - y$

$$(-\csc^2 y)y' = 1 - y'$$

$$y' = \frac{1}{1 - \csc^2 y} = \frac{1}{-\cot^2 y} = -\tan^2 y$$

16.  $x = \sec \frac{1}{y}$

$$1 = -\frac{y'}{y^2} \sec \frac{1}{y} \tan \frac{1}{y}$$

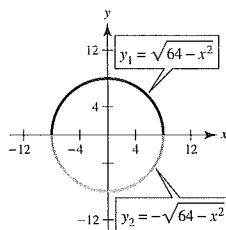
$$y' = \frac{-y^2}{\sec(1/y) \tan(1/y)} = -y^2 \cos\left(\frac{1}{y}\right) \cot\left(\frac{1}{y}\right)$$

17. (a)  $x^2 + y^2 = 64$

$$y^2 = 64 - x^2$$

$$y = \pm\sqrt{64 - x^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm \frac{1}{2}(64 - x^2)^{-1/2}(-2x) = \frac{\mp x}{\sqrt{64 - x^2}} = \frac{-x}{\pm\sqrt{64 - x^2}} = -\frac{x}{y}$$

 (d) Implicitly:  $2x + 2yy' = 0$ 

$$y' = -\frac{x}{y}$$

18. (a)  $(x^2 - 4x + 4) + (y^2 + 6y + 9) = -9 + 4 + 9$

$$(x - 2)^2 + (y + 3)^2 = 4, \quad \text{Circle}$$

$$(y + 3)^2 = 4 - (x - 2)^2$$

$$y = -3 \pm \sqrt{4 - (x - 2)^2}$$

(c) Explicitly:

$$\begin{aligned} \frac{dy}{dx} &= \pm \frac{1}{2} [4 - (x - 2)^2]^{-1/2} (-2)(x - 2) = \frac{\mp(x - 2)}{\sqrt{4 - (x - 2)^2}} = \frac{-(x - 2)}{\pm\sqrt{4 - (x - 2)^2}} \\ &= \frac{-(x - 2)}{-3 \pm \sqrt{4 - (x - 2)^2} + 3} = \frac{-(x - 2)}{y + 3} \end{aligned}$$

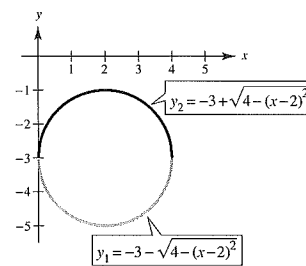
(d) Implicitly:

$$2x + 2yy' - 4 + 6y' = 0$$

$$(2y + 6)y' = -2(x - 2)$$

$$y' = \frac{-(x - 2)}{y + 3}$$

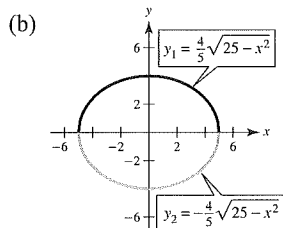
(b)



19. (a)  $16y^2 = 400 - 25x^2$

$$y^2 = \frac{1}{25}(400 - 16x^2) = \frac{16}{25}(25 - x^2)$$

$$y = \pm \frac{4}{5}\sqrt{25 - x^2}$$



(c) Explicitly:

$$\begin{aligned} \frac{dy}{dx} &= \pm \frac{4}{10}(25 - x^2)^{-1/2}(-2x) \\ &= \mp \frac{4x}{5\sqrt{25 - x^2}} = \frac{-4x}{5(5/4)y} = -\frac{16x}{25y} \end{aligned}$$

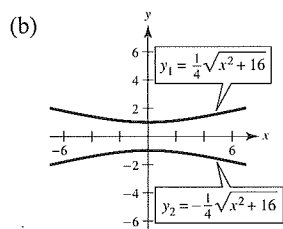
(d) Implicitly:  $32x + 50yy' = 0$ 

$$y' = -\frac{16x}{25y}$$

20. (a)  $16y^2 = x^2 + 16$

$$y^2 = \frac{x^2}{16} + 1 = \frac{x^2 + 16}{16}$$

$$y = \pm \frac{\sqrt{x^2 + 16}}{4}$$



(c) Explicitly:

$$\begin{aligned} \frac{dy}{dx} &= \frac{\pm \frac{1}{2}(x^2 + 16)^{-1/2}(2x)}{4} \\ &= \frac{\pm x}{4\sqrt{x^2 + 16}} = \frac{\pm x}{4(\pm 4y)} = \frac{x}{16y} \end{aligned}$$

(d) Implicitly:  $16y^2 - x^2 = 16$ 

$$32yy' - 2x = 0$$

$$32yy' = 2x$$

$$y' = \frac{2x}{32y} = \frac{x}{16y}$$

21.  $xy = 6$

$$xy' + y(1) = 0$$

$$xy' = -y$$

$$y' = -\frac{y}{x}$$

At  $(-6, -1)$ :  $y' = -\frac{1}{6}$

22.  $x^2 - y^3 = 0$

$$2x - 3y^2y' = 0$$

$$y' = \frac{2x}{3y^2}$$

At  $(1, 1)$ :  $y' = \frac{2}{3}$

23.  $y^2 = \frac{x^2 - 49}{x^2 + 49}$

$$2yy' = \frac{(x^2 + 49)(2x) - (x^2 - 49)(2x)}{(x^2 + 49)^2}$$

$$2yy' = \frac{196x}{(x^2 + 49)^2}$$

$$y' = \frac{98x}{y(x^2 + 49)^2}$$

At  $(7, 0)$ :  $y'$  is undefined.

24.  $(x + y)^3 = x^3 + y^3$

$$x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3$$

$$3x^2y + 3xy^2 = 0$$

$$x^2y + xy^2 = 0$$

$$x^2y' + 2xy + 2xyy' + y^2 = 0$$

$$(x^2 + 2xy)y' = -(y^2 + 2xy)$$

$$y' = -\frac{y(y + 2x)}{x(x + 2y)}$$

At  $(-1, 1)$ :  $y' = -1$ 

25.  $x^{2/3} + y^{2/3} = 5$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\sqrt[3]{\frac{y}{x}}$$

At  $(8, 1)$ :  $y' = -\frac{1}{2}$

$$26. \quad x^3 + y^3 = 6xy - 1$$

$$3x^2 + 3y^2y' = 6xy' + 6y$$

$$(3y^2 - 6x)y' = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\text{At } (2, 3), y' = \frac{18 - 12}{27 - 12} = \frac{6}{15} = \frac{2}{5}$$

$$27. \quad \tan(x + y) = x$$

$$(1 + y') \sec^2(x + y) = 1$$

$$y' = \frac{1 - \sec^2(x + y)}{\sec^2(x + y)}$$

$$= \frac{-\tan^2(x + y)}{\tan^2(x + y) + 1}$$

$$= -\sin^2(x + y)$$

$$= -\frac{x^2}{x^2 + 1}$$

$$\text{At } (0, 0): y' = 0$$

$$28. \quad x \cos y = 1$$

$$x[-y' \sin y] + \cos y = 0$$

$$y' = \frac{\cos y}{x \sin y}$$

$$= \frac{1}{x} \cot y$$

$$= \frac{\cot y}{x}$$

$$\text{At } \left(2, \frac{\pi}{3}\right): y' = \frac{1}{2\sqrt{3}}$$

$$29. \quad (x^2 + 4)y = 8$$

$$(x^2 + 4)y' + y(2x) = 0$$

$$y' = \frac{-2xy}{x^2 + 4}$$

$$= \frac{-2x[8/(x^2 + 4)]}{x^2 + 4}$$

$$= \frac{-16x}{(x^2 + 4)^2}$$

$$\text{At } (2, 1): y' = \frac{-32}{64} = -\frac{1}{2}$$

$$\left(\text{Or, you could just solve for } y: y = \frac{8}{x^2 + 4}\right)$$

$$30. \quad (4 - x)y^2 = x^3$$

$$(4 - x)(2yy') + y^2(-1) = 3x^2$$

$$y' = \frac{3x^2 + y^2}{2y(4 - x)}$$

$$\text{At } (2, 2): y' = 2$$

$$31. \quad (x^2 + y^2)^2 = 4x^2y$$

$$2(x^2 + y^2)(2x + 2yy') = 4x^2y' + y(8x)$$

$$4x^3 + 4x^2yy' + 4xy^2 + 4y^3y' = 4x^2y' + 8xy$$

$$4x^2yy' + 4y^3y' - 4x^2y' = 8xy - 4x^3 - 4xy^2$$

$$4y'(x^2y + y^3 - x^2) = 4(2xy - x^3 - xy^2)$$

$$y' = \frac{2xy - x^3 - xy^2}{x^2y + y^3 - x^2}$$

$$\text{At } (1, 1): y' = 0$$

$$32. \quad x^3 + y^3 - 6xy = 0$$

$$3x^2 + 3y^2y' - 6xy' - 6y = 0$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

$$\text{At } \left(\frac{4}{3}, \frac{8}{3}\right): y' = \frac{(16/3) - (16/9)}{(64/9) - (8/3)} = \frac{32}{40} = \frac{4}{5}$$

$$33. \quad (y - 3)^2 = 4(x - 5), \quad (6, 1)$$

$$2(y - 3)y' = 4$$

$$y' = \frac{2}{y - 3}$$

$$\text{At } (6, 1), y' = \frac{2}{1 - 3} = -1$$

$$\text{Tangent line: } y - 1 = -1(x - 6)$$

$$y = -x + 7$$

$$34. \quad (x + 2)^2 + (y - 3)^2 = 37, \quad (4, 4)$$

$$2(x + 2) + 2(y - 3)y' = 0$$

$$(y - 3)y' = -(x + 2)$$

$$y' = -\frac{(x + 2)}{y - 3}$$

$$\text{At } (4, 4), y' = -\frac{6}{1} = -6$$

$$\text{Tangent line: } y - 4 = -6(x - 4)$$

$$y = -6x + 28$$

35.  $xy = 1, (1, 1)$

$xy' + y = 0$

$y' = \frac{-y}{x}$

At  $(1, 1)$ :  $y' = -1$

Tangent line:  $y - 1 = -1(x - 1)$

$y = -x + 2$

36.  $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0, (\sqrt{3}, 1)$

$14x - 6\sqrt{3}xy' - 6\sqrt{3}y + 26yy' = 0$

$y' = \frac{6\sqrt{3}y - 14x}{26y - 6\sqrt{3}x}$

At  $(\sqrt{3}, 1)$ :  $y' = \frac{6\sqrt{3} - 14\sqrt{3}}{26 - 6\sqrt{3}\sqrt{3}} = \frac{-8\sqrt{3}}{8} = -\sqrt{3}$

Tangent line:  $y - 1 = -\sqrt{3}(x - \sqrt{3})$

$y = -\sqrt{3}x + 4$

37.  $x^2y^2 - 9x^2 - 4y^2 = 0, (-4, 2\sqrt{3})$

$x^22yy' + 2xy^2 - 18x - 8yy' = 0$

$y' = \frac{18x - 2xy^2}{2x^2y - 8y}$

At  $(-4, 2\sqrt{3})$ :  $y' = \frac{18(-4) - 2(-4)(12)}{2(16)(2\sqrt{3}) - 16\sqrt{3}}$   
 $= \frac{24}{48\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

Tangent line:  $y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x + 4)$

$y = \frac{\sqrt{3}}{6}x + \frac{8}{3}\sqrt{3}$

38.  $x^{2/3} + y^{2/3} = 5, (8, 1)$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$

$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$

At  $(8, 1)$ :  $y' = -\frac{1}{2}$

Tangent line:  $y - 1 = -\frac{1}{2}(x - 8)$

$y = -\frac{1}{2}x + 5$

39.  $3(x^2 + y^2)^2 = 100(x^2 - y^2), (4, 2)$

$6(x^2 + y^2)(2x + 2yy') = 100(2x - 2yy')$

At  $(4, 2)$ :

$6(16 + 4)(8 + 4y') = 100(8 - 4y')$

$960 + 480y' = 800 - 400y'$

$880y' = -160$

$y' = -\frac{2}{11}$

Tangent line:  $y - 2 = -\frac{2}{11}(x - 4)$

$11y + 2x - 30 = 0$

$y = -\frac{2}{11}x + \frac{30}{11}$

40.  $y^2(x^2 + y^2) = 2x^2, (1, 1)$

$y^2x^2 + y^4 = 2x^2$

$2yy'x^2 + 2xy^2 + 4y^3y' = 4x$

At  $(1, 1)$ :

$2y' + 2 + 4y' = 4$

$6y' = 2$

$y' = \frac{1}{3}$

Tangent line:  $y - 1 = \frac{1}{3}(x - 1)$

$y = \frac{1}{3}x + \frac{2}{3}$

41. (a)  $\frac{x^2}{2} + \frac{y^2}{8} = 1, (1, 2)$

$x + \frac{yy'}{4} = 0$

$y' = -\frac{4x}{y}$

At  $(1, 2)$ :  $y' = -2$

Tangent line:  $y - 2 = -2(x - 1)$

$y = -2x + 4$

(b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = \frac{-b^2x}{a^2y}$

$y - y_0 = \frac{-b^2x_0}{a^2y_0}(x - x_0)$ , Tangent line at  $(x_0, y_0)$

$\frac{y_0y}{b^2} - \frac{y_0^2}{b^2} = \frac{-x_0x}{a^2} + \frac{x_0^2}{a^2}$

Because  $\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$ , you have  $\frac{y_0y}{b^2} + \frac{x_0x}{a^2} = 1$ .

**Note:** From part (a),

$\frac{1(x)}{2} + \frac{2(y)}{8} = 1 \Rightarrow \frac{1}{4}y = -\frac{1}{2}x + 1 \Rightarrow y = -2x + 4$ ,

Tangent line.

42. (a)  $\frac{x^2}{6} - \frac{y^2}{8} = 1, (3, -2)$

$$\frac{x}{3} - \frac{y}{4}y' = 0$$

$$\frac{y}{4}y' = \frac{x}{3}$$

$$y' = \frac{4x}{3y}$$

At  $(3, -2)$ :  $y' = \frac{4(3)}{3(-2)} = -2$

Tangent line:  $y + 2 = -2(x - 3)$

$$y = -2x + 4$$

(b)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \Rightarrow y' = \frac{xb^2}{ya^2}$

$y - y_0 = \frac{x_0 b^2}{y_0 a^2}(x - x_0)$ , Tangent line at  $(x_0, y_0)$

$$\frac{yy_0}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0 x}{a^2} - \frac{x_0^2}{a^2}$$

Because  $\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$ , you have  $\frac{x_0 x}{a^2} - \frac{yy_0}{b^2} = 1$ .

**Note:** From part (a),

$$\frac{3x}{6} - \frac{(-2)y}{8} = 1 \Rightarrow \frac{1}{2}x + \frac{y}{4} = 1 \Rightarrow y = -2x + 4,$$

Tangent line.

45.  $x^2 + y^2 = 4$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y}$$

$$y'' = \frac{y(-1) + xy'}{y^2} = \frac{-y + x(-x/y)}{y^2} = \frac{-y^2 - x^2}{y^3} = -\frac{4}{y^3}$$

46.

$$x^2 y^2 - 2x = 3$$

$$2x^2 y y' + 2x y^2 - 2 = 0$$

$$x^2 y y' + x y^2 - 1 = 0$$

$$y' = \frac{1 - x y^2}{x^2 y}$$

$$2x y y' + x^2 (y')^2 + x^2 y y'' + 2x y y' + y^2 = 0$$

$$4x y y' + x^2 (y')^2 + x^2 y y'' + y^2 = 0$$

$$\frac{4 - 4x y^2}{x} + \frac{(1 - x y^2)^2}{x^2 y^2} + x^2 y y'' + y^2 = 0$$

$$4x y^2 - 4x^2 y^4 + 1 - 2x y^2 + x^2 y^4 + x^4 y^3 y'' + x^2 y^4 = 0$$

$$x^4 y^3 y'' = 2x^2 y^4 - 2x y^2 - 1$$

$$y'' = \frac{2x^2 y^4 - 2x y^2 - 1}{x^4 y^3}$$

43.  $\tan y = x$

$$y' \sec^2 y = 1$$

$$y' = \frac{1}{\sec^2 y} = \cos^2 y, -\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$\sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$y' = \frac{1}{1 + x^2}$$

44.  $\cos y = x$

$$-\sin y \cdot y' = 1$$

$$y' = \frac{-1}{\sin y}, 0 < y < \pi$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin^2 y = 1 - \cos^2 y$$

$$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

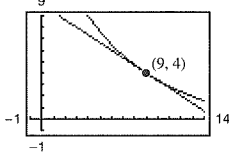
$$y' = \frac{-1}{\sqrt{1 - x^2}}, -1 < x < 1$$

$$\begin{aligned}
 47. \quad x^2 - y^2 &= 36 \\
 2x - 2yy' &= 0 \\
 y' &= \frac{x}{y} \\
 x - yy' &= 0 \\
 1 - yy'' - (y')^2 &= 0 \\
 1 - yy'' - \left(\frac{x}{y}\right)^2 &= 0 \\
 y^2 - y^3y'' &= x^2 \\
 y'' &= \frac{y^2 - x^2}{y^3} = -\frac{36}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad 1 - xy &= x - y \\
 y - xy &= x - 1 \\
 y &= \frac{x-1}{1-x} = -1 \\
 y' &= 0 \\
 y'' &= 0
 \end{aligned}$$

$$\begin{aligned}
 49. \quad y^2 &= x^3 \\
 2yy' &= 3x^2 \\
 y' &= \frac{3x^2}{2y} = \frac{3x^2}{2y} \cdot \frac{xy}{xy} = \frac{3y}{2x} \cdot \frac{x^3}{y^2} = \frac{3y}{2x} \\
 y'' &= \frac{2x(3y') - 3y(2)}{4x^2} \\
 &= \frac{2x[3 \cdot (3y/2x)] - 6y}{4x^2} = \frac{3y}{4x^2} = \frac{3x}{4y}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad y^2 &= 10x \\
 2yy' &= 10 \\
 y' &= \frac{5}{y} \\
 y'' &= -5y^{-2}y' = \left[\frac{-5}{y^2}\right] \cdot \frac{5}{y} = -\frac{25}{y^3}
 \end{aligned}$$

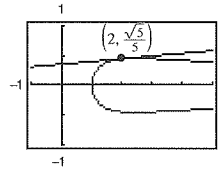
$$\begin{aligned}
 51. \quad \sqrt{x} + \sqrt{y} &= 5 \\
 \frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' &= 0 \\
 y' &= \frac{-\sqrt{y}}{\sqrt{x}}
 \end{aligned}$$


At (9, 4):  $y' = -\frac{2}{3}$

Tangent line:  $y - 4 = -\frac{2}{3}(x - 9)$

$$2x + 3y - 30 = 0$$

$$\begin{aligned}
 52. \quad y^2 &= \frac{x-1}{x^2+1} \\
 2yy' &= \frac{(x^2+1)(1) - (x-1)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2+2x}{(x^2+1)^2} \\
 y' &= \frac{1+2x-x^2}{2y(x^2+1)^2}
 \end{aligned}$$



At  $\left(2, \frac{\sqrt{5}}{5}\right)$ :  $y' = \frac{1+4-4}{\left[\frac{(2\sqrt{5})}{5}\right](4+1)^2} = \frac{1}{10\sqrt{5}}$

Tangent line:  $y - \frac{\sqrt{5}}{5} = \frac{1}{10\sqrt{5}}(x - 2)$

$$10\sqrt{5}y - 10 = x - 2$$

$$x - 10\sqrt{5}y + 8 = 0$$

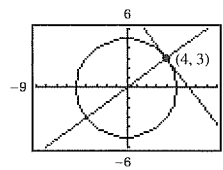
$$\begin{aligned}
 53. \quad x^2 + y^2 &= 25 \\
 2x + 2yy' &= 0 \\
 y' &= \frac{-x}{y}
 \end{aligned}$$

At (4, 3):

Tangent line:

$$y - 3 = \frac{-4}{3}(x - 4) \Rightarrow 4x + 3y - 25 = 0$$

Normal line:  $y - 3 = \frac{3}{4}(x - 4) \Rightarrow 3x - 4y = 0$

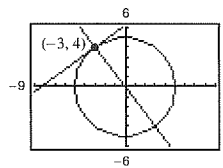


At (-3, 4):

Tangent line:

$$y - 4 = \frac{3}{4}(x + 3) \Rightarrow 3x - 4y + 25 = 0$$

Normal line:  $y - 4 = \frac{-4}{3}(x + 3) \Rightarrow 4x + 3y = 0$



54.  $x^2 + y^2 = 36$

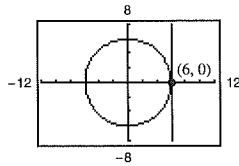
$2x + 2yy' = 0$

$$y' = -\frac{x}{y}$$

At  $(6, 0)$ ; slope is undefined.

Tangent line:  $x = 6$

Normal line:  $y = 0$



At  $(5, \sqrt{11})$ , slope is  $-\frac{5}{\sqrt{11}}$

Tangent line:  $y - \sqrt{11} = \frac{-5}{\sqrt{11}}(x - 5)$

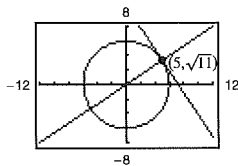
$$\sqrt{11}y - 11 = -5x + 25$$

$$5x + \sqrt{11}y - 36 = 0$$

Normal line:  $y - \sqrt{11} = \frac{\sqrt{11}}{5}(x - 5)$

$$5y - 5\sqrt{11} = \sqrt{11}x - 5\sqrt{11}$$

$$5y - \sqrt{11}x = 0$$



55.  $x^2 + y^2 = r^2$

$2x + 2yy' = 0$

$$y' = \frac{-x}{y} = \text{slope of tangent line}$$

$$\frac{y}{x} = \text{slope of normal line}$$

Let  $(x_0, y_0)$  be a point on the circle. If  $x_0 = 0$ , then the tangent line is horizontal, the normal line is vertical and, hence, passes through the origin. If  $x_0 \neq 0$ , then the equation of the normal line is

$$y - y_0 = \frac{y_0}{x_0}(x - x_0)$$

$$y = \frac{y_0}{x_0}x$$

which passes through the origin.

56.  $y^2 = 4x$

$2yy' = 4$

$$y' = \frac{2}{y} = 1 \text{ at } (1, 2)$$

Equation of normal line at  $(1, 2)$  is

$y - 2 = -1(x - 1)$ ,  $y = 3 - x$ . The centers of the circles must be on the normal line and at a distance of 4 units from  $(1, 2)$ . Therefore,

$$(x - 1)^2 + [(3 - x) - 2]^2 = 16$$

$$2(x - 1)^2 = 16$$

$$x = 1 \pm 2\sqrt{2}$$

Centers of the circles:  $(1 + 2\sqrt{2}, 2 - 2\sqrt{2})$  and

$$(1 - 2\sqrt{2}, 2 + 2\sqrt{2})$$

Equations:  $(x - 1 - 2\sqrt{2})^2 + (y - 2 + 2\sqrt{2})^2 = 16$

$$(x - 1 + 2\sqrt{2})^2 + (y - 2 - 2\sqrt{2})^2 = 16$$

57.  $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32yy' + 200 - 160y' = 0$

$$y' = \frac{200 + 50x}{160 - 32y}$$

Horizontal tangents occur when  $x = -4$ :

$$25(16) + 16y^2 + 200(-4) - 160y + 400 = 0$$

$$y(y - 10) = 0 \Rightarrow y = 0, 10$$

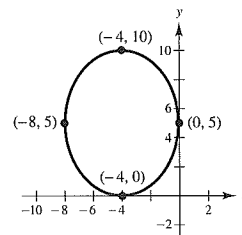
Horizontal tangents:  $(-4, 0)$ ,  $(-4, 10)$

Vertical tangents occur when  $y = 5$ :

$$25x^2 + 400 + 200x - 800 + 400 = 0$$

$$25x(x + 8) = 0 \Rightarrow x = 0, -8$$

Vertical tangents:  $(0, 5)$ ,  $(-8, 5)$



58.  $4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2yy' - 8 + 4y' = 0$

$$y' = \frac{8 - 8x}{2y + 4} = \frac{4 - 4x}{y + 2}$$

Horizontal tangents occur when  $x = 1$ :

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = y(y + 4) = 0 \Rightarrow y = 0, -4$

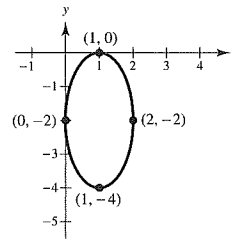
Horizontal tangents:  $(1, 0), (1, -4)$

Vertical tangents occur when  $y = -2$ :

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$4x^2 - 8x = 4x(x - 2) = 0 \Rightarrow x = 0, 2$

Vertical tangents:  $(0, -2), (2, -2)$



59. Find the points of intersection by letting  $y^2 = 4x$  in the equation  $2x^2 + y^2 = 6$ .

$2x^2 + 4x = 6$  and  $(x + 3)(x - 1) = 0$

The curves intersect at  $(1, \pm 2)$ .

Ellipse:

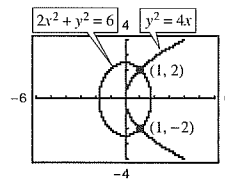
$4x + 2yy' = 0$

$y' = -\frac{2x}{y}$

Parabola:

$2yy' = 4$

$y' = \frac{2}{y}$



At  $(1, 2)$ , the slopes are:

$y' = -1$

$y' = 1$

At  $(1, -2)$ , the slopes are:

$y' = 1$

$y' = -1$

Tangents are perpendicular.

60. Find the points of intersection by letting  $y^2 = x^3$  in the equation  $2x^2 + 3y^2 = 5$ .

$2x^2 + 3x^3 = 5$  and  $3x^3 + 2x^2 - 5 = 0$

Intersect when  $x = 1$ .

Points of intersection:  $(1, \pm 1)$

$y^2 = x^3$ :

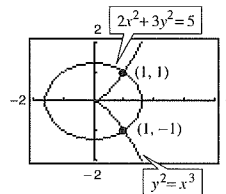
$2yy' = 3x^2$

$y' = \frac{3x^2}{2y}$

$2x^2 + 3y^2 = 5$ :

$4x + 6yy' = 0$

$y' = -\frac{2x}{3y}$



At  $(1, 1)$ , the slopes are:

$y' = \frac{3}{2}$

$y' = -\frac{2}{3}$

At  $(1, -1)$ , the slopes are:

$y' = -\frac{3}{2}$

$y' = \frac{2}{3}$

Tangents are perpendicular.

61.  $y = -x$  and  $x = \sin y$

 Point of intersection:  $(0, 0)$ 

$$y = -x:$$

$$y' = -1$$

$$x = \sin y:$$

$$1 = y' \cos y$$

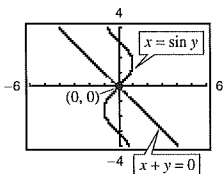
$$y' = \sec y$$

 At  $(0, 0)$ , the slopes are:

$$y' = -1$$

$$y' = 1$$

Tangents are perpendicular.



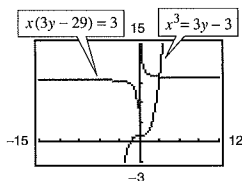
62. Rewriting each equation and differentiating:

$$x^3 = 3(y - 1) \quad x(3y - 29) = 3$$

$$y = \frac{x^3}{3} + 1 \quad y = \frac{1}{3} \left( \frac{3}{x} + 29 \right)$$

$$y' = x^2$$

$$y' = -\frac{1}{x^2}$$

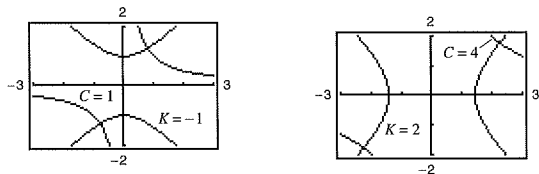

 For each value of  $x$ , the derivatives are negative reciprocals of each other. So, the tangent lines are orthogonal at both points of intersection.

63.  $xy = C \quad x^2 - y^2 = K$

$$xy' + y = 0 \quad 2x - 2yy' = 0$$

$$y' = -\frac{y}{x}$$

$$y' = \frac{x}{y}$$

 At any point of intersection  $(x, y)$  the product of the slopes is  $(-y/x)(x/y) = -1$ . The curves are orthogonal.


68.  $4 \sin x \cos y = 1$

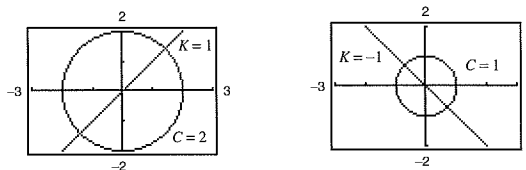
(a)  $4 \sin x (-\sin y) y' + 4 \cos x \cos y = 0$

$$y' = \frac{\cos x \cos y}{\sin x \sin y}$$

64.  $x^2 + y^2 = C^2 \quad y = Kx$

$$2x + 2yy' = 0 \quad y' = K$$

$$y' = -\frac{x}{y}$$

 At the point of intersection  $(x, y)$ , the product of the slopes is  $(-x/y)(K) = (-x/Kx)(K) = -1$ . The curves are orthogonal.


65.  $2y^2 - 3x^4 = 0$

(a)  $4yy' - 12x^3 = 0$

$$4yy' = 12x^3$$

$$y' = \frac{12x^3}{4y} = \frac{3x^3}{y}$$

(b)  $4y \frac{dy}{dt} - 12x^3 \frac{dx}{dt} = 0$

$$y \frac{dy}{dt} = 3x^3 \frac{dx}{dt}$$

66.  $x^2 - 3xy^2 + y^3 = 10$

(a)  $2x - 3y^2 - 6xyy' + 3y^2y' = 0$

$$(-6xy + 3y^2)y' = 3y^2 - 2x$$

$$y' = \frac{3y^2 - 2x}{3y^2 - 6xy}$$

(b)  $2x \frac{dx}{dt} - 3y^2 \frac{dx}{dt} - 6xy \frac{dy}{dt} + 3y^2 \frac{dy}{dt} = 0$

$$(2x - 3y^2) \frac{dx}{dt} = (6xy - 3y^2) \frac{dy}{dt}$$

67.  $\cos \pi y - 3 \sin \pi x = 1$

(a)  $-\pi \sin \pi y (y') - 3\pi \cos \pi x = 0$

$$y' = \frac{-3 \cos \pi x}{\sin \pi y}$$

(b)  $-\pi \sin \pi y \left( \frac{dy}{dt} \right) - 3\pi \cos \pi x \left( \frac{dx}{dt} \right) = 0$

$$-\sin \pi y \left( \frac{dy}{dt} \right) = 3 \cos \pi x \left( \frac{dx}{dt} \right)$$

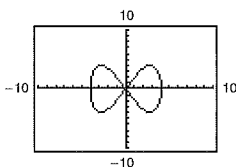
(a)  $4 \sin x (-\sin y) \left( \frac{dy}{dt} \right) + 4 \cos x \left( \frac{dx}{dt} \right) \cos y = 0$

$$\cos x \cos y \left( \frac{dx}{dt} \right) = \sin x \sin y \left( \frac{dy}{dt} \right)$$

69. Answers will vary. *Sample answer:* In the explicit form of a function, the variable is explicitly written as a function of  $x$ . In an implicit equation, the function is only implied by an equation. An example of an implicit function is  $x^2 + xy = 5$ . In explicit form it would be  $y = (5 - x^2)/x$ .

70. Answers will vary. *Sample answer:* Given an implicit equation, first differentiate both sides with respect to  $x$ . Collect all terms involving  $y'$  on the left, and all other terms to the right. Factor out  $y'$  on the left side. Finally, divide both sides by the left-hand factor that does not contain  $y'$ .

73. (a)  $x^4 = 4(4x^2 - y^2)$   
 $4y^2 = 16x^2 - x^4$   
 $y^2 = 4x^2 - \frac{1}{4}x^4$   
 $y = \pm\sqrt{4x^2 - \frac{1}{4}x^4}$

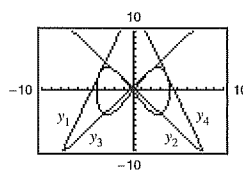


(b)  $y = 3 \Rightarrow 9 = 4x^2 - \frac{1}{4}x^4$

$$36 = 16x^2 - x^4$$

$$x^4 - 16x^2 + 36 = 0$$

$$x^2 = \frac{16 \pm \sqrt{256 - 144}}{2} = 8 \pm \sqrt{28}$$



Note that  $x^2 = 8 \pm \sqrt{28} = 8 \pm 2\sqrt{7} = (1 \pm \sqrt{7})^2$ . So, there are four values of  $x$ :

$$-1 - \sqrt{7}, 1 - \sqrt{7}, -1 + \sqrt{7}, 1 + \sqrt{7}$$

To find the slope,  $2yy' = 8x - x^3 \Rightarrow y' = \frac{x(8 - x^2)}{2(3)}$ .

For  $x = -1 - \sqrt{7}$ ,  $y' = \frac{1}{3}(\sqrt{7} + 7)$ , and the line is

$$y_1 = \frac{1}{3}(\sqrt{7} + 7)(x + 1 + \sqrt{7}) + 3 = \frac{1}{3}[(\sqrt{7} + 7)x + 8\sqrt{7} + 23].$$

For  $x = 1 - \sqrt{7}$ ,  $y' = \frac{1}{3}(\sqrt{7} - 7)$ , and the line is

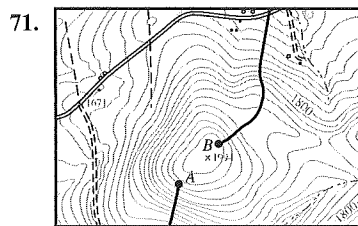
$$y_2 = \frac{1}{3}(\sqrt{7} - 7)(x - 1 + \sqrt{7}) + 3 = \frac{1}{3}[(\sqrt{7} - 7)x + 23 - 8\sqrt{7}].$$

For  $x = -1 + \sqrt{7}$ ,  $y' = -\frac{1}{3}(\sqrt{7} - 7)$ , and the line is

$$y_3 = -\frac{1}{3}(\sqrt{7} - 7)(x + 1 - \sqrt{7}) + 3 = -\frac{1}{3}[(\sqrt{7} - 7)x - (23 - 8\sqrt{7})].$$

For  $x = 1 + \sqrt{7}$ ,  $y' = -\frac{1}{3}(\sqrt{7} + 7)$ , and the line is

$$y_4 = -\frac{1}{3}(\sqrt{7} + 7)(x - 1 - \sqrt{7}) + 3 = -\frac{1}{3}[(\sqrt{7} + 7)x - (8\sqrt{7} + 23)].$$



Use starting point B.

72. Highest wind speed near L

(c) Equating  $y_3$  and  $y_4$ :

$$\begin{aligned} -\frac{1}{3}(\sqrt{7}-7)(x+1-\sqrt{7})+3 &= -\frac{1}{3}(\sqrt{7}+7)(x-1-\sqrt{7})+3 \\ (\sqrt{7}-7)(x+1-\sqrt{7}) &= (\sqrt{7}+7)(x-1-\sqrt{7}) \\ \sqrt{7}x+\sqrt{7}-7-7x-7+7\sqrt{7} &= \sqrt{7}x-\sqrt{7}-7+7x-7-7\sqrt{7} \\ 16\sqrt{7} &= 14x \\ x &= \frac{8\sqrt{7}}{7} \end{aligned}$$

If  $x = \frac{8\sqrt{7}}{7}$ , then  $y = 5$  and the lines intersect at  $\left(\frac{8\sqrt{7}}{7}, 5\right)$ .

74. (a) True

(b) False.  $\frac{d}{dy}\cos(y^2) = -2y\sin(y^2)$ .

(c) False.  $\frac{d}{dx}\cos(y^2) = -2yy'\sin(y^2)$ .

75.  $\sqrt{x} + \sqrt{y} = \sqrt{c}$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

Tangent line at  $(x_0, y_0)$ :  $y - y_0 = -\frac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0)$

x-intercept:  $(x_0 + \sqrt{x_0}\sqrt{y_0}, 0)$

y-intercept:  $(0, y_0 + \sqrt{x_0}\sqrt{y_0})$

Sum of intercepts:

$$(x_0 + \sqrt{x_0}\sqrt{y_0}) + (y_0 + \sqrt{x_0}\sqrt{y_0}) = x_0 + 2\sqrt{x_0}\sqrt{y_0} + y_0 = (\sqrt{x_0} + \sqrt{y_0})^2 = (\sqrt{c})^2 = c$$

76.  $y = x^{p/q}$ ;  $p, q$  integers and  $q > 0$

$$y^q = x^p$$

$$qy^{q-1}y' = px^{p-1}$$

$$y' = \frac{p}{q} \cdot \frac{x^{p-1}}{y^{q-1}} = \frac{p}{q} \cdot \frac{x^{p-1}y}{y^q}$$

$$= \frac{p}{q} \cdot \frac{x^{p-1}}{x^p} x^{p/q} = \frac{p}{q} x^{p/q-1}$$

So, if  $y = x^n$ ,  $n = p/q$ , then  $y' = nx^{n-1}$ .

77.  $x^2 + y^2 = 100$ , slope =  $\frac{3}{4}$

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = \frac{3}{4} \Rightarrow y = -\frac{4}{3}x$$

$$x^2 + \left(\frac{16}{9}x^2\right) = 100$$

$$\frac{25}{9}x^2 = 100$$

$$x = \pm 6$$

Points:  $(6, -8)$  and  $(-6, 8)$

$$78. \quad y^4 = y^2 - x^2$$

$$4y^3y' = 2yy' - 2x$$

$$2x = (2y - 4y^3)y'$$

$$y' = \frac{2x}{2y - 4y^3} = 0 \Rightarrow x = 0$$

Horizontal tangents at  $(0, 1)$  and  $(0, -1)$

**Note:**  $y^4 - y^2 + x^2 = 0$

$$y^2 = \frac{1 \pm \sqrt{1 - 4x^2}}{2}$$

If you graph these four equations, you will see that these are horizontal tangents at  $(0, \pm 1)$ , but not at  $(0, 0)$ .

$$79. \quad \frac{x^2}{4} + \frac{y^2}{9} = 1, \quad (4, 0)$$

$$\frac{2x}{4} + \frac{2yy'}{9} = 0$$

$$y' = \frac{-9x}{4y}$$

$$\frac{-9x}{4y} = \frac{y - 0}{x - 4}$$

$$-9x(x - 4) = 4y^2$$

But,  $9x^2 + 4y^2 = 36 \Rightarrow 4y^2 = 36 - 9x^2$ . So,

$$-9x^2 + 36x = 4y^2 = 36 - 9x^2 \Rightarrow x = 1.$$

Points on ellipse:  $\left(1, \pm \frac{3}{2}\sqrt{3}\right)$

$$\text{At } \left(1, \frac{3}{2}\sqrt{3}\right): y' = \frac{-9x}{4y} = \frac{-9}{4\left[\frac{3}{2}\sqrt{3}\right]} = -\frac{\sqrt{3}}{2}$$

$$\text{At } \left(1, -\frac{3}{2}\sqrt{3}\right): y' = \frac{\sqrt{3}}{2}$$

$$\text{Tangent lines: } y = -\frac{\sqrt{3}}{2}(x - 4) = -\frac{\sqrt{3}}{2}x + 2\sqrt{3}$$

$$y = \frac{\sqrt{3}}{2}(x - 4) = \frac{\sqrt{3}}{2}x - 2\sqrt{3}$$

$$81. \text{ (a)} \quad \frac{x^2}{32} + \frac{y^2}{8} = 1$$

$$\frac{2x}{32} + \frac{2yy'}{8} = 0 \Rightarrow y' = \frac{-x}{4y}$$

$$\text{At } (4, 2): y' = \frac{-4}{4(2)} = -\frac{1}{2}$$

Slope of normal line is 2.

$$y - 2 = 2(x - 4)$$

$$y = 2x - 6$$

$$80. \quad x = y^2$$

$$1 = 2yy'$$

$$y' = \frac{1}{2y}, \quad \text{slope of tangent line}$$

Consider the slope of the normal line joining  $(x_0, 0)$  and

$(x, y) = (y^2, y)$  on the parabola.

$$-2y = \frac{y - 0}{y^2 - x_0}$$

$$y^2 - x_0 = -\frac{1}{2}$$

$$y^2 = x_0 - \frac{1}{2}$$

(a) If  $x_0 = \frac{1}{4}$ , then  $y^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ , which is impossible. So, the only normal line is the  $x$ -axis ( $y = 0$ ).

(b) If  $x_0 = \frac{1}{2}$ , then  $y^2 = 0 \Rightarrow y = 0$ . Same as part (a).

(c) If  $x_0 = 1$ , then  $y^2 = \frac{1}{2} = x$  and there are three normal lines.

The  $x$ -axis, the line joining  $(x_0, 0)$  and  $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ , and

the line joining  $(x_0, 0)$  and  $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$

If two normals are perpendicular, then their slopes are  $-1$  and  $1$ . So,

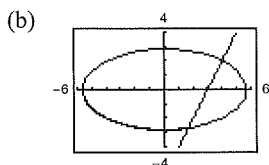
$$-2y = -1 = \frac{y - 0}{y^2 - x_0} \Rightarrow y = \frac{1}{2}$$

and

$$\frac{1/2}{(1/4) - x_0} = -1 \Rightarrow \frac{1}{4} - x_0 = -\frac{1}{2} \Rightarrow x_0 = \frac{3}{4}$$

The perpendicular normal lines are  $y = -x + \frac{3}{4}$  and

$$y = x - \frac{3}{4}.$$



(c) 
$$\frac{x^2}{32} + \frac{(2x-6)^2}{8} = 1$$

$$x^2 + 4(4x^2 - 24x + 36) = 32$$

$$17x^2 - 96x + 112 = 0$$

$$(17x - 28)(x - 4) = 0 \Rightarrow x = 4, \frac{28}{17}$$

Second point:  $\left(\frac{28}{17}, -\frac{46}{17}\right)$

## Section 2.6 Related Rates

1.  $y = \sqrt{x}$

$$\frac{dy}{dt} = \left(\frac{1}{2\sqrt{x}}\right) \frac{dx}{dt}$$

$$\frac{dx}{dt} = 2\sqrt{x} \frac{dy}{dt}$$

(a) When  $x = 4$  and  $dx/dt = 3$ ,

$$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}$$

(b) When  $x = 25$  and  $dy/dt = 2$ ,

$$\frac{dx}{dt} = 2\sqrt{25}(2) = 20$$

2.  $y = 4(x^2 - 5x)$

$$\frac{dy}{dt} = (8x - 20) \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{8x - 20} \frac{dy}{dt}$$

(a) When  $x = 3$  and  $dx/dt = 2$ ,

$$\frac{dy}{dt} = [8(3) - 20](2) = 8$$

(b) When  $x = 1$  and  $dy/dt = 5$ ,

$$\frac{dx}{dt} = \frac{1}{8(1) - 20}(5) = -\frac{5}{12}$$

3.  $xy = 4$

$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{y}{x}\right) \frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{x}{y}\right) \frac{dy}{dt}$$

(a) When  $x = 8$ ,  $y = 1/2$ , and  $dx/dt = 10$ ,

$$\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}$$

(b) When  $x = 1$ ,  $y = 4$ , and  $dy/dt = -6$ ,

$$\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}$$

4.  $x^2 + y^2 = 25$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{x}{y}\right) \frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{y}{x}\right) \frac{dy}{dt}$$

(a) When  $x = 3$ ,  $y = 4$ , and  $dx/dt = 8$ ,

$$\frac{dy}{dt} = -\frac{3}{4}(8) = -6$$

(b) When  $x = 4$ ,  $y = 3$ , and  $dy/dt = -2$ ,

$$\frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}$$

5.  $y = 2x^2 + 1$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 4x \frac{dx}{dt}$$

(a) When  $x = -1$ ,

$$\frac{dy}{dt} = 4(-1)(2) = -8 \text{ cm/sec.}$$

(b) When  $x = 0$ ,

$$\frac{dy}{dt} = 4(0)(2) = 0 \text{ cm/sec.}$$

(c) When  $x = 1$ ,

$$\frac{dy}{dt} = 4(1)(2) = 8 \text{ cm/sec.}$$

6.  $y = \frac{1}{1+x^2}$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \left[ \frac{-2x}{(1+x^2)^2} \right] \frac{dx}{dt}$$

(a) When  $x = -2$ ,

$$\frac{dy}{dt} = \frac{-2(-2)(2)}{25} = \frac{8}{25} \text{ cm/sec.}$$

(b) When  $x = 0$ ,

$$\frac{dy}{dt} = 0 \text{ cm/sec.}$$

(c) When  $x = 2$ ,

$$\frac{dy}{dt} = \frac{-2(2)(2)}{25} = \frac{-8}{25} \text{ cm/sec.}$$

7.  $y = \tan x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \sec^2 x \frac{dx}{dt}$$

(a) When  $x = -\pi/3$ ,

$$\frac{dy}{dt} = (2)^2(2) = 8 \text{ cm/sec.}$$

(b) When  $x = -\pi/4$ ,

$$\frac{dy}{dt} = (\sqrt{2})^2(2) = 4 \text{ cm/sec.}$$

(c) When  $x = 0$ ,

$$\frac{dy}{dt} = (1)^2(2) = 2 \text{ cm/sec.}$$

8.  $y = \cos x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = -\sin x \frac{dx}{dt}$$

(a) When  $x = \pi/6$ ,

$$\frac{dy}{dt} = \left( -\sin \frac{\pi}{6} \right) (2) = -1 \text{ cm/sec.}$$

(b) When  $x = \pi/4$ ,

$$\frac{dy}{dt} = \left( -\sin \frac{\pi}{4} \right) (2) = -\sqrt{2} \text{ cm/sec.}$$

(c) When  $x = \pi/3$ ,

$$\frac{dy}{dt} = \left( -\sin \frac{\pi}{3} \right) (2) = -\sqrt{3} \text{ cm/sec.}$$

9. Yes,  $y$  changes at a constant rate.

$$\frac{dy}{dt} = a \cdot \frac{dx}{dt}$$

No, the rate  $dy/dt$  is a multiple of  $dx/dt$ .

10. Answers will vary. See page 150.

11.  $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x^2 + 1)^2} = \sqrt{x^4 + 3x^2 + 1}$

$$\frac{dx}{dt} = 2$$

$$\frac{dD}{dt} = \frac{1}{2}(x^4 + 3x^2 + 1)^{-1/2} (4x^3 + 6x) \frac{dx}{dt}$$

$$= \frac{2x^3 + 3x}{\sqrt{x^4 + 3x^2 + 1}} \frac{dx}{dt}$$

$$= \frac{4x^3 + 6x}{\sqrt{x^4 + 3x^2 + 1}}$$

12.  $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + \sin^2 x}$

$$\frac{dx}{dt} = 2$$

$$\frac{dD}{dt} = \frac{1}{2}(x^2 + \sin^2 x)^{-1/2} (2x + 2 \sin x \cos x) \frac{dx}{dt}$$

$$= \frac{x + \sin x \cos x}{\sqrt{x^2 + \sin^2 x}} \frac{dx}{dt}$$

$$= \frac{2x + 2 \sin x \cos x}{\sqrt{x^2 + \sin^2 x}}$$

13.  $A = \pi r^2$

$$\frac{dr}{dt} = 4$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

(a) When  $r = 8$ ,  $\frac{dA}{dt} = 2\pi(8)(4) = 64\pi \text{ cm}^2/\text{min}$ .

(b) When  $r = 32$ ,  $\frac{dA}{dt} = 2\pi(32)(4) = 256\pi \text{ cm}^2/\text{min}$ .

14.  $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

If  $dr/dt$  is constant,  $dA/dt$  is not constant.

$dA/dt$  depends on  $r$  and  $dr/dt$ .

15. (a)  $\sin \frac{\theta}{2} = \frac{(1/2)b}{s} \Rightarrow b = 2s \sin \frac{\theta}{2}$

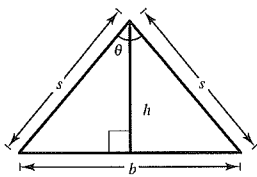
$$\cos \frac{\theta}{2} = \frac{h}{s} \Rightarrow h = s \cos \frac{\theta}{2}$$

$$\begin{aligned} A &= \frac{1}{2}bh = \frac{1}{2}\left(2s \sin \frac{\theta}{2}\right)\left(s \cos \frac{\theta}{2}\right) \\ &= \frac{s^2}{2}\left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}\right) = \frac{s^2}{2} \sin \theta \end{aligned}$$

(b)  $\frac{dA}{dt} = \frac{s^2}{2} \cos \theta \frac{d\theta}{dt}$  where  $\frac{d\theta}{dt} = \frac{1}{2} \text{ rad/min}$ .

When  $\theta = \frac{\pi}{6}$ ,  $\frac{dA}{dt} = \frac{s^2}{2} \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right) = \frac{\sqrt{3}s^2}{8}$ .

When  $\theta = \frac{\pi}{3}$ ,  $\frac{dA}{dt} = \frac{s^2}{2} \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) = \frac{s^2}{8}$ .



(c) If  $s$  and  $\frac{d\theta}{dt}$  is constant,  $\frac{dA}{dt}$  is proportional to  $\cos \theta$ .

16.  $V = \frac{4}{3}\pi r^3$

$$\frac{dr}{dt} = 3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

(a) When  $r = 9$ ,

$$\frac{dV}{dt} = 4\pi(9)^2(3) = 972\pi \text{ in.}^3/\text{min}$$

When  $r = 36$ ,

$$\frac{dV}{dt} = 4\pi(36)^2(3) = 15,552\pi \text{ in.}^3/\text{min}$$

(b) If  $dr/dt$  is constant,  $dV/dt$  is proportional to  $r^2$ .

17.  $V = \frac{4}{3}\pi r^3$ ,  $\frac{dV}{dt} = 800$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} \left(\frac{dV}{dt}\right) = \frac{1}{4\pi r^2}(800)$$

(a) When  $r = 30$ ,

$$\frac{dr}{dt} = \frac{1}{4\pi(30)^2}(800) = \frac{2}{9\pi} \text{ cm/min}$$

(b) When  $r = 60$ ,

$$\frac{dr}{dt} = \frac{1}{4\pi(60)^2}(800) = \frac{1}{18\pi} \text{ cm/min}$$

18.  $V = x^3$

$$\frac{dx}{dt} = 6$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

(a) When  $x = 2$ ,

$$\frac{dV}{dt} = 3(2)^2(6) = 72 \text{ cm}^3/\text{sec}$$

(b) When  $x = 10$ ,

$$\frac{dV}{dt} = 3(10)^2(6) = 1800 \text{ cm}^3/\text{sec}$$

19.  $s = 6x^2$

$$\frac{dx}{dt} = 6$$

$$\frac{ds}{dt} = 12x \frac{dx}{dt}$$

(a) When  $x = 2$ ,

$$\frac{ds}{dt} = 12(2)(6) = 144 \text{ cm}^2/\text{sec.}$$

(b) When  $x = 10$ ,

$$\frac{ds}{dt} = 12(10)(6) = 720 \text{ cm}^2/\text{sec.}$$

20.  $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$

$$\frac{dr}{dt} = 2$$

$$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$$

(a) When  $r = 6$ ,

$$\frac{dV}{dt} = 3\pi(6)^2(2) = 216\pi \text{ in.}^3/\text{min.}$$

(b) When  $r = 24$ ,

$$\frac{dV}{dt} = 3\pi(24)^2(2) = 3456\pi \text{ in.}^3/\text{min.}$$

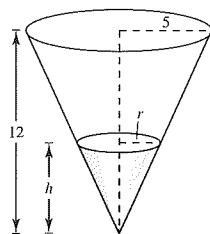
22.  $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{25}{144} h^3 = \frac{25\pi}{3(144)} h^3$

 $\left( \text{By similar triangles, } \frac{r}{5} = \frac{h}{12} \Rightarrow r = \frac{5}{12}h. \right)$ 

$$\frac{dV}{dt} = 10$$

$$\frac{dV}{dt} = \frac{25\pi}{144} h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left( \frac{144}{25\pi h^2} \right) \frac{dV}{dt}$$

$$\text{When } h = 8, \frac{dh}{dt} = \frac{144}{25\pi(64)}(10) = \frac{9}{10\pi} \text{ ft/min.}$$



21.  $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left( \frac{9}{4} h^2 \right) h$  [because  $2r = 3h$ ]

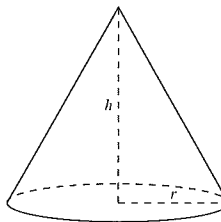
$$= \frac{3\pi}{4} h^3$$

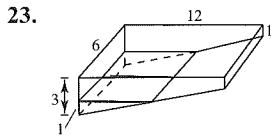
$$\frac{dV}{dt} = 10$$

$$\frac{dV}{dt} = \frac{9\pi}{4} h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$$

When  $h = 15$ ,

$$\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi} \text{ ft/min.}$$





(a) Total volume of pool =  $\frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144 \text{ m}^3$

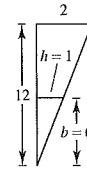
Volume of 1 m of water =  $\frac{1}{2}(1)(6)(6) = 18 \text{ m}^3$  (see similar triangle diagram)

% pool filled =  $\frac{18}{144}(100\%) = 12.5\%$

(b) Because for  $0 \leq h \leq 2$ ,  $b = 6h$ , you have

$$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$$

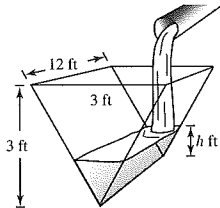
$$\frac{dV}{dt} = 36h \frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144} \text{ m/min.}$$



24.  $V = \frac{1}{2}bh(12) = 6bh = 6h^2$  (since  $b = h$ )

(a)  $\frac{dV}{dt} = 12h \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h} \frac{dV}{dt}$

When  $h = 1$  and  $\frac{dV}{dt} = 2$ ,  $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6} \text{ ft/min.}$



(b) If  $\frac{dh}{dt} = \frac{3}{8} \text{ in./min} = \frac{1}{32} \text{ ft/min}$  and  $h = 2 \text{ ft}$ , then  $\frac{dV}{dt} = (12)(2)\left(\frac{1}{32}\right) = \frac{3}{4} \text{ ft}^3/\text{min.}$

25.  $x^2 + y^2 = 25^2$

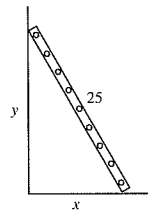
$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x}{y} \cdot \frac{dx}{dt} = \frac{-2x}{y} \quad \text{because } \frac{dx}{dt} = 2.$$

(a) When  $x = 7$ ,  $y = \sqrt{576} = 24$ ,  $\frac{dy}{dt} = \frac{-2(7)}{24} = -\frac{7}{12} \text{ ft/sec.}$

When  $x = 15$ ,  $y = \sqrt{400} = 20$ ,  $\frac{dy}{dt} = \frac{-2(15)}{20} = -\frac{3}{2} \text{ ft/sec.}$

When  $x = 24$ ,  $y = 7$ ,  $\frac{dy}{dt} = \frac{-2(24)}{7} = -\frac{48}{7} \text{ ft/sec.}$



$$(b) \quad A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2} \left( x \frac{dy}{dt} + y \frac{dx}{dt} \right)$$

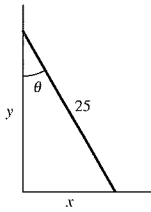
From part (a) you have  $x = 7$ ,  $y = 24$ ,  $\frac{dx}{dt} = 2$ , and  $\frac{dy}{dt} = -\frac{7}{12}$ . So,

$$\frac{dA}{dt} = \frac{1}{2} \left[ 7 \left( -\frac{7}{12} \right) + 24(2) \right] = \frac{527}{24} \text{ ft}^2/\text{sec}.$$

$$(c) \quad \tan \theta = \frac{x}{y}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{y} \cdot \frac{dx}{dt} - \frac{x}{y^2} \cdot \frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2 \theta \left[ \frac{1}{y} \cdot \frac{dx}{dt} - \frac{x}{y^2} \cdot \frac{dy}{dt} \right]$$



Using  $x = 7$ ,  $y = 24$ ,  $\frac{dx}{dt} = 2$ ,  $\frac{dy}{dt} = -\frac{7}{12}$  and  $\cos \theta = \frac{24}{25}$ , you have

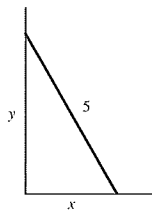
$$\frac{d\theta}{dt} = \left( \frac{24}{25} \right)^2 \left[ \frac{1}{24}(2) - \frac{7}{(24)^2} \left( -\frac{7}{12} \right) \right] = \frac{1}{12} \text{ rad/sec}.$$

$$26. \quad x^2 + y^2 = 25$$

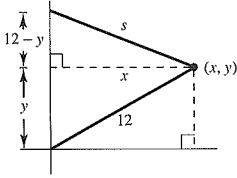
$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x} \cdot \frac{dy}{dt} = -\frac{0.15y}{x} \quad \left( \text{because } \frac{dy}{dt} = 0.15 \right)$$

When  $x = 2.5$ ,  $y = \sqrt{18.75}$ ,  $\frac{dx}{dt} = -\frac{\sqrt{18.75}}{2.5} 0.15 \approx -0.26 \text{ m/sec}$ .



27. When  $y = 6$ ,  $x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$ , and  $s = \sqrt{x^2 + (12 - y)^2} = \sqrt{108 + 36} = 12$ .



$$x^2 + (12 - y)^2 = s^2$$

$$2x \frac{dx}{dt} + 2(12 - y)(-1) \frac{dy}{dt} = 2s \frac{ds}{dt}$$

$$x \frac{dx}{dt} + (y - 12) \frac{dy}{dt} = s \frac{ds}{dt}$$

Also,  $x^2 + y^2 = 12^2$ .

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$\text{So, } x \frac{dx}{dt} + (y - 12) \left( -\frac{x}{y} \frac{dx}{dt} \right) = s \frac{ds}{dt}$$

$$\frac{dx}{dt} \left[ x - x + \frac{12x}{y} \right] = s \frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x} \cdot \frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})} (-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

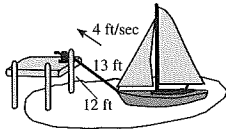
$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = \frac{-6\sqrt{3}}{6} \cdot \frac{(-\sqrt{3})}{15} = \frac{1}{5} \text{ m/sec (vertical)}$$

28. Let  $L$  be the length of the rope.

$$(a) \quad L^2 = 144 + x^2$$

$$2L \frac{dL}{dt} = 2x \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{L}{x} \cdot \frac{dL}{dt} = -\frac{4L}{x} \quad \left( \text{since } \frac{dL}{dt} = -4 \text{ ft/sec} \right)$$



When  $L = 13$ :

$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$

$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec}$$

Speed of the boat increases as it approaches the dock.

(b) If  $\frac{dx}{dt} = -4$ , and  $L = 13$ :

$$\frac{dL}{dt} = \frac{x}{L} \frac{dx}{dt} = \frac{5}{13} (-4) = \frac{-20}{13} \text{ ft/sec}$$

$$\frac{dL}{dt} = \frac{x}{L} \frac{dx}{dt} = \frac{\sqrt{L^2 - 144}}{L} (-4)$$

$$\lim_{L \rightarrow 12^+} \frac{dL}{dt} = \lim_{L \rightarrow 12^+} \frac{-4\sqrt{L^2 - 144}}{L} = 0$$

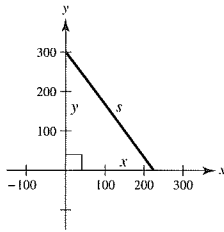
29. (a)  $s^2 = x^2 + y^2$

$$\frac{dx}{dt} = -450$$

$$\frac{dy}{dt} = -600$$

$$2s \frac{ds}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$$



When  $x = 225$  and  $y = 300$ ,  $s = 375$  and

$$\frac{ds}{dt} = \frac{225(-450) + 300(-600)}{375} = -750 \text{ mi/h.}$$

(b)  $t = \frac{375}{750} = \frac{1}{2} \text{ h} = 30 \text{ min}$

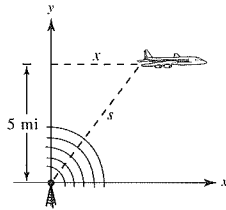
30.  $x^2 + y^2 = s^2$

$$2x \frac{dx}{dt} + 0 = 2s \frac{ds}{dt} \quad \left( \text{because } \frac{dy}{dt} = 0 \right)$$

$$\frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}$$

When  $s = 10$ ,  $x = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$ ,

$$\frac{dx}{dt} = \frac{10}{5\sqrt{3}} (240) = \frac{480}{\sqrt{3}} = 160\sqrt{3} \approx 277.13 \text{ mi/h.}$$



31.  $s^2 = 90^2 + x^2$

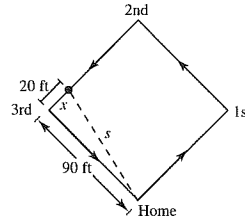
$$x = 30$$

$$\frac{dx}{dt} = -25$$

$$2s \frac{ds}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$$

When  $x = 20$ ,  $s = \sqrt{90^2 + 20^2} = 10\sqrt{85}$ ,

$$\frac{ds}{dt} = \frac{20}{10\sqrt{85}}(-25) = \frac{-50}{\sqrt{85}} \approx -5.42 \text{ ft/sec.}$$



32.  $s^2 = 90^2 + x^2$

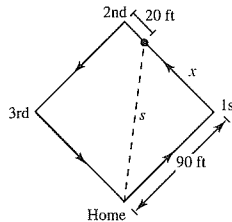
$$x = 90 - 20 = 70$$

$$\frac{dx}{dt} = 25$$

$$\frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$$

When  $x = 70$ ,  $s = \sqrt{90^2 + 70^2} = 10\sqrt{130}$ ,

$$\frac{ds}{dt} = \frac{70}{10\sqrt{130}}(25) = \frac{175}{\sqrt{130}} \approx 15.35 \text{ ft/sec.}$$

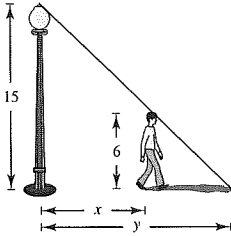


33. (a)  $\frac{15}{6} = \frac{y}{y-x} \Rightarrow 15y - 15x = 6y$

$y = \frac{5}{3}x$

$\frac{dx}{dt} = 5$

$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$  ft/sec



(b)  $\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3}$  ft/sec

34. (a)  $\frac{20}{6} = \frac{y}{y-x}$

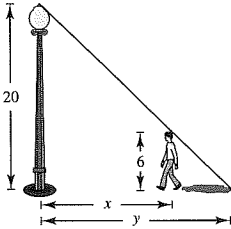
$20y - 20x = 6y$

$14y = 20x$

$y = \frac{10}{7}x$

$\frac{dx}{dt} = -5$

$\frac{dy}{dt} = \frac{10}{7} \frac{dx}{dt} = \frac{10}{7}(-5) = -\frac{50}{7}$  ft/sec



(b)  $\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt}$   
 $= \frac{-50}{7} - (-5)$   
 $= \frac{-50}{7} + \frac{35}{7} = \frac{-15}{7}$  ft/sec

35.  $x(t) = \frac{1}{2} \sin \frac{\pi t}{6}, x^2 + y^2 = 1$

(a) Period:  $\frac{2\pi}{\pi/6} = 12$  seconds

(b) When  $x = \frac{1}{2}, y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$  m.

Lowest point:  $\left(0, \frac{\sqrt{3}}{2}\right)$

(c) When

$x = \frac{1}{4}, y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$  and  $t = 1$ :

$\frac{dx}{dt} = \frac{1}{2} \left(\frac{\pi}{6}\right) \cos \frac{\pi t}{6} = \frac{\pi}{12} \cos \frac{\pi t}{6}$

$x^2 + y^2 = 1$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$

So,  $\frac{dy}{dt} = -\frac{1/4}{\sqrt{15}/4} \cdot \frac{\pi}{12} \cos\left(\frac{\pi}{6}\right)$

$= \frac{-\pi}{\sqrt{15}} \left(\frac{1}{12}\right) \frac{\sqrt{3}}{2} = \frac{-\pi}{24} \frac{1}{\sqrt{5}} = \frac{-\sqrt{5}\pi}{120}$ .

Speed =  $\left| \frac{-\sqrt{5}\pi}{120} \right| = \frac{\sqrt{5}\pi}{120}$  m/sec

36.  $x(t) = \frac{3}{5} \sin \pi t, x^2 + y^2 = 1$

(a) Period:  $\frac{2\pi}{\pi} = 2$  seconds

(b) When  $x = \frac{3}{5}, y = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$  m.

Lowest point:  $\left(0, \frac{4}{5}\right)$

(c) When  $x = \frac{3}{10}, y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$  and  $\frac{3}{10} = \frac{3}{5} \sin \pi t \Rightarrow \sin \pi t = \frac{1}{2} \Rightarrow t = \frac{1}{6}$ :

$$\frac{dx}{dt} = \frac{3}{5} \pi \cos \pi t$$

$$x^2 + y^2 = 1$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y} \frac{dx}{dt}$$

So,  $\frac{dy}{dt} = \frac{-3/10}{\sqrt{15}/4} \cdot \frac{3}{5} \pi \cos\left(\frac{\pi}{6}\right)$

$$= \frac{-9\pi}{25\sqrt{5}} = \frac{-9\sqrt{5}\pi}{125}$$

Speed =  $\left| \frac{-9\sqrt{5}\pi}{125} \right| \approx 0.5058$  m/sec

37. Because the evaporation rate is proportional to the surface area,  $dV/dt = k(4\pi r^2)$ . However, because

$$V = (4/3)\pi r^3, \text{ you have}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Therefore,  $k(4\pi r^2) = 4\pi r^2 \frac{dr}{dt} \Rightarrow k = \frac{dr}{dt}$ .

38 (i) (a)  $\frac{dx}{dt}$  negative  $\Rightarrow \frac{dy}{dt}$  positive

(b)  $\frac{dy}{dt}$  positive  $\Rightarrow \frac{dx}{dt}$  negative

(ii) (a)  $\frac{dx}{dt}$  negative  $\Rightarrow \frac{dy}{dt}$  negative

(b)  $\frac{dy}{dt}$  positive  $\Rightarrow \frac{dx}{dt}$  positive

39.  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{dR_1}{dt} = 1$$

$$\frac{dR_2}{dt} = 1.5$$

$$\frac{1}{R^2} \cdot \frac{dR}{dt} = \frac{1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{1}{R_2^2} \cdot \frac{dR_2}{dt}$$

When  $R_1 = 50$  and  $R_2 = 75$ :

$$R = 30$$

$$\frac{dR}{dt} = (30)^2 \left[ \frac{1}{(50)^2}(1) + \frac{1}{(75)^2}(1.5) \right] = 0.6 \text{ ohm/sec}$$

40.  $pV^{1.3} = k$

$$1.3pV^{0.3} \frac{dV}{dt} + V^{1.3} \frac{dp}{dt} = 0$$

$$V^{0.3} \left( 1.3p \frac{dV}{dt} + V \frac{dp}{dt} \right) = 0$$

$$1.3p \frac{dV}{dt} = -V \frac{dp}{dt}$$

41.  $rg \tan \theta = v^2$   
 $32r \tan \theta = v^2$ ,  $r$  is a constant.

$$32r \sec^2 \theta \frac{d\theta}{dt} = 2v \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{16r}{v} \sec^2 \theta \frac{d\theta}{dt}$$

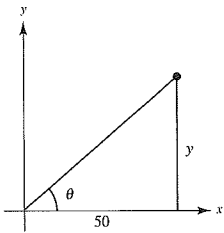
Likewise,  $\frac{d\theta}{dt} = \frac{v}{16r} \cos^2 \theta \frac{dv}{dt}$ .

42.  $\tan \theta = \frac{y}{50}$

$$\frac{dy}{dt} = 4 \text{ m/sec}$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{50} \frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{50} \cos^2 \theta \cdot \frac{dy}{dt}$$



When  $y = 50$ ,  $\theta = \frac{\pi}{4}$ , and  $\cos \theta = \frac{\sqrt{2}}{2}$ . So,

$$\frac{d\theta}{dt} = \frac{1}{50} \left( \frac{\sqrt{2}}{2} \right)^2 (4) = \frac{1}{25} \text{ rad/sec.}$$

43.  $\sin \theta = \frac{10}{x}$

$$\frac{dx}{dt} = (-1) \text{ ft/sec}$$

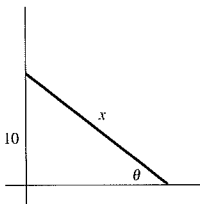
$$\cos \theta \left( \frac{d\theta}{dt} \right) = \frac{-10}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-10}{x^2} \frac{dx}{dt} (\sec \theta)$$

$$= \frac{-10}{25^2} (-1) \frac{25}{\sqrt{25^2 - 10^2}}$$

$$= \frac{10}{25} \frac{1}{5\sqrt{21}} = \frac{2}{25\sqrt{21}}$$

$$= \frac{2\sqrt{21}}{525} \approx 0.017 \text{ rad/sec}$$



44.  $\tan \theta = \frac{y}{x}$ ,  $y = 5$

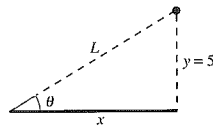
$$\frac{dx}{dt} = -600 \text{ mi/h}$$

$$(\sec^2 \theta) \frac{d\theta}{dt} = -\frac{5}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2 \theta \left( -\frac{5}{x^2} \right) \frac{dx}{dt} = \frac{x^2}{L^2} \left( -\frac{5}{x^2} \right) \frac{dx}{dt}$$

$$= \left( -\frac{5^2}{L^2} \right) \left( \frac{1}{5} \right) \frac{dx}{dt}$$

$$= (-\sin^2 \theta) \left( \frac{1}{5} \right) (-600) = 120 \sin^2 \theta$$



(a) When  $\theta = 30^\circ$ ,

$$\frac{d\theta}{dt} = \frac{120}{4} = 30 \text{ rad/h} = \frac{1}{2} \text{ rad/min.}$$

(b) When  $\theta = 60^\circ$ ,

$$\frac{d\theta}{dt} = 120 \left( \frac{3}{4} \right) = 90 \text{ rad/h} = \frac{3}{2} \text{ rad/min.}$$

(c) When  $\theta = 75^\circ$ ,

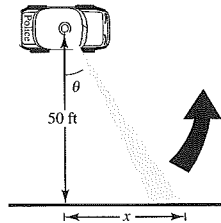
$$\frac{d\theta}{dt} = 120 \sin^2 75^\circ \approx 111.96 \text{ rad/h} \approx 1.87 \text{ rad/min.}$$

45.  $\tan \theta = \frac{x}{50}$

$$\frac{d\theta}{dt} = 30(2\pi) = 60\pi \text{ rad/min} = \pi \text{ rad/sec}$$

$$\sec^2 \theta \left( \frac{d\theta}{dt} \right) = \frac{1}{50} \left( \frac{dx}{dt} \right)$$

$$\frac{dx}{dt} = 50 \sec^2 \theta \left( \frac{d\theta}{dt} \right)$$



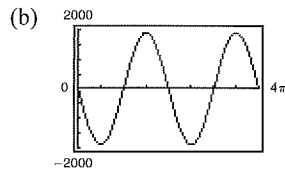
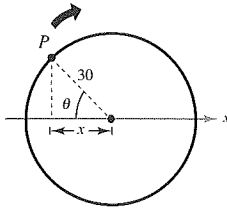
(a) When  $\theta = 30^\circ$ ,  $\frac{dx}{dt} = \frac{200\pi}{3} \text{ ft/sec.}$

(b) When  $\theta = 60^\circ$ ,  $\frac{dx}{dt} = 200\pi \text{ ft/sec.}$

(c) When  $\theta = 70^\circ$ ,  $\frac{dx}{dt} \approx 427.43\pi \text{ ft/sec.}$

46.  $\frac{d\theta}{dt} = (10 \text{ rev/sec})(2\pi \text{ rad/rev}) = 20\pi \text{ rad/sec}$

(a)  $\cos \theta = \frac{x}{30}$   
 $-\sin \theta \frac{d\theta}{dt} = \frac{1}{30} \frac{dx}{dt}$   
 $\frac{dx}{dt} = -30 \sin \theta \frac{d\theta}{dt}$   
 $= -30 \sin \theta (20\pi)$   
 $= -600\pi \sin \theta$



(c)  $|dx/dt| = |-600\pi \sin \theta|$  is greatest when

$$|\sin \theta| = 1 \Rightarrow \theta = \frac{\pi}{2} + n\pi \quad (\text{or } 90^\circ + n \cdot 180^\circ).$$

$$|dx/dt| \text{ is least when } \theta = n\pi \quad (\text{or } n \cdot 180^\circ).$$

(d) For  $\theta = 30^\circ$ ,

$$\frac{dx}{dt} = -600\pi \sin(30^\circ) = -600\pi \cdot \frac{1}{2} = -300\pi \text{ cm/sec.}$$

For  $\theta = 60^\circ$ ,

$$\frac{dx}{dt} = -600\pi \sin(60^\circ)$$

$$= -600\pi \frac{\sqrt{3}}{2} = -300\sqrt{3}\pi \text{ cm/sec.}$$

50.  $x^2 + y^2 = 25$ ; acceleration of the top of the ladder =  $\frac{d^2y}{dt^2}$

First derivative:  $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

Second derivative:  $x \frac{d^2x}{dt^2} + \frac{dx}{dt} \cdot \frac{dx}{dt} + y \frac{d^2y}{dt^2} + \frac{dy}{dt} \cdot \frac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right) \left[ -x \frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2 \right]$$

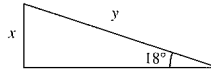
When  $x = 7, y = 24, \frac{dy}{dt} = -\frac{7}{12}$ , and  $\frac{dx}{dt} = 2$  (see Exercise 25). Because  $\frac{dx}{dt}$  is constant,  $\frac{d^2x}{dt^2} = 0$ .

$$\frac{d^2y}{dt^2} = \frac{1}{24} \left[ -7(0) - (2)^2 - \left(-\frac{7}{12}\right)^2 \right] = \frac{1}{24} \left[ -4 - \frac{49}{144} \right] = \frac{1}{24} \left[ -\frac{625}{144} \right] \approx -0.1808 \text{ ft/sec}^2$$

47.  $\sin 18^\circ = \frac{x}{y}$

$$0 = -\frac{x}{y^2} \cdot \frac{dy}{dt} + \frac{1}{y} \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{y} \cdot \frac{dy}{dt} = (\sin 18^\circ)(275) \approx 84.9797 \text{ mi/hr}$$



48.  $\tan \theta = \frac{x}{50} \Rightarrow x = 50 \tan \theta$

$$\frac{dx}{dt} = 50 \sec^2 \theta \frac{d\theta}{dt}$$

$$2 = 50 \sec^2 \theta \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{25} \cos^2 \theta, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

49. (a)  $dy/dt = 3(dx/dt)$  means that  $y$  changes three times as fast as  $x$  changes.

(b)  $y$  changes slowly when  $x \approx 0$  or  $x \approx L$ .  $y$  changes more rapidly when  $x$  is near the middle of the interval.

51.  $L^2 = 144 + x^2$ ; acceleration of the boat =  $\frac{d^2x}{dt^2}$

First derivative:  $2L \frac{dL}{dt} = 2x \frac{dx}{dt}$

$$L \frac{dL}{dt} = x \frac{dx}{dt}$$

Second derivative:  $L \frac{d^2L}{dt^2} + \frac{dL}{dt} \cdot \frac{dL}{dt} = x \frac{d^2x}{dt^2} + \frac{dx}{dt} \cdot \frac{dx}{dt}$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{x}\right) \left[ L \frac{d^2L}{dt^2} + \left(\frac{dL}{dt}\right)^2 - \left(\frac{dx}{dt}\right)^2 \right]$$

When  $L = 13$ ,  $x = 5$ ,  $\frac{dx}{dt} = -10.4$ , and  $\frac{dL}{dt} = -4$  (see Exercise 28). Because  $\frac{dL}{dt}$  is constant,  $\frac{d^2L}{dt^2} = 0$ .

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{1}{5} \left[ 13(0) + (-4)^2 - (-10.4)^2 \right] \\ &= \frac{1}{5} [16 - 108.16] = \frac{1}{5} [-92.16] = -18.432 \text{ ft/sec}^2 \end{aligned}$$

52. (a) Using a graphing utility,

$$m(s) = -0.4988 s^3 + 26.625 s^2 - 472.09 s + 2815.0.$$

(b)  $\frac{dm}{dt} = (-1.4964 s^2 + 53.25 s - 472.09) \frac{ds}{dt}$

If  $\frac{ds}{dt} = 0.75$  and  $t = 10$ , then  $s = 17.8$  and

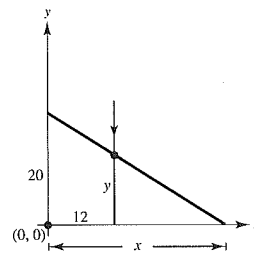
$$\frac{dm}{dt} \approx 1.23 \text{ million/yr.}$$

53.  $y(t) = -4.9t^2 + 20$

$$\frac{dy}{dt} = -9.8t$$

$$y(1) = -4.9 + 20 = 15.1$$

$$y'(1) = -9.8$$



By similar triangles:  $\frac{20}{x} = \frac{y}{x-12}$   
 $20x - 240 = xy$

When  $y = 15.1$ :  $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}$$

$$20x - 240 = xy$$

$$20 \frac{dx}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y} \frac{dy}{dt}$$

At  $t = 1$ ,  $\frac{dx}{dt} = \frac{240/4.9}{20 - 15.1} (-9.8) \approx -97.96 \text{ m/sec.}$

## Review Exercises for Chapter 2

1.  $f(x) = x^2 - 4x + 5$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{[(x + \Delta x)^2 - 4(x + \Delta x) + 5] - [x^2 - 4x + 5]}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x^2 + 2x(\Delta x) + (\Delta x)^2 - 4x - 4(\Delta x) + 5) - (x^2 - 4x + 5)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{2x(\Delta x) + (\Delta x)^2 - 4(\Delta x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 4) = 2x - 4
 \end{aligned}$$

2.  $f(x) = \sqrt{x} + 1$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(\sqrt{x + \Delta x} + 1) - (\sqrt{x} + 1)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x) - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
 \end{aligned}$$

3.  $f(x) = \frac{x + 1}{x - 1}$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{x + \Delta x + 1}{x + \Delta x - 1} - \frac{x + 1}{x - 1}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x + 1)(x - 1) - (x + \Delta x - 1)(x + 1)}{\Delta x(x + \Delta x - 1)(x - 1)} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x^2 + x\Delta x + x - x - \Delta x - 1) - (x^2 + x\Delta x - x + x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} = \lim_{\Delta x \rightarrow 0} \frac{-2}{(x + \Delta x - 1)(x - 1)} = \frac{-2}{(x - 1)^2}
 \end{aligned}$$

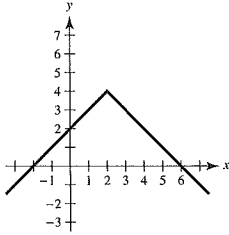
4.  $f(x) = \frac{6}{x}$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\frac{6}{x + \Delta x} - \frac{6}{x}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{6x - (6x + 6\Delta x)}{\Delta x(x + \Delta x)x} = \lim_{\Delta x \rightarrow 0} \frac{-6\Delta x}{\Delta x(x + \Delta x)x} = \lim_{\Delta x \rightarrow 0} \frac{-6}{(x + \Delta x)x} = \frac{-6}{x^2}
 \end{aligned}$$

5.  $f$  is differentiable for all  $x \neq 3$ .6.  $f$  is differentiable for all  $x \neq -1$ .

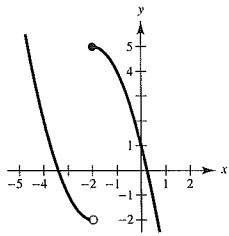
7.  $f(x) = 4 - |x - 2|$

- (a) Continuous at  $x = 2$   
 (b) Not differentiable at  $x = 2$  because of the sharp turn in the graph. Also, the derivatives from the left and right are not equal.



8.  $f(x) = \begin{cases} x^2 + 4x + 2, & \text{if } x < -2 \\ 1 - 4x - x^2, & \text{if } x \geq -2 \end{cases}$

- (a) Nonremovable discontinuity at  $x = -2$   
 (b) Not differentiable at  $x = -2$  because the function is discontinuous there.



9. Using the limit definition, you obtain  $g'(x) = \frac{4}{3}x - \frac{1}{6}$ . At  $x = -1$ ,

$$g'(-1) = -\frac{4}{3} - \frac{1}{6} = -\frac{3}{2}.$$

10. Using the limit definition, you obtain  $h'(x) = \frac{3}{8} - 4x$ . At

$$x = -2,$$

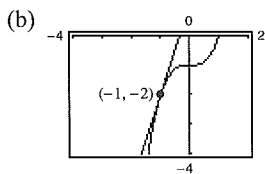
$$h'(-2) = \frac{3}{8} - 4(-2) = \frac{67}{8}.$$

11. (a) Using the limit definition,  $f'(x) = 3x^2$ . At

$$x = -1, f'(-1) = 3. \text{ The tangent line is}$$

$$y - (-2) = 3(x - (-1))$$

$$y = 3x + 1.$$



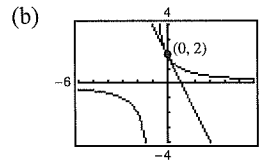
12. (a) Using the limit definition,

$$f'(x) = \frac{-2}{(x+1)^2}.$$

At  $x = 0, f'(0) = -2$ . The tangent line is

$$y - 2 = -2(x - 0)$$

$$y = -2x + 2.$$



13.  $g'(2) = \lim_{x \rightarrow 2} \frac{g(x) - g(2)}{x - 2}$

$$= \lim_{x \rightarrow 2} \frac{x^2(x-1) - 4}{x-2}$$

$$= \lim_{x \rightarrow 2} \frac{x^3 - x^2 - 4}{x-2}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + x + 2)}{x-2}$$

$$= \lim_{x \rightarrow 2} (x^2 + x + 2) = 8$$

14.  $f'(3) = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$

$$= \lim_{x \rightarrow 3} \frac{\frac{1}{x+4} - \frac{1}{7}}{x-3}$$

$$= \lim_{x \rightarrow 3} \frac{7 - x - 4}{(x-3)(x+4)7}$$

$$= \lim_{x \rightarrow 3} \frac{-1}{(x+4)7} = -\frac{1}{49}$$

15.  $y = 25$

$$y' = 0$$

16.  $y = -30$

$$y' = 0$$

17.  $f(x) = x^8$

$$f'(x) = 8x^7$$

18.  $g(x) = x^{20}$

$$g'(x) = 20x^{19}$$

19.  $h(t) = 13t^4$

$$h'(t) = 52t^3$$

20.  $f(t) = -8t^5$   
 $f'(t) = -40t^4$

21.  $f(x) = x^3 - 11x^2$   
 $f'(x) = 3x^2 - 22x$

22.  $g(s) = 4s^4 - 5s^2$   
 $g'(s) = 16s^3 - 10s$

23.  $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$   
 $h'(x) = 3x^{-1/2} + x^{-2/3} = \frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$

24.  $f(x) = x^{1/2} - x^{-1/2}$   
 $f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{x+1}{2x^{3/2}}$

25.  $g(t) = \frac{2}{3}t^{-2}$   
 $g'(t) = \frac{-4}{3}t^{-3} = -\frac{4}{3t^3}$

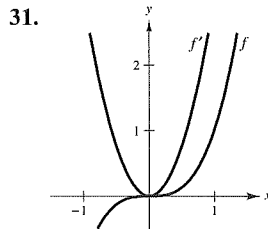
26.  $h(x) = \frac{10}{49}x^{-2}$   
 $h'(x) = \frac{-20}{49}x^{-3} = -\frac{20}{49x^3}$

27.  $f(\theta) = 4\theta - 5 \sin \theta$   
 $f'(\theta) = 4 - 5 \cos \theta$

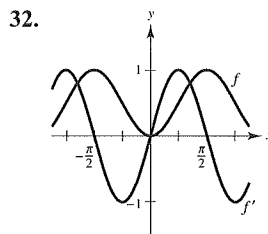
28.  $g(\alpha) = 4 \cos \alpha + 6$   
 $g'(\alpha) = -4 \sin \alpha$

29.  $f(\theta) = 3 \cos \theta - \frac{\sin \theta}{4}$   
 $f'(\theta) = -3 \sin \theta - \frac{\cos \theta}{4}$

30.  $g(\alpha) = \frac{5 \sin \alpha}{3} - 2\alpha$   
 $g'(\alpha) = \frac{5 \cos \alpha}{3} - 2$



31.  $f' > 0$  where the slopes of tangent lines to the graph of  $f$  are positive.



32.  $f' = 0$  where the slopes of tangent lines to the graph of  $f$  are 0.

33.  $F = 200\sqrt{T}$   
 $F'(t) = \frac{100}{\sqrt{T}}$

(a) When  $T = 4$ ,  $F'(4) = 50$  vibrations/sec/lb.

(b) When  $T = 9$ ,  $F'(9) = 33\frac{1}{3}$  vibrations/sec/lb.

34.  $s = -16t^2 + s_0$

First ball:

$$-16t^2 + 100 = 0$$

$$t = \sqrt{\frac{100}{16}}$$

$$= \frac{10}{4} = 2.5 \text{ seconds to hit ground}$$

Second ball:

$$-16t^2 + 75 = 0$$

$$t^2 = \sqrt{\frac{75}{16}}$$

$$= \frac{5\sqrt{3}}{4} \approx 2.165 \text{ seconds to hit ground}$$

Because the second ball was released one second after the first ball, the first ball will hit the ground first.

The second ball will hit the ground

$$3.165 - 2.5 = 0.665 \text{ second later.}$$

35.  $s(t) = -16t^2 + s_0$

$$s(9.2) = -16(9.2)^2 + s_0 = 0$$

$$s_0 = 1354.24$$

The building is approximately 1354 feet high (or 415 m).

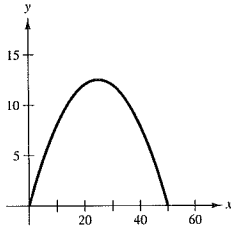
36.  $s(t) = -16t^2 + 14,400 = 0$

$16t^2 = 14,400$

$t = 30 \text{ sec}$

Because  $600 \text{ mi/h} = \frac{1}{6} \text{ mi/sec}$ , in 30 seconds the bomb will move horizontally  $(\frac{1}{6})(30) = 5 \text{ miles}$ .

37. (a)



Total horizontal distance: 50

(b)  $0 = x - 0.02x^2$

$0 = x\left(1 - \frac{x}{50}\right)$  implies  $x = 50$ .

 (c) Ball reaches maximum height when  $x = 25$ .

(d)  $y = x - 0.02x^2$

$y' = 1 - 0.04x$

$y'(0) = 1$

$y'(10) = 0.6$

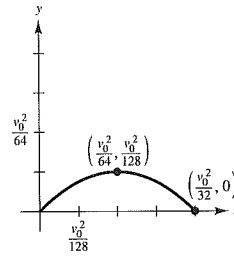
$y'(25) = 0$

$y'(30) = -0.2$

$y'(50) = -1$

(e)  $y'(25) = 0$

38.



(a)  $y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right)$   
 $= 0$  if  $x = 0$  or  $x = \frac{v_0^2}{32}$

Projectile strikes the ground when  $x = v_0^2/32$ .

Projectile reaches its maximum height at  $x = v_0^2/64$  (one-half the distance).

(b)  $y' = 1 - \frac{64}{v_0^2}x$

When  $x = \frac{v_0^2}{64}$ ,  $y' = 1 - \frac{64}{v_0^2}\left(\frac{v_0^2}{64}\right) = 0$ .

(c)  $y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right) = 0$

when  $x = 0$  and  $x = v_0^2/32$ . Therefore, the range is  $x = v_0^2/32$ . When the initial velocity is doubled the range is

$$x = \frac{(2v_0)^2}{32} = \frac{4v_0^2}{32}$$

or four times the initial range. From part (a), the maximum height occurs when  $x = v_0^2/64$ . The maximum height is

$$y\left(\frac{v_0^2}{64}\right) = \frac{v_0^2}{64} - \frac{32}{v_0^2}\left(\frac{v_0^2}{64}\right)^2 = \frac{v_0^2}{64} - \frac{v_0^2}{128} = \frac{v_0^2}{128}$$

If the initial velocity is doubled, the maximum height is

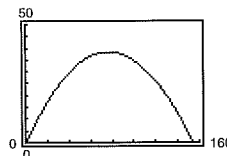
$$y\left[\frac{(2v_0)^2}{64}\right] = \frac{(2v_0)^2}{128} = 4\left(\frac{v_0^2}{128}\right)$$

or four times the original maximum height.

(d)  $v_0 = 70 \text{ ft/sec}$

Range:  $x = \frac{v_0^2}{32} = \frac{(70)^2}{32} = 153.125 \text{ ft}$

Maximum height:  $y = \frac{v_0^2}{128} = \frac{(70)^2}{128} \approx 38.28 \text{ ft}$



$$39. x(t) = t^2 - 3t + 2 = (t - 2)(t - 1)$$

$$(a) v(t) = x'(t) = 2t - 3$$

$$(b) v(t) < 0 \text{ for } t < \frac{3}{2}$$

$$(c) v(t) = 0 \text{ for } t = \frac{3}{2}$$

$$x = \left(\frac{3}{2} - 2\right)\left(\frac{3}{2} - 1\right) = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{1}{4}$$

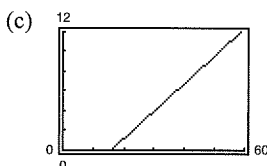
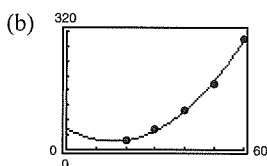
$$(d) x(t) = 0 \text{ for } t = 1, 2$$

$$|v(1)| = |2(1) - 3| = 1$$

$$|v(2)| = |2(2) - 3| = 1$$

The speed is 1 when the position is 0.

$$40. (a) y = 0.14x^2 - 4.43x + 58.4$$



$$(d) \text{ If } x = 65, y \approx 362 \text{ feet.}$$

(e) As the speed increases, the stopping distance increases at an increasing rate.

$$41. f(x) = (5x^2 + 8)(x^2 - 4x - 6)$$

$$\begin{aligned} f'(x) &= (5x^2 + 8)(2x - 4) + (x^2 - 4x - 6)(10x) \\ &= 10x^3 + 16x - 20x^2 - 32 + 10x^3 - 40x^2 - 60x \\ &= 20x^3 - 60x^2 - 44x - 32 \\ &= 4(5x^3 - 15x^2 - 11x - 8) \end{aligned}$$

$$42. g(x) = (x^3 + 7x)(x + 3)$$

$$\begin{aligned} g'(x) &= (x^3 + 7x)(1) + (x + 3)(3x^2 + 7) \\ &= x^3 + 7x + 3x^3 + 9x^2 + 7x + 21 \\ &= 4x^3 + 9x^2 + 14x + 21 \end{aligned}$$

$$43. h(x) = \sqrt{x} \sin x = x^{1/2} \sin x$$

$$h'(x) = \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$$

$$44. f(t) = 2t^5 \cos t$$

$$\begin{aligned} f'(t) &= 2t^5(-\sin t) + \cos t(10t^4) \\ &= -2t^5 \sin t + 10t^4 \cos t \end{aligned}$$

$$45. f(x) = \frac{x^2 + x - 1}{x^2 - 1}$$

$$\begin{aligned} f'(x) &= \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2} \\ &= \frac{-(x^2 + 1)}{(x^2 - 1)^2} \end{aligned}$$

$$46. f(x) = \frac{6x - 5}{x^2 + 1}$$

$$\begin{aligned} f'(x) &= \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2} \\ &= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2} \end{aligned}$$

$$47. f(x) = (9 - 4x^2)^{-1}$$

$$f'(x) = -(9 - 4x^2)^{-2}(-8x) = \frac{8x}{(9 - 4x^2)^2}$$

$$48. f(x) = 9(3x^2 - 2x)^{-1}$$

$$f'(x) = -9(3x^2 - 2x)^{-2}(6x - 2) = \frac{18(1 - 3x)}{(3x^2 - 2x)^2}$$

$$49. y = \frac{x^4}{\cos x}$$

$$\begin{aligned} y' &= \frac{(\cos x) 4x^3 - x^4(-\sin x)}{\cos^2 x} \\ &= \frac{4x^3 \cos x + x^4 \sin x}{\cos^2 x} \end{aligned}$$

$$50. y = \frac{\sin x}{x^4}$$

$$y' = \frac{(x^4) \cos x - (\sin x)(4x^3)}{(x^4)^2} = \frac{x \cos x - 4 \sin x}{x^5}$$

$$51. y = 3x^2 \sec x$$

$$y' = 3x^2 \sec x \tan x + 6x \sec x$$

$$52. y = 2x - x^2 \tan x$$

$$y' = 2 - x^2 \sec^2 x - 2x \tan x$$

$$53. y = x \cos x - \sin x$$

$$y' = -x \sin x + \cos x - \cos x = -x \sin x$$

$$54. \quad g(x) = 3x \sin x + x^2 \cos x$$

$$g'(x) = 3x \cos x + 3 \sin x - x^2 \sin x + 2x \cos x$$

$$= 5x \cos x + (3 - x^2) \sin x$$

$$55. \quad f(x) = \frac{2x^3 - 1}{x^2} = 2x - x^{-2}, \quad (1, 1)$$

$$f'(x) = 2 + 2x^{-3}$$

$$f'(1) = 4$$

$$\text{Tangent line: } y - 1 = 4(x - 1)$$

$$y = 4x - 3$$

$$56. \quad f(x) = \frac{x+1}{x-1}, \left(\frac{1}{2}, -3\right)$$

$$f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{-2}{(1/4)} = -8$$

$$\text{Tangent line: } y + 3 = -8\left(x - \frac{1}{2}\right)$$

$$y = -8x + 1$$

$$57. \quad f(x) = -x \tan x, \quad (0, 0)$$

$$f'(x) = -x \sec^2 x - \tan x$$

$$f'(0) = 0$$

$$\text{Tangent line: } y - 0 = 0(x - 0)$$

$$y = 0$$

$$58. \quad f(x) = \frac{1 + \cos x}{1 - \cos x}, \left(\frac{\pi}{2}, 1\right)$$

$$f'(x) = \frac{(1 - \cos x)(-\sin x) - (1 + \cos x)(\sin x)}{(1 - \cos x)^2}$$

$$= \frac{-2 \sin x}{(1 - \cos x)^2}$$

$$f'\left(\frac{\pi}{2}\right) = \frac{-2}{1} = -2$$

$$\text{Tangent line: } y - 1 = -2\left(x - \frac{\pi}{2}\right)$$

$$y = -2x + 1 + \pi$$

$$59. \quad v(t) = 36 - t^2, \quad 0 \leq t \leq 6$$

$$a(t) = v'(t) = -2t$$

$$v(4) = 36 - 16 = 20 \text{ m/sec}$$

$$a(4) = -8 \text{ m/sec}^2$$

$$60. \quad v(t) = \frac{90t}{4t + 10}$$

$$a(t) = \frac{(4t + 10)90 - 90t(4)}{(4t + 10)^2}$$

$$= \frac{900}{(4t + 10)^2} = \frac{225}{(2t + 5)^2}$$

$$(a) \quad v(1) = \frac{90}{14} \approx 6.43 \text{ ft/sec}$$

$$a(1) = \frac{225}{49} \approx 4.59 \text{ ft/sec}^2$$

$$(b) \quad v(5) = \frac{90(5)}{30} = 15 \text{ ft/sec}$$

$$a(5) = \frac{225}{15^2} = 1 \text{ ft/sec}^2$$

$$(c) \quad v(10) = \frac{90(10)}{50} = 18 \text{ ft/sec}$$

$$a(10) = \frac{225}{25^2} = 0.36 \text{ ft/sec}^2$$

$$61. \quad g(t) = -8t^3 - 5t + 12$$

$$g'(t) = -24t^2 - 5$$

$$g''(t) = -48t$$

$$62. \quad h(x) = 21x^{-3} + 3x$$

$$h'(x) = -63x^{-4} + 3$$

$$h''(x) = 252x^{-5} = \frac{252}{x^5}$$

$$63. \quad f(x) = 15x^{5/2}$$

$$f'(x) = \frac{75}{2}x^{3/2}$$

$$f''(x) = \frac{225}{4}x^{1/2} = \frac{225}{4}\sqrt{x}$$

$$64. \quad f(x) = 20\sqrt[5]{x} = 20x^{1/5}$$

$$f'(x) = 4x^{-4/5}$$

$$f''(x) = \frac{-16}{5}x^{-9/5} = -\frac{16}{5x^{9/5}}$$

$$65. \quad f(\theta) = 3 \tan \theta$$

$$f'(\theta) = 3 \sec^2 \theta$$

$$f''(\theta) = 6 \sec \theta (\sec \theta \tan \theta)$$

$$= 6 \sec^2 \theta \tan \theta$$

$$66. \quad h(t) = 10 \cos t - 15 \sin t$$

$$h'(t) = -10 \sin t - 15 \cos t$$

$$h''(t) = -10 \cos t + 15 \sin t$$

$$\begin{aligned}
 67. \quad y &= 2 \sin x + 3 \cos x \\
 y' &= 2 \cos x - 3 \sin x \\
 y'' &= -2 \sin x - 3 \cos x \\
 y'' + y &= -(2 \sin x + 3 \cos x) + (2 \sin x + 3 \cos x) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 68. \quad y &= \frac{(10 - \cos x)}{x} \\
 xy + \cos x &= 10 \\
 xy' + y - \sin x &= 0 \\
 xy' + y &= \sin x
 \end{aligned}$$

$$\begin{aligned}
 69. \quad h(x) &= \left( \frac{x+5}{x^2+3} \right)^2 \\
 h'(x) &= 2 \left( \frac{x+5}{x^2+3} \right) \left( \frac{(x^2+3)(1) - (x+5)(2x)}{(x^2+3)^2} \right) \\
 &= \frac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}
 \end{aligned}$$

$$\begin{aligned}
 70. \quad f(x) &= \left( x^2 + \frac{1}{x} \right)^5 \\
 f'(x) &= 5 \left( x^2 + \frac{1}{x} \right)^4 \left( 2x - \frac{1}{x^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 71. \quad f(s) &= (s^2 - 1)^{5/2} (s^3 + 5) \\
 f'(s) &= (s^2 - 1)^{5/2} (3s^2) + (s^3 + 5) \left( \frac{5}{2} \right) (s^2 - 1)^{3/2} (2s) \\
 &= s(s^2 - 1)^{3/2} [3s(s^2 - 1) + 5(s^3 + 5)] \\
 &= s(s^2 - 1)^{3/2} (8s^3 - 3s + 25)
 \end{aligned}$$

$$\begin{aligned}
 72. \quad h(\theta) &= \frac{\theta}{(1 - \theta)^3} \\
 h'(\theta) &= \frac{(1 - \theta)^3 - \theta [3(1 - \theta)^2(-1)]}{(1 - \theta)^6} \\
 &= \frac{(1 - \theta)^2(1 - \theta + 3\theta)}{(1 - \theta)^6} = \frac{2\theta + 1}{(1 - \theta)^4}
 \end{aligned}$$

$$\begin{aligned}
 73. \quad y &= 5 \cos(9x + 1) \\
 y' &= -5 \sin(9x + 1)(9) = -45 \sin(9x + 1)
 \end{aligned}$$

$$\begin{aligned}
 74. \quad y &= 1 - \cos 2x + 2 \cos^2 x \\
 y' &= 2 \sin 2x - 4 \cos x \sin x \\
 &= 2[2 \sin x \cos x] - 4 \sin x \cos x \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 75. \quad y &= \frac{x}{2} - \frac{\sin 2x}{4} \\
 y' &= \frac{1}{2} - \frac{1}{4} \cos 2x(2) = \frac{1}{2}(1 - \cos 2x) = \sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 76. \quad y &= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} \\
 y' &= \sec^6 x (\sec x \tan x) - \sec^4 x (\sec x \tan x) \\
 &= \sec^5 x \tan x (\sec^2 x - 1) \\
 &= \sec^5 x \tan^3 x
 \end{aligned}$$

$$\begin{aligned}
 77. \quad y &= \frac{2}{3} \sin^{3/2} x - \frac{2}{7} \sin^{7/2} x \\
 y' &= \sin^{1/2} x \cos x - \sin^{5/2} x \cos x \\
 &= (\cos x) \sqrt{\sin x} (1 - \sin^2 x) \\
 &= \cos^3 x \sqrt{\sin x}
 \end{aligned}$$

$$\begin{aligned}
 78. \quad f(x) &= \frac{3x}{\sqrt{x^2 + 1}} \\
 f'(x) &= \frac{3(x^2 + 1)^{1/2} - 3x \cdot \frac{1}{2}(x^2 + 1)^{-1/2}(2x)}{x^2 + 1} \\
 &= \frac{3(x^2 + 1) - 3x^2}{(x^2 + 1)^{3/2}} = \frac{3}{(x^2 + 1)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 79. \quad y &= \frac{\sin \pi x}{x + 2} \\
 y' &= \frac{(x + 2)\pi \cos \pi x - \sin \pi x}{(x + 2)^2}
 \end{aligned}$$

$$\begin{aligned}
 80. \quad y &= \frac{\cos(x - 1)}{x - 1} \\
 y' &= \frac{-(x - 1) \sin(x - 1) - \cos(x - 1)(1)}{(x - 1)^2} \\
 &= -\frac{1}{(x - 1)^2} [(x - 1) \sin(x - 1) + \cos(x - 1)]
 \end{aligned}$$

$$\begin{aligned}
 81. \quad f(x) &= \sqrt{1 - x^3} \\
 f'(x) &= \frac{1}{2}(1 - x^3)^{-1/2} (-3x^2) = \frac{-3x^2}{2\sqrt{1 - x^3}} \\
 f'(-2) &= \frac{-12}{2(3)} = -2
 \end{aligned}$$

$$\begin{aligned}
 82. \quad f(x) &= \sqrt[3]{x^2 - 1} \\
 f'(x) &= \frac{1}{3}(x^2 - 1)^{-2/3} (2x) = \frac{2x}{3(x^2 - 1)^{2/3}} \\
 f'(3) &= \frac{2(3)}{3(4)} = \frac{1}{2}
 \end{aligned}$$

$$83. \quad y = \frac{1}{2} \csc 2x$$

$$y' = -\csc 2x \cot 2x$$

$$y'\left(\frac{\pi}{4}\right) = 0$$

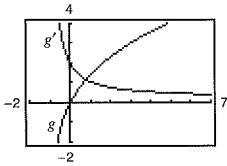
$$84. \quad y = \csc 3x + \cot 3x$$

$$y' = -3 \csc 3x \cot 3x - 3 \csc^2 3x$$

$$y'\left(\frac{\pi}{6}\right) = 0 - 3 = -3$$

$$85. \quad g(x) = 2x(x+1)^{-1/2}$$

$$g'(x) = \frac{x+2}{(x+1)^{3/2}}$$



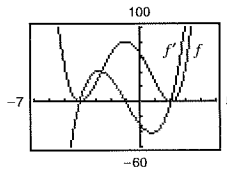
$g'$  does not equal zero for any value of  $x$  in the domain. The graph of  $g$  has no horizontal tangent lines.

$$86. \quad f(x) = [(x-2)(x+4)]^2 = (x^2 + 2x - 8)^2$$

$$f'(x) = 4(x^3 + 3x^2 - 6x - 8)$$

$$= 4(x-2)(x+1)(x+4)$$

The zeros of  $f'$  correspond to the points on the graph of  $f$  where the tangent line is horizontal.

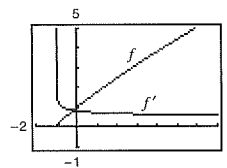


$$87. \quad f(t) = \sqrt{t+1} \sqrt[3]{t+1}$$

$$f(t) = (t+1)^{1/2} (t+1)^{1/3} = (t+1)^{5/6}$$

$$f'(t) = \frac{5}{6(t+1)^{1/6}}$$

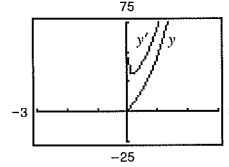
$f'$  does not equal zero for any  $x$  in the domain. The graph of  $f$  has no horizontal tangent lines.



$$88. \quad y = \sqrt{3x}(x+2)^3$$

$$y' = \frac{3(x+2)^2(7x+2)}{2\sqrt{3x}}$$

$y'$  does not equal zero for any  $x$  in the domain. The graph has no horizontal tangent lines.



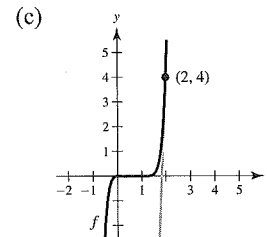
$$89. \quad f(t) = t^2(t-1)^5, \quad (2, 4)$$

$$(a) \quad f'(t) = t(t-1)^4(7t-2)$$

$$f'(2) = 24$$

$$(b) \quad y - 4 = 24(t - 2)$$

$$y = 24t - 44$$



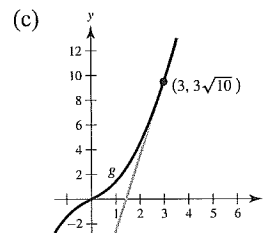
$$90. \quad g(x) = x\sqrt{x^2+1}, \quad (3, 3\sqrt{10})$$

$$(a) \quad g'(x) = \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}$$

$$g'(3) = \frac{19\sqrt{10}}{10}$$

$$(b) \quad y - 3\sqrt{10} = \frac{19\sqrt{10}}{10}(x - 3)$$

$$y = \frac{19\sqrt{10}}{10}x - \frac{27\sqrt{10}}{10}$$



91.  $f(x) = \tan \sqrt{1-x}, \quad (-2, \tan \sqrt{3})$

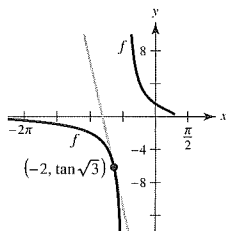
(a)  $f'(x) = \frac{-1}{2\sqrt{1-x} \cos^2 \sqrt{1-x}}$

$$f'(-2) = \frac{-\sqrt{3}}{6 \cos^2 \sqrt{3}} \approx -11.1983$$

(b)  $y - \tan \sqrt{3} = \frac{-\sqrt{3}}{6 \cos^2 \sqrt{3}}(x + 2)$

$$y = -\frac{(x+2)\sqrt{3}}{6 \cos^2 \sqrt{3}} + \tan \sqrt{3}$$

(c)



92.  $f(x) = 2 \csc^3(\sqrt{x}) = \frac{2}{\sin^3(\sqrt{x})}, \quad (1, 2 \csc^3(1))$

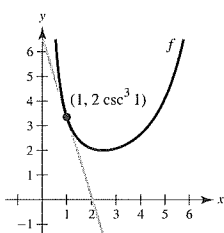
(a)  $f'(x) = \frac{-3 \cos(\sqrt{x})}{\sqrt{x} \sin^4(\sqrt{x})}$

$$f'(1) = \frac{-3 \cos(1)}{\sin^4(1)}$$

(b)  $y - 2 \csc^3(1) = \frac{-3 \cos(1)}{\sin^4(1)}(x - 1)$

$$y = \frac{-3 \cos(1)}{\sin^4(1)}x + \frac{2}{\sin^3(1)} + \frac{3 \cos(1)}{\sin^4(1)}$$

(c)



93.  $y = 7x^2 + \cos 2x$   
 $y' = 14x - 2 \sin 2x$   
 $y'' = 14 - 4 \cos 2x$

94.  $y = x^{-1} + \tan x$   
 $y' = -x^{-2} + \sec^2 x$   
 $y'' = 2x^{-3} + 2 \sec x(\sec x \tan x)$   
 $= \frac{2}{x^3} + 2 \sec^2 x \tan x$

95.  $f(x) = \cot x$   
 $f'(x) = -\csc^2 x$   
 $f''(x) = -2 \csc x(-\csc x \cdot \cot x)$   
 $= 2 \csc^2 x \cot x$

96.  $y = \sin^2 x$   
 $y' = 2 \sin x \cos x = \sin 2x$   
 $y'' = 2 \cos 2x$

97.  $f(t) = \frac{4t^2}{(1-t)^2}$

$$f'(t) = \frac{8t}{(1-t)^3}$$

$$f''(t) = \frac{8(2t+1)}{(1-t)^4}$$

98.  $g(x) = \frac{6x-5}{x^2+1}$

$$g'(x) = \frac{2(-3x^2+5x+3)}{(x^2+1)^2}$$

$$g''(x) = \frac{2(6x^3-15x^2-18x+5)}{(x^2+1)^3}$$

99.  $g(\theta) = \tan 3\theta - \sin(\theta - 1)$   
 $g'(\theta) = 3 \sec^2 3\theta - \cos(\theta - 1)$   
 $g''(\theta) = 18 \sec^2 3\theta \tan 3\theta + \sin(\theta - 1)$

100.  $h(x) = 5x\sqrt{x^2-16}$

$$h'(x) = \frac{10x^2-80}{\sqrt{x^2-16}}$$

$$h''(x) = \frac{10x^3-240x}{(x^2-16)^{3/2}}$$

$$101. T = \frac{700}{t^2 + 4t + 10}$$

$$T = 700(t^2 + 4t + 10)^{-1}$$

$$T' = \frac{-1400(t + 2)}{(t^2 + 4t + 10)^2}$$

(a) When  $t = 1$ ,

$$T' = \frac{-1400(1 + 2)}{(1 + 4 + 10)^2} \approx -18.667 \text{ deg/h.}$$

(b) When  $t = 3$ ,

$$T' = \frac{-1400(3 + 2)}{(9 + 12 + 10)^2} \approx -7.284 \text{ deg/h.}$$

(c) When  $t = 5$ ,

$$T' = \frac{-1400(5 + 2)}{(25 + 20 + 10)^2} \approx -3.240 \text{ deg/h.}$$

(d) When  $t = 10$ ,

$$T' = \frac{-1400(10 + 2)}{(100 + 40 + 10)^2} \approx -0.747 \text{ deg/h.}$$

$$105. \quad \sqrt{xy} = x - 4y$$

$$\frac{\sqrt{x}}{2\sqrt{y}}y' + \frac{\sqrt{y}}{2\sqrt{x}} = 1 - 4y'$$

$$xy' + y = 2\sqrt{xy} - 8\sqrt{xy}y'$$

$$x + 8\sqrt{xy}y' = 2\sqrt{xy} - y$$

$$y' = \frac{2\sqrt{xy} - y}{x + 8\sqrt{xy}} = \frac{2(x - 4y) - y}{x + 8(x - 4y)} = \frac{2x - 9y}{9x - 32y}$$

$$106. \quad y\sqrt{x} - x\sqrt{y} = 25$$

$$y\left(\frac{1}{2}x^{-1/2}\right) + x^{1/2}y' - x\left(\frac{1}{2}y^{-1/2}y'\right) - y^{1/2} = 0$$

$$\left(\sqrt{x} - \frac{x}{2\sqrt{y}}\right)y' = \sqrt{y} - \frac{y}{2\sqrt{x}}$$

$$\left(\frac{2\sqrt{xy} - x}{2\sqrt{y}}\right)y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}}$$

$$y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{2\sqrt{xy} - x}$$

$$= \frac{2y\sqrt{x} - y\sqrt{y}}{2x\sqrt{y} - x\sqrt{x}}$$

$$102. \quad v = \sqrt{2gh} = \sqrt{2(32)h} = 8\sqrt{h}$$

$$\frac{dv}{dh} = \frac{4}{\sqrt{h}}$$

(a) When  $h = 9$ ,  $\frac{dv}{dh} = \frac{4}{3}$  ft/sec.

(b) When  $h = 4$ ,  $\frac{dv}{dh} = 2$  ft/sec.

$$103. \quad x^2 + 3xy + y^3 = 10$$

$$2x + 3xy' + 3y + 3y^2y' = 0$$

$$3(x + y^2)y' = -(2x + 3y)$$

$$y' = \frac{-(2x + 3y)}{3(x + y^2)}$$

$$104. \quad y^2 = x^3 - x^2y + xy - y^2$$

$$0 = x^3 - x^2y + xy - 2y^2$$

$$0 = 3x^2 - x^2y' - 2xy + xy' + y - 4yy'$$

$$(x^2 - x + 4y)y' = 3x^2 - 2xy + y$$

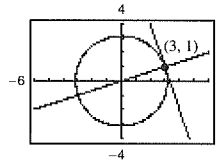
$$y' = \frac{3x^2 - 2xy + y}{x^2 - x + 4y}$$

107.  $x \sin y = y \cos x$   
 $(x \cos y)y' + \sin y = -y \sin x + y' \cos x$   
 $y'(x \cos y - \cos x) = -y \sin x - \sin y$   
 $y' = \frac{y \sin x + \sin y}{\cos x - x \cos y}$

109.  $x^2 + y^2 = 10$   
 $2x + 2yy' = 0$   
 $y' = \frac{-x}{y}$

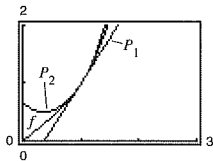
At (3, 1),  $y' = -3$   
 Tangent line:  $y - 1 = -3(x - 3) \Rightarrow 3x + y - 10 = 0$   
 Normal line:  $y - 1 = \frac{1}{3}(x - 3) \Rightarrow x - 3y = 0$

108.  $\cos(x + y) = x$   
 $-(1 + y') \sin(x + y) = 1$   
 $-y' \sin(x + y) = 1 + \sin(x + y)$   
 $y' = \frac{1 + \sin(x + y)}{\sin(x + y)} = -\csc(x + y) - 1$



110.  $x^2 - y^2 = 20$   
 $2x - 2yy' = 0$   
 $y' = \frac{x}{y}$   
 At (6, 4),  $y' = \frac{3}{2}$

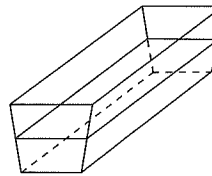
Tangent line:  $y - 4 = \frac{3}{2}(x - 6)$   
 $y = \frac{3}{2}x - 5$   
 $2y - 3x + 10 = 0$   
 Normal line:  $y - 4 = -\frac{2}{3}(x - 6)$   
 $y = -\frac{2}{3}x + 8$   
 $3y + 2x - 24 = 0$



111.  $y = \sqrt{x}$   
 $\frac{dy}{dt} = 2$  units/sec  
 $\frac{dy}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2\sqrt{x} \frac{dy}{dt} = 4\sqrt{x}$   
 (a) When  $x = \frac{1}{2}$ ,  $\frac{dx}{dt} = 2\sqrt{2}$  units/sec.  
 (b) When  $x = 1$ ,  $\frac{dx}{dt} = 4$  units/sec.  
 (c) When  $x = 4$ ,  $\frac{dx}{dt} = 8$  units/sec.

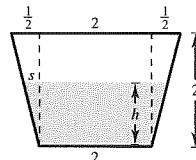
112. Surface area =  $A = 6x^2$ ,  $x$  = length of edge  
 $\frac{dx}{dt} = 8$   
 $\frac{dA}{dt} = 12x \frac{dx}{dt} = 12(6.5)(8) = 624 \text{ cm}^2/\text{sec}$

113.  $\frac{s}{h} = \frac{1}{2}$   
 $s = \frac{1}{4}h$   
 $\frac{dV}{dt} = 1$

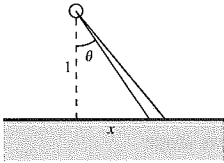


Width of water at depth  $h$ :  
 $w = 2 + 2s = 2 + 2\left(\frac{1}{4}h\right) = \frac{4 + h}{2}$   
 $V = \frac{5}{2}\left(2 + \frac{4 + h}{2}\right)h = \frac{5}{4}(8 + h)h$   
 $\frac{dV}{dt} = \frac{5}{2}(4 + h)\frac{dh}{dt}$   
 $\frac{dh}{dt} = \frac{2(dV/dt)}{5(4 + h)}$

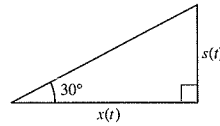
When  $h = 1$ ,  $\frac{dh}{dt} = \frac{2}{25}$  m/min.



114.  $\tan\theta = x$   
 $\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$   
 $\sec^2\theta \left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$   
 $\frac{dx}{dt} = (\tan^2\theta + 1)(6\pi) = 6\pi(x^2 + 1)$   
 When  $x = \frac{1}{2}$ ,  
 $\frac{dx}{dt} = 6\pi\left(\frac{1}{4} + 1\right) = \frac{15\pi}{2} \text{ km/min} = 450\pi \text{ km/h.}$



115.  $s(t) = 60 - 4.9t^2$   
 $s'(t) = -9.8t$   
 $s = 35 = 60 - 4.9t^2$   
 $4.9t^2 = 25$   
 $t = \frac{5}{\sqrt{4.9}}$   
 $\tan 30 = \frac{1}{\sqrt{3}} = \frac{s(t)}{x(t)}$   
 $x(t) = \sqrt{3}s(t)$   
 $\frac{dx}{dt} = \sqrt{3} \frac{ds}{dt} = \sqrt{3}(-9.8) \frac{5}{\sqrt{4.9}} \approx -38.34 \text{ m/sec}$

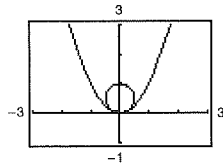


**Problem Solving for Chapter 2**

1. (a)  $x^2 + (y - r)^2 = r^2$ , Circle  
 $x^2 = y$ , Parabola

Substituting:

$(y - r)^2 = r^2 - y$   
 $y^2 - 2ry + r^2 = r^2 - y$   
 $y^2 - 2ry + y = 0$   
 $y(y - 2r + 1) = 0$



Because you want only one solution, let  $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$ . Graph  $y = x^2$  and  $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$ .

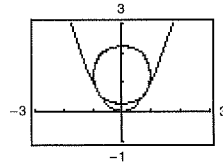
- (b) Let  $(x, y)$  be a point of tangency:

$x^2 + (y - b)^2 = 1 \Rightarrow 2x + 2(y - b)y' = 0 \Rightarrow y' = \frac{x}{b - y}$ , Circle

$y = x^2 \Rightarrow y' = 2x$ , Parabola

Equating:

$2x = \frac{x}{b - y}$   
 $2(b - y) = 1$   
 $b - y = \frac{1}{2} \Rightarrow b = y + \frac{1}{2}$



Also,  $x^2 + (y - b)^2 = 1$  and  $y = x^2$  imply:

$y + (y - b)^2 = 1 \Rightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right]^2 = 1 \Rightarrow y + \frac{1}{4} = 1 \Rightarrow y = \frac{3}{4}$  and  $b = \frac{5}{4}$

Center:  $\left(0, \frac{5}{4}\right)$

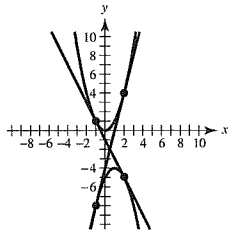
Graph  $y = x^2$  and  $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$ .

2. Let  $(a, a^2)$  and  $(b, -b^2 + 2b - 5)$  be the points of tangency. For  $y = x^2$ ,  $y' = 2x$  and for  $y = -x^2 + 2x - 5$ ,  $y' = -2x + 2$ . So,  $2a = -2b + 2 \Rightarrow a + b = 1$ , or  $a = 1 - b$ . Furthermore, the slope of the common tangent line is

$$\begin{aligned} \frac{a^2 - (-b^2 + 2b - 5)}{a - b} &= \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2 \\ &\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2 \\ &\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2 \\ &\Rightarrow 2b^2 - 2b - 4 = 0 \\ &\Rightarrow b^2 - b - 2 = 0 \\ &\Rightarrow (b - 2)(b + 1) = 0 \\ &b = 2, -1 \end{aligned}$$

For  $b = 2$ ,  $a = 1 - b = -1$  and the points of tangency are  $(-1, 1)$  and  $(2, -5)$ . The tangent line has slope  $-2$ :  $y - 1 = -2(x + 1) \Rightarrow y = -2x - 1$

For  $b = -1$ ,  $a = 1 - b = 2$  and the points of tangency are  $(2, 4)$  and  $(-1, -8)$ . The tangent line has slope  $4$ :  $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$



3. (a)  $f(x) = \cos x$        $P_1(x) = a_0 + a_1x$   
 $f(0) = 1$        $P_1(0) = a_0 \Rightarrow a_0 = 1$   
 $f'(0) = 0$        $P_1'(0) = a_1 \Rightarrow a_1 = 0$   
 $P_1(x) = 1$
- (b)  $f(x) = \cos x$        $P_2(x) = a_0 + a_1x + a_2x^2$   
 $f(0) = 1$        $P_2(0) = a_0 \Rightarrow a_0 = 1$   
 $f'(0) = 0$        $P_2'(0) = a_1 \Rightarrow a_1 = 0$   
 $f''(0) = -1$        $P_2''(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$   
 $P_2(x) = 1 - \frac{1}{2}x^2$

(c)

$x$	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	$\approx 1$	1	$\approx 1$	0.9950	0.5403
$P_2(x)$	0.5	0.9950	$\approx 1$	1	$\approx 1$	0.9950	0.5

$P_2(x)$  is a good approximation of  $f(x) = \cos x$  when  $x$  is near 0.

- (d)  $f(x) = \sin x$        $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$   
 $f(0) = 0$        $P_3(0) = a_0 \Rightarrow a_0 = 0$   
 $f'(0) = 1$        $P_3'(0) = a_1 \Rightarrow a_1 = 1$   
 $f''(0) = 0$        $P_3''(0) = 2a_2 \Rightarrow a_2 = 0$   
 $f'''(0) = -1$        $P_3'''(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$   
 $P_3(x) = x - \frac{1}{6}x^3$