Chapter 5

1) a) $v(t)=0 \quad t=.701,\ 2.879$

$\boxed{\text{Left on } (.701, 2.879) \text{ b/c } v(t)<0}$

v	$+$	$-$	$+$
0	$.701$	2.879	4

b) $$\frac{\int_0^4 v(t)\,dt}{4-0} = -.103$$

c) $v(2) = -.849 \qquad a(2) = .699$

Speed is dec b/c $v(2)<0$ and $a(2)>0$

d) $t = 2.879$

$= .701$

t	$x(t)$
0	2
$.701$	$2+\int_0^{.701} v(t)\,dt = 2.433$
2.879	$2+\int_0^{2.879} v(t)\,dt = 1.0697$
4	$2+\int_0^4 v(t)\,dt = 1.588$

Furthest Left @ $t=2.879$

Furthest Right @ $t=.701$

2) a) $f(x) = \dfrac{e^x+e^{-x}}{2} \qquad f'(x)=0 \qquad x=0$

f'	$-$	$+$
	0	

Rel. Min @ $x=0$
b/c f' changes $-$ to $+$.

b) $\frac{1}{2}\int_{-1}^{1} f(x)\,dx = 1.175$

c) no POIs b/c $f''(x)>0$ $\forall x$.

3) In Rate: $f(t) = -2\sqrt{t} + 5$ Out Rate: $g(t) = 5(1 - e^{-.5t})$

a) $\int_0^2 f(t)\,dt = 6.229$ b) $f(2) - g(2) = -.989$

c) $[0, 5]$ $V(t) = 15 + \int_0^t f(x) - g(x)\,dx$

$V'(t) = f(t) - g(t) = 0$

$t = 1.457$

t	V(t)
0	15
1.457	$15 + \int_0^{1.457} f(t) - g(t)\,dt = 17.829$
5	9.272

max Volume is 17.829 ft^3

@ $t = 1.457$ hrs.

4) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$ $g(x) = \begin{cases} f(x), & -2 \le x \le 0 \\ x + \frac{1}{2}, & 0 < x \le 2 \end{cases}$

a) $\lim_{x \to 0^-} g(x) = f(0) = \frac{1}{2}$ $\lim_{x \to 0^+} g(x) = 0 + \frac{1}{2} = \frac{1}{2}$

$g(0) = f(0) = \frac{1}{2}$ $\therefore$ g is cont @ $x = 0$ b/c $\lim_{x \to 0} g(x) = g(0)$.

b) $\int_0^1 f(x)\,dx = .524$

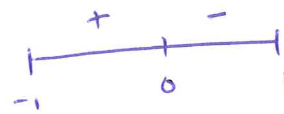
c) $\int_{-1}^{1} g(x)\,dx = \int_{-1}^{0} f(x) + \int_0^1 x + \frac{1}{2}\,dx = 1.524$

5) a) $f(x) = \arccos x$ $\qquad g(x) = x^2$

$h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$= \dfrac{-1}{\sqrt{1-x^4}} \cdot 2x = 0$



$x = 0$

Rel. Max @ $x = 0$ b/c

$h'(x)$ changes $+$ to $-$.

b) $h(x) = \frac{\pi}{3}$

$x = \pm .707$

$$\int_{-.707}^{.707} h(x) - \frac{\pi}{3}\, dx =$$

5c) $f(x) = \frac{\pi}{3}$ $\quad$ $\arccos x = \frac{\pi}{3}$ $\quad$ $f'(x) = \frac{-1}{\sqrt{1-x^2}}$

$\cos\frac{\pi}{3} = x$

$f^{-1}\left(\frac{\pi}{3}\right) = \frac{1}{2}$ $\quad$ $x = \frac{1}{2}$

$$\frac{d}{dx} f^{-1}\left(\frac{\pi}{3}\right) = \frac{1}{f'\left(f^{-1}\left(\frac{\pi}{3}\right)\right)} = \frac{-1}{\sqrt{1-\frac{1}{4}}} = \frac{-1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

6) a) $f(x) = e^x - x$ $\quad$ $e^x = 1$

$f'(x) = e^x - 1 = 0$ $\quad$ $x = \ln 1 = 0$

f': $-$ | $+$ at 0

critical pt. @ $x=0$

local min @ $x=0$

b/c f' changes $-$ to $+$.

b) $f'(1) = e - 1$ $\quad$ $y - (e-1) = (e-1)(x-1)$

$f(1) = e - 1$

c) $\int_0^a f(x) = f'(a)$

$$e^x - \frac{x^2}{2}\Big|_0^a = e^a - 1$$

$$\cancel{e^a} - \frac{a^2}{2} - (e^0 - 0) = \cancel{e^a} - 1$$

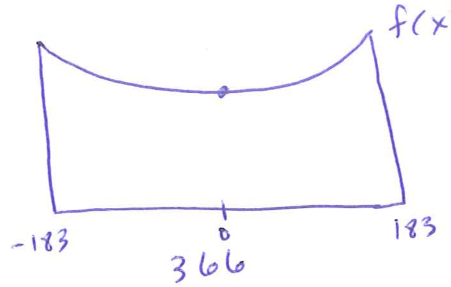
$$-\frac{a^2}{2} - 1 = -1$$

$$-\frac{a^2}{2} = 0$$

$$a = 0$$

7)

$f(x) = \frac{125}{2}\left(e^{x/250} + e^{-x/250}\right)$



a) $f(183) = 160.011$ ft.

$\boxed{160 \text{ ft}}$

b) $f'(100) = .265$ $f'(100)$ is the slope of the cable 100 ft from the middle of catenary.

c) $$\frac{\int_{-183}^{183} f(x)\,dx}{183+183} = 136.466 \text{ ft.}$$

d) $f(x) = 136.466$

$x = 106.277$

$f'(106.277) = .219$

8) a) $f'(1.5) \approx \frac{-1-3}{2-1} = -4$

b) $\frac{f(3)-f(1)}{3-1} = \frac{5-3}{3-1} = \frac{2}{2} = 1$

since f is diff on $(1,3)$ and

cont on $[1,3]$, and $\frac{f(3)-f(1)}{3-1} = 1$,

$\exists$ a c on $(1,3)$ s.t. $f'(c) = 1$

by MVT.

c) $\frac{1}{4-1}\int_1^4 f(x)\,dx \approx \dfrac{\frac{3-1}{2}(1) + \frac{-1+5}{2}(1) + \frac{5-2}{2}(1)}{3}$

$= \frac{1+2+3/2}{3} = \frac{9}{6} = \frac{3}{2}$

d) $(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(3)} = \boxed{\frac{1}{-2}}$

$f^{-1}(5) = 3$

$f(x) = 5$

$x = 3$

9) $g'(3) = \dfrac{1}{f'(g(3))} = \dfrac{1}{f'(-2)} = \dfrac{1}{4}$

(B)

$g(3) = -2$

$f(x) = 3$

$x = -2$

10) $\cos x = \ln y$

$e^{\cos x} = e^{\ln y}$

$-\sin x = \dfrac{y'}{y}$

$y = e^{\cos x}$

$y' = -y \sin x$

$y' = -e^{\cos x} \cdot \sin x$

(A)

Chapter 6

1) a) $\frac{dy}{dx} = \frac{1}{xy}$ $(1,2)$ $\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{2}$

$$\frac{d^2y}{dx^2} = \frac{(xy)\cdot 0 - 1\cdot(xy' + y)}{(xy)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{-\left(\frac{1}{2}+2\right)}{4} = \frac{-5}{8}$$

b) $y - 2 = \frac{1}{2}(x-1)$

$y = 2 + \frac{1}{2}(x-1)$

$f(1.1) \approx 2 + \frac{1}{2}(1.1-1) = \boxed{2.05}$

c) $\frac{dy}{dx} = \frac{1}{xy}$

$\int y\,dy = \int \frac{1}{x}\,dx$

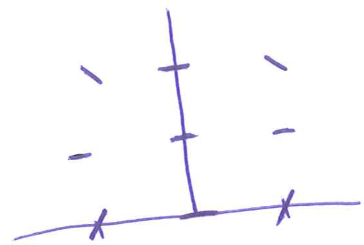
$\frac{y^2}{2} = \ln|x| + c$ $y^2 = 2\ln|x| + 4$

$y^2 = 2\ln|x| + c$ $\boxed{y = +\sqrt{2\ln|x| + 4}}$

$4 = 2\ln 1 + c$

$c = 4$

2) a) $\frac{dy}{dx} = x^2(1-y)$



b) $y<1,\ x>0$

$y<1,\ x<0$

c) $\int \frac{dy}{1-y} = \int x^2\,dx$

$-\ln|1-y| = \frac{x^3}{3} + c$

$\cancel{\ln}|1-y| = e^{-\frac{x^3}{3}+c}$

$1-y = Ce^{-x^3/3}$ $\quad (0,2)$

$1-2 = Ce^0$

$c=-1$

$1-y = -e^{-x^3/3}$

$\boxed{y = 1 + e^{x^3/3}}$

5) a) $\frac{dy}{dt} = \frac{1}{2}y$ $\quad (0,200)$ $\quad \boxed{y = 200e^{1/2t}}$

$\int \frac{dy}{y} = \int \frac{1}{2}dt$

$\cancel{\ln}|y| = e^{\frac{1}{2}t+c}$

$y = Ce^{1/2t}$

$200 = Ce^0$

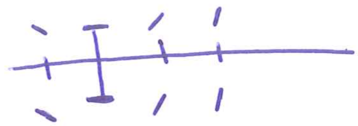
$c = 200$

b) $\frac{1}{10-0}\int_0^{10} 200e^{1/2t}\,dt$

c) $\frac{dy}{dt} = 100e^{1/2t}$

$\frac{1}{10-0}\int_0^{10} 100e^{1/2t}\,dt$ bacteria/hr.

8) $\frac{dy}{dx} = \frac{x}{y^2}$

a) 

c) $\int y^2 dy = \int x\, dx$

$\frac{y^3}{3} = \frac{x^2}{2} + c$

$y^3 = \frac{3}{2}x^2 + c$

b) $\frac{d^2y}{dx^2} = \frac{y^2 \cdot 1 - x \cdot 2yy'}{y^4}$

$= \frac{y^2 - 2xy \cdot \frac{x}{y^2}}{y^4}$

9) $\frac{dy}{dx} = 2xy^2$ $(-1, 2)$

$\int \frac{dy}{y^2} = \int 2x\, dx$

$\frac{y^{-1}}{-1} = x^2 + c$

$-\frac{1}{y} = x^2 + c$

$-\frac{1}{2} = 1 + c$

$-\frac{3}{2} = c$

$-\frac{1}{y} = x^2 - \frac{3}{2}$

$\frac{1}{y} = \frac{3}{2} - x^2 = \frac{3-2x^2}{2}$

$\boxed{y = \frac{2}{3-2x^2}}$

$y(2) = \frac{2}{3-8} = -\frac{2}{5}$ D

10) $\frac{dy}{dx} = \frac{3y}{x}$ $(1, -1)$

$\int \frac{dy}{y} = \int \frac{3}{x}\, dx$

$e^{\ln|y|} = e^{3\ln|x| + c}$

$y = C|x|^3$

$-1 = C(1)$

$c = -1$

$y = -|x|^3$

B