AP CALCULUS – SECTION 5.1 – SUMMATION EXPRESSIONS

In Exercises 1–6, find the sum. Use the summation capabilities of a graphing utility to verify your result.

1. $\sum_{i=1}^{6}(3i+2)$ $=75$

2. $\sum_{k=5}^{8}k(k-4)$ $=70$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$ $=\frac{158}{85}$

4. $\sum_{j=4}^{7}\frac{2}{j}$ $=\frac{319}{210}$

5. $\sum_{k=1}^{4}c$ $=4c$

6. $\sum_{i=1}^{4}[(i-1)^2+(i+1)^3]$ $=238$

In Exercises 7–14, use sigma notation to write the sum.

7. $\frac{1}{5(1)}+\frac{1}{5(2)}+\frac{1}{5(3)}+\cdots+\frac{1}{5(11)}$ $\sum_{i=1}^{11}\frac{1}{5i}$

8. $\frac{9}{1+1}+\frac{9}{1+2}+\frac{9}{1+3}+\cdots+\frac{9}{1+14}$ $\sum_{i=1}^{14}\frac{9}{1+i}$

9. $\left[7\left(\frac{1}{6}\right)+5\right]+\left[7\left(\frac{2}{6}\right)+5\right]+\cdots+\left[7\left(\frac{6}{6}\right)+5\right]$ $\sum_{j=1}^{6}\left[7\left(\frac{j}{6}\right)+5\right]$

10. $\left[1-\left(\frac{1}{4}\right)^2\right]+\left[1-\left(\frac{2}{4}\right)^2\right]+\cdots+\left[1-\left(\frac{4}{4}\right)^2\right]$ $\sum_{j=1}^{4}\left[1-\left(\frac{j}{4}\right)^2\right]$

11. $\left[\left(\frac{2}{n}\right)^3-\frac{2}{n}\right]\left(\frac{2}{n}\right)+\cdots+\left[\left(\frac{2n}{n}\right)^3-\frac{2n}{n}\right]\left(\frac{2}{n}\right)$ $\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3-\left(\frac{2i}{n}\right)\right]$

12. $\left[1-\left(\frac{2}{n}-1\right)^2\right]\left(\frac{2}{n}\right)+\cdots+\left[1-\left(\frac{2n}{n}-1\right)^2\right]\left(\frac{2}{n}\right)$ $\frac{2}{n}\sum_{i=1}^{n}\left[1-\left(\frac{2i}{n}-1\right)^2\right]$

In Exercises 45–48, use the summation formulas to rewrite the expression without the summation notation. Use the result to find the sums for $n = 10, 100, 1000,$ and $10{,}000$.

45. $\sum_{i=1}^{n}\frac{2i+1}{n^2}$ $1+2n$

46. $\sum_{j=1}^{n}\frac{4j+3}{n^2}$ $2+\frac{5}{n}$ or $\frac{2n+5}{n}$

47. $\sum_{k=1}^{n}\frac{6k(k-1)}{n^3}$ $2-\frac{2}{n^2}$

48. $\sum_{i=1}^{n}\frac{4i^2(i-1)}{n^4}$ $\frac{1}{3n^3}\left[3n^3+2n^2-3n-2\right]$

In Exercises 49–54, find a formula for the sum of n terms. Use the formula to find the limit as $n\to\infty$.

49. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{24i}{n^2}$ 12

50. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$ 2

51. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n^3}(i-1)^2$ $\frac{1}{3}$

52. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$ $\frac{26}{3}$

53. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right)$ 3

54. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$ 20